Analog Kerr Black hole and Penrose effect in a Bose-Einstein Condensate

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Overview

- Analog physics: possibility to **simulate black holes in the lab** [1].
- Polariton condensates perfect for analog physics: 1) precise optical/lithography **configuration control** 2) advanced optical detection techniques for condensate **wave function reconstruction** (amplitude + phase).
- We implement a **Kerr black hole** in a polariton condensate.
- Topological defects of the condensate (quantum vortices) – test particles following the **time-like geodesics of the Kerr metric** [2].
- We observe the **Penrose effect** [3] using a vortex-antivortex pair, with an antivortex **falling** into the black hole and reducing its angular momentum, and a vortex **escaping** from the black hole to the infinity.

Metric induced by condensate w.f.

Gross-Pitaevskii equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + \alpha|\psi|^2\psi + U\psi - \mu\psi$$

Relativistic wave equation for the phase:

$$\partial_\nu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) = 0$$

Metric in general case:

$$g_{\mu\nu} = \frac{\rho_0}{c}\begin{pmatrix} -(c^2-\mathbf{v}^2) & \vdots & -\mathbf{v} \\ \cdots\cdots\cdots & \cdot & \cdots\cdots \\ -\mathbf{v} & \vdots & \delta_{ij} \end{pmatrix}$$

Kerr metric:

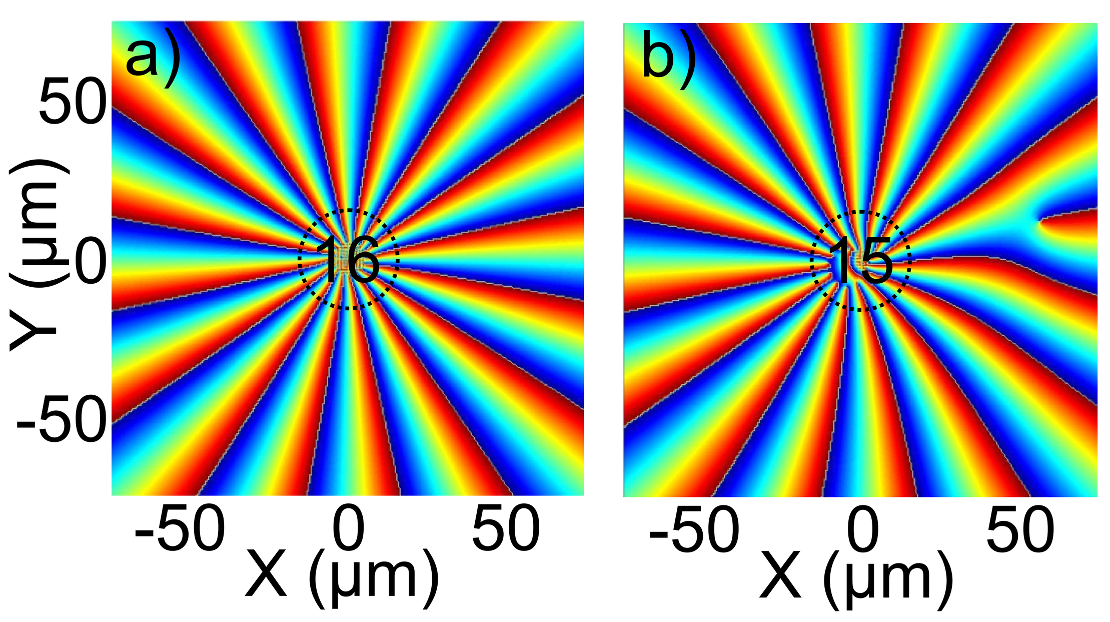
$$g_{\mu\nu}^{Kerr} = \begin{pmatrix} -\left(1-\frac{2M}{r}\right) & 0 & -\frac{4aM}{r} \\ 0 & \frac{r^2}{r^2-2Mr+a^2} & 0 \\ -\frac{4aM}{r} & 0 & \left(r^2+a^2+\frac{2a^2M}{r}\right) \end{pmatrix}$$

Rotating condensate metric (+ inward flow):

$$g_{\mu\nu} = \frac{\rho_0}{c}\begin{pmatrix} -(c^2-v^2) & 0 & -2rv_\phi \\ 0 & \left(1-\frac{v_r^2}{c^2}\right)^{-1} & 0 \\ -2rv_\phi & 0 & r^2 \end{pmatrix}$$

Black hole during Penrose process

Black hole before a) and after b) Penrose process. On panel b) one sees the escaping vortex as a phase dislocation.



Derivation of condensate wave function

KERR BLACK HOLE = + **Angular momentum** (Gauss Laguerre beam) **Inward flow** (region of reduced lifetime)

Analytical solution: series expansion for $r \gg \xi$ ($\xi = \hbar/\sqrt{2\alpha n m}$ – healing length).
Equations for the flow:

$$\nabla\times\mathbf{v} = 2\pi\nu\frac{\hbar}{m}\delta_{2D}(\mathbf{r}) \qquad \nabla\cdot\mathbf{v} = 2\pi\zeta\frac{\hbar}{m}\delta_{2D}(\mathbf{r})$$

Vorticity ν and decay ζ localized in the center.

Approximate **solution for the wavefunction** of the condensate:

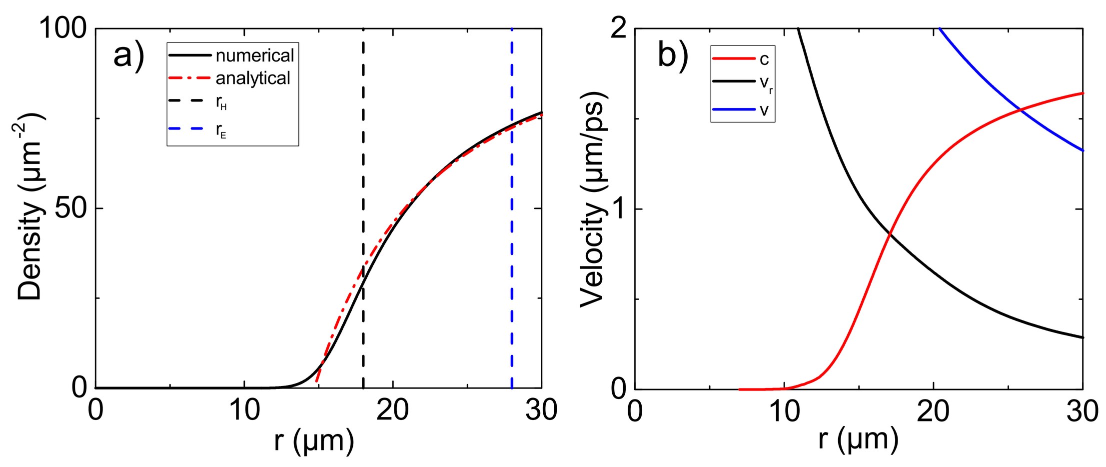
$$\psi(r,\phi) = \sqrt{n_\infty}\left(1-\xi^2\frac{\nu^2+\zeta^2}{\rho^2}\right)\exp\left(i\left(\zeta\ln(\rho)+\nu\phi\right)\right)$$

Event horizon (change of the g_{rr} sign, $v_r = c$):

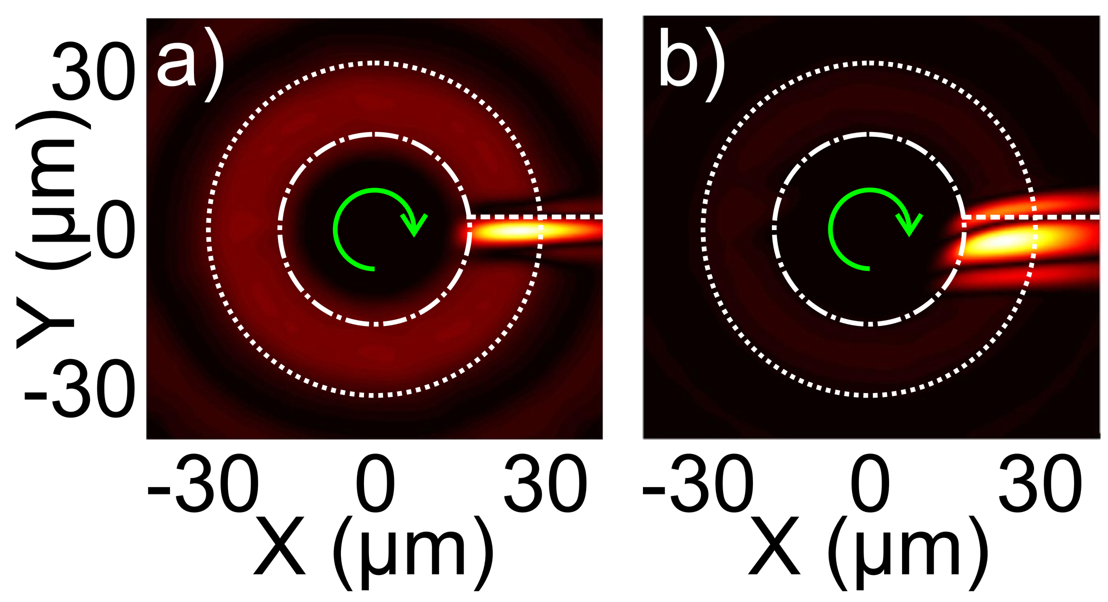
$$r_H = \frac{\xi}{\sqrt{2}}\left(\zeta+\sqrt{3\zeta^2+2\nu^2}\right)$$

Static limit/ergosphere size (change of g_{tt} sign, $v = c$):

$$r_E = \frac{1+\sqrt{3}}{\sqrt{2}}\xi\sqrt{\zeta^2+\nu^2}$$



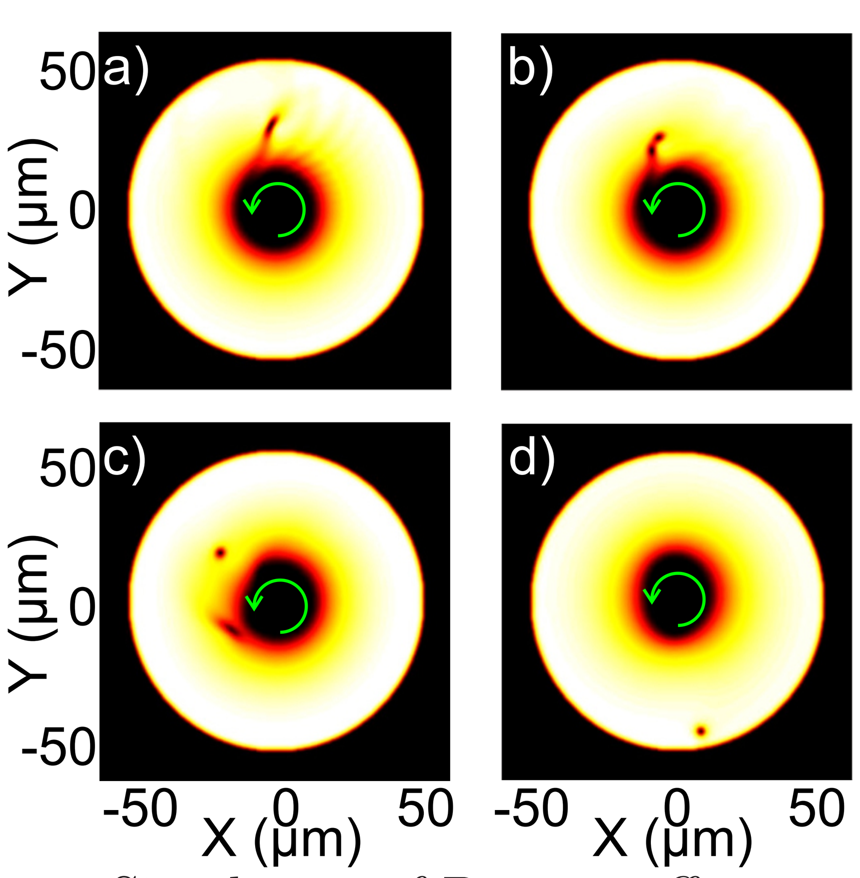
Wave function density and velocity profiles. Analytics vs Simulation



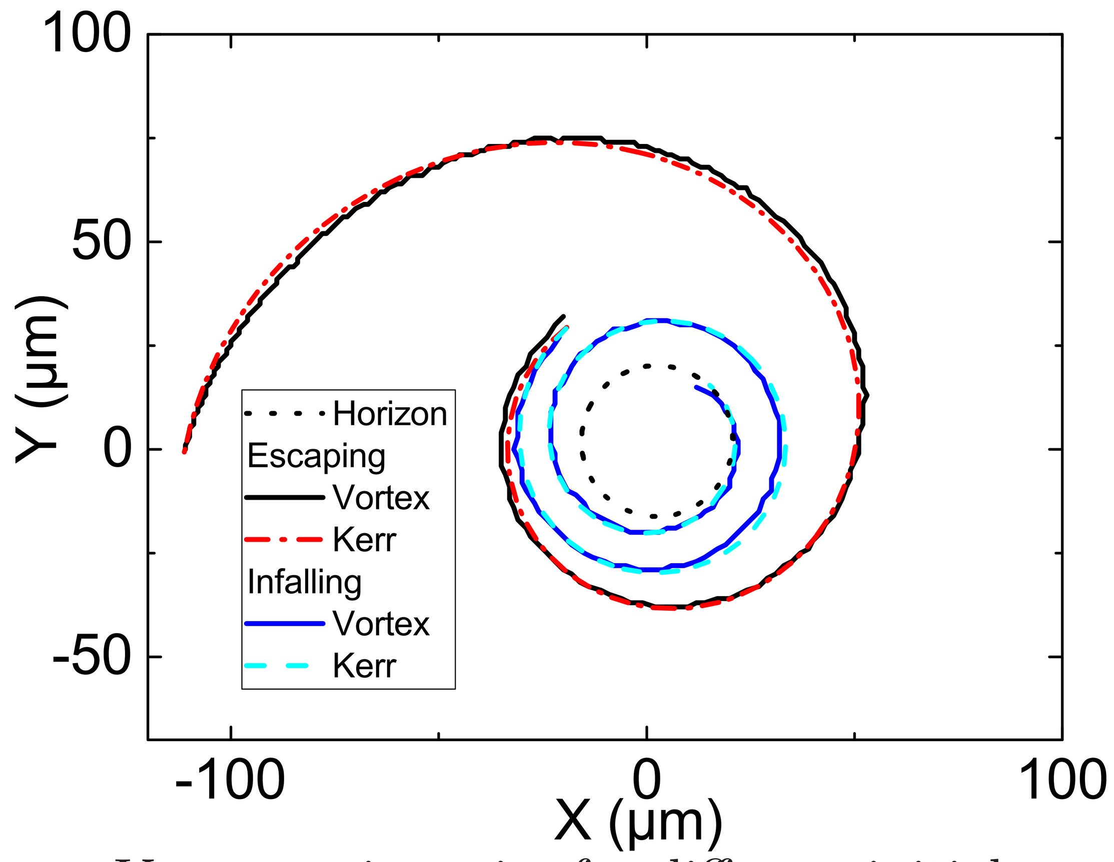
Frame dragging effect and static limit existence

The Penrose effect

a) Creating a **density minimum** by a pulsed potential. b) The minimum was **torn into a vortex-antivortex pair**. c) The **antivortex** has fallen to the black hole and d) the **vortex escaped**. The black hole **loses one vorticity quantum** due to annihilation with antivortex:



Simulation of Penrose effect



Vortex trajectories for different initial conditions fitted by time-like Kerr geodesics

Equations for time-like Kerr geodesics from General Relativity:

$$\dot{r} = \frac{\Delta}{\Sigma}p_r \qquad \dot{p}_r = -\left(\frac{\Delta}{2\Sigma}\right)'p_r^2 + \left(\frac{R}{2\Delta\Sigma}\right)' \quad \dot{\phi} = -\frac{1}{2\Delta\Sigma}\frac{\partial}{\partial L}R$$

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References

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[3] R. Penrose and R. M. Floyd. Extraction of rotational energy from a black hole. *Nature Physical Science*, 229:177, Feb 1971.