Nonadiabatic anomalous Hall effect for exciton-polaritons

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PHYSICAL REVIEW LETTERS 121, 020401 (2018)

Two-level system in an in-plane rotating magnetic field

Time dependent Hamiltonian $H=\vec{\Omega}(t).\vec{\sigma}$ $\quad \vec{\Omega}=\Omega(\cos\omega t,\sin\omega t,0)^T$

Adiabatic limit, instantaneous eigenstate $|\psi_{ad}(t)\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega t}\\1\end{pmatrix}$
→Berry phase [1]

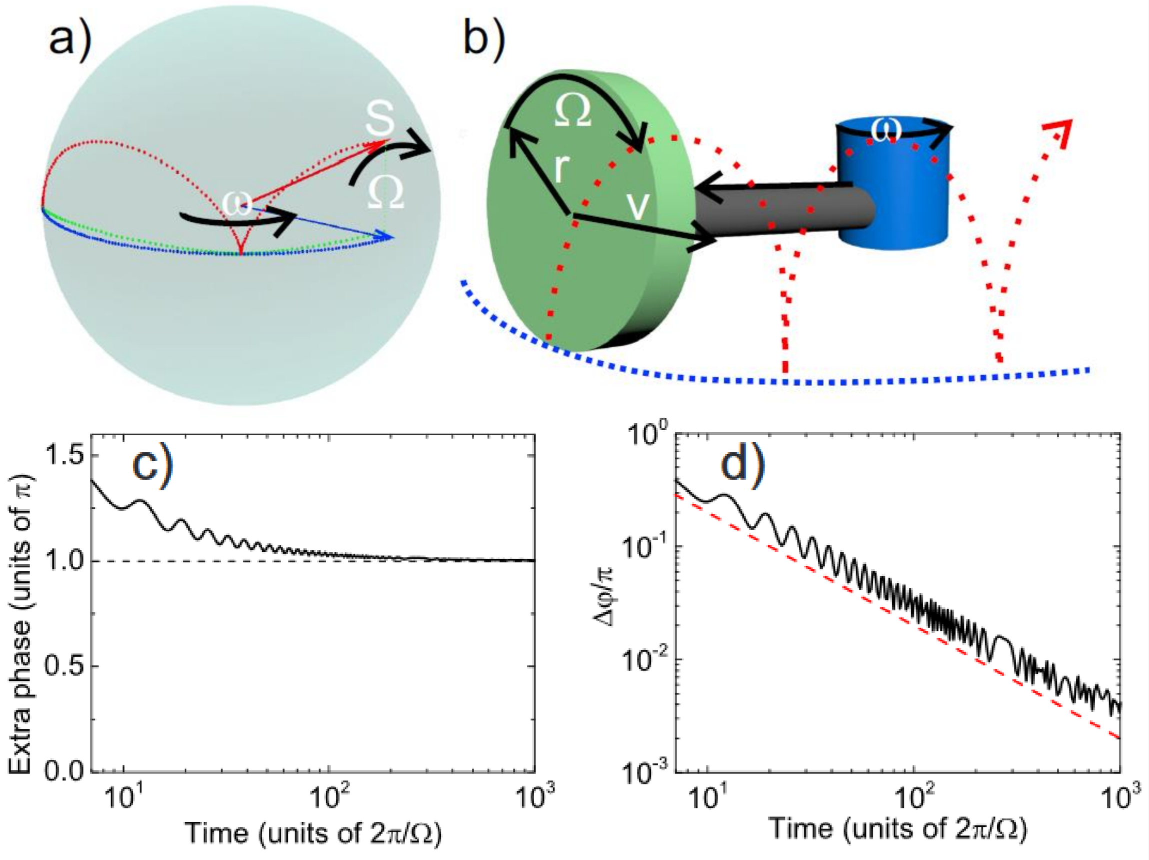
for 1 full rotation of $\vec{\Omega}$: $\varphi_B=i\int_0^T\langle\psi_{ad}|\frac{\partial}{\partial t}|\psi_{ad}\rangle dt=\pm\pi$

Due to finite time experiment, perfect adiabaticity never reached
(Cycloidal trajectory of the spin)

$$\varphi_{cor}=\pi\left(1+2\frac{\omega}{\Omega}\right)$$

→ Correction to the geometric phase (see also [2])
→ Non-adiabatic fraction

$$\Delta\varphi_{cor}/\varphi_B=4\pi/\Omega T$$

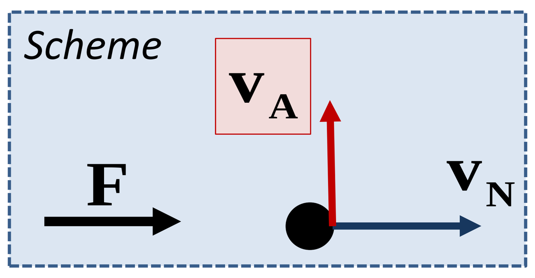


Anomalous Hall effect [3,4]

Semiclassical equations of motion for wavepacket center of mass

$$\hbar\frac{\partial\mathbf{k}}{\partial t}=\mathbf{F},\qquad \textit{External constant force}$$
$$\hbar\frac{\partial\mathbf{r}}{\partial t}=\frac{\partial E_n}{\partial\mathbf{k}}-\hbar\frac{\partial\mathbf{k}}{\partial t}\times\mathbf{B}_n(\mathbf{k})\qquad(1)$$

Anomalous velocity



Valid in the **adiabatic limit**

Nonadiabatic correction to the semiclassical equations for a 2-band system

$$\Omega_{NA,ij}(\mathbf{k})=\left(1+\sum_{l,m}\frac{g_{lm}}{(\epsilon_0-\epsilon_1)^2}\frac{\partial k_l}{\partial t}\frac{\partial k_m}{\partial t}\right)\Omega_{ij}(\mathbf{k})\qquad \hbar\frac{\partial\mathbf{r}}{\partial t}=\frac{\partial\tilde{\epsilon}}{\partial\mathbf{k}}-\hbar\frac{\partial\mathbf{k}}{\partial t}\times\mathbf{B}_{NA}\qquad(2)$$

Ω_{ij}: *adiabatic Berry curvature*
g_{ij} : *quantum metric component*

$$\mathbf{B}_{NA}=(\Omega_{NA,yz},\Omega_{NA,zx},\Omega_{NA,xy})^T$$

Time evolution of a wavepacket in an energy band using both real and imaginary part of QGT computed with the instantaneous Hamiltonian

Valid on geodesics in momentum space, where 1st order corrections are zero

Quantum geometric tensor (QGT) in k space

Introduced in 1980 to define metric in arbitrary parameter space in QM [5]

$$T_{ij}^{(n)}(\mathbf{k})=\langle\partial_{k_i}u_n|\partial_{k_j}u_n\rangle-\langle\partial_{k_i}u_n|u_n\rangle\langle u_n|\partial_{k_j}u_n\rangle=g_{ij}^{(n)}-i\frac{\Omega_{ij}^{(n)}}{2}$$

→ Contains both quantum metric + Berry curvature

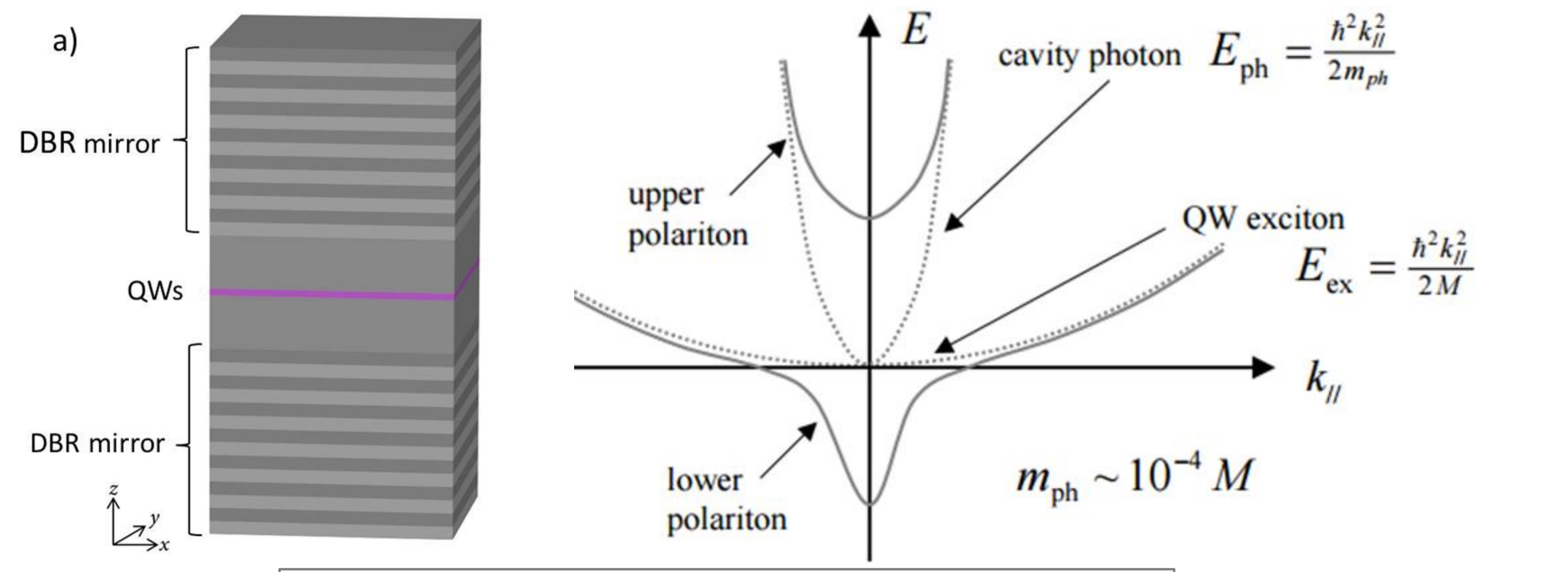
- Real part = metric $\quad ds^2=\Re[T_{ij}]dk_idk_j=g_{ij}^{(n)}dk_idk_j$

$$ds^2=1-|\langle u_n(\mathbf{k})|u_n(\mathbf{k}+\delta\mathbf{k})\rangle|^2$$

- Imaginary part = Berry curvature

$$\Omega_{ij}^{(n)}=-2\Im[T_{ij}^{(n)}]\qquad \mathbf{B}_n=(\Omega_{yz}^{(n)},\Omega_{zx}^{(n)},\Omega_{xy}^{(n)})^T$$

Cavity exciton-polaritons: strong light-matter coupling



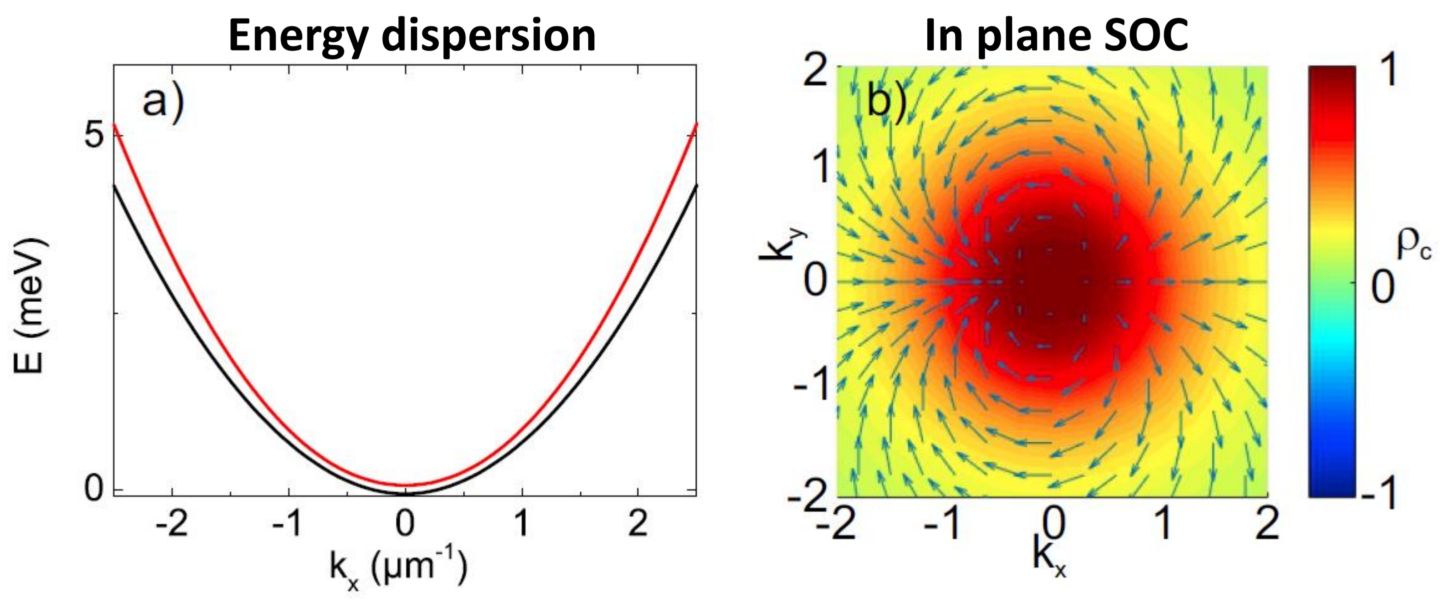
→ 2D neutral bosonic quasi-particles

Quantum geometry of planar cavity polaritons

- LPB parabolic approximation
- TE-TM SOC + Zeeman field

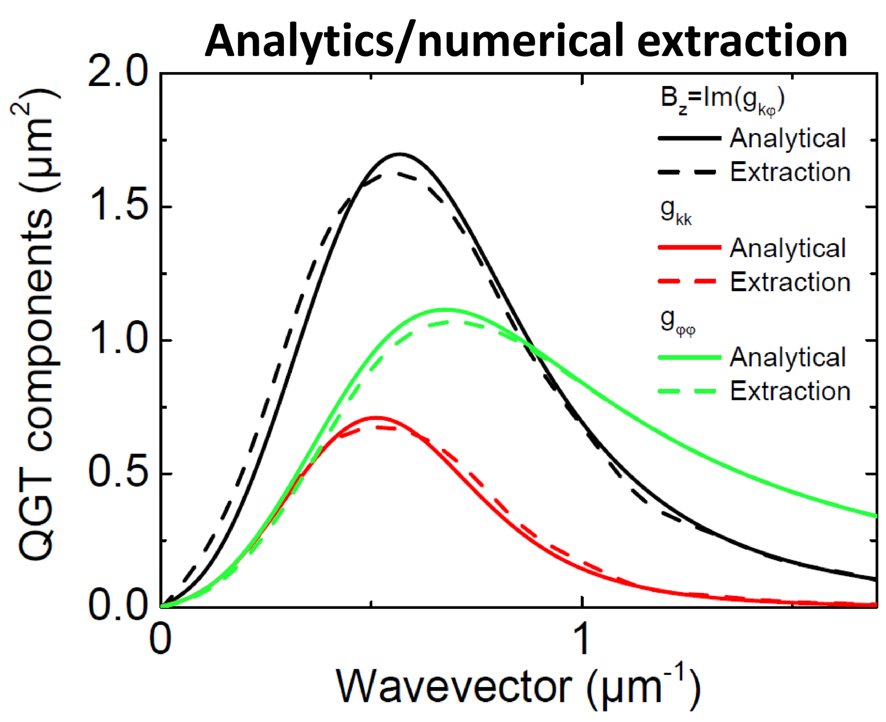
→ Effective **2-band system**

$$H=\begin{pmatrix}\frac{\hbar^2k^2}{2m^*}+\Delta & \beta k^2e^{2i\phi}\\ \beta k^2e^{-2i\phi} & \frac{\hbar^2k^2}{2m^*}-\Delta\end{pmatrix}$$



QGT components

$$g_{kk}=\frac{\Delta^2k^2\beta^2}{(\Delta^2+\beta^2k^4)^2},$$
$$g_{\phi\phi}=\frac{k^2\beta^2}{\Delta^2+\beta^2k^4},$$
$$g_{k\phi}=g_{\phi k}=0$$
$$\mathbf{B}=\frac{2\Delta k^2\beta^2}{(\Delta^2+k^4\beta^2)^{3/2}}\mathbf{e}_z$$

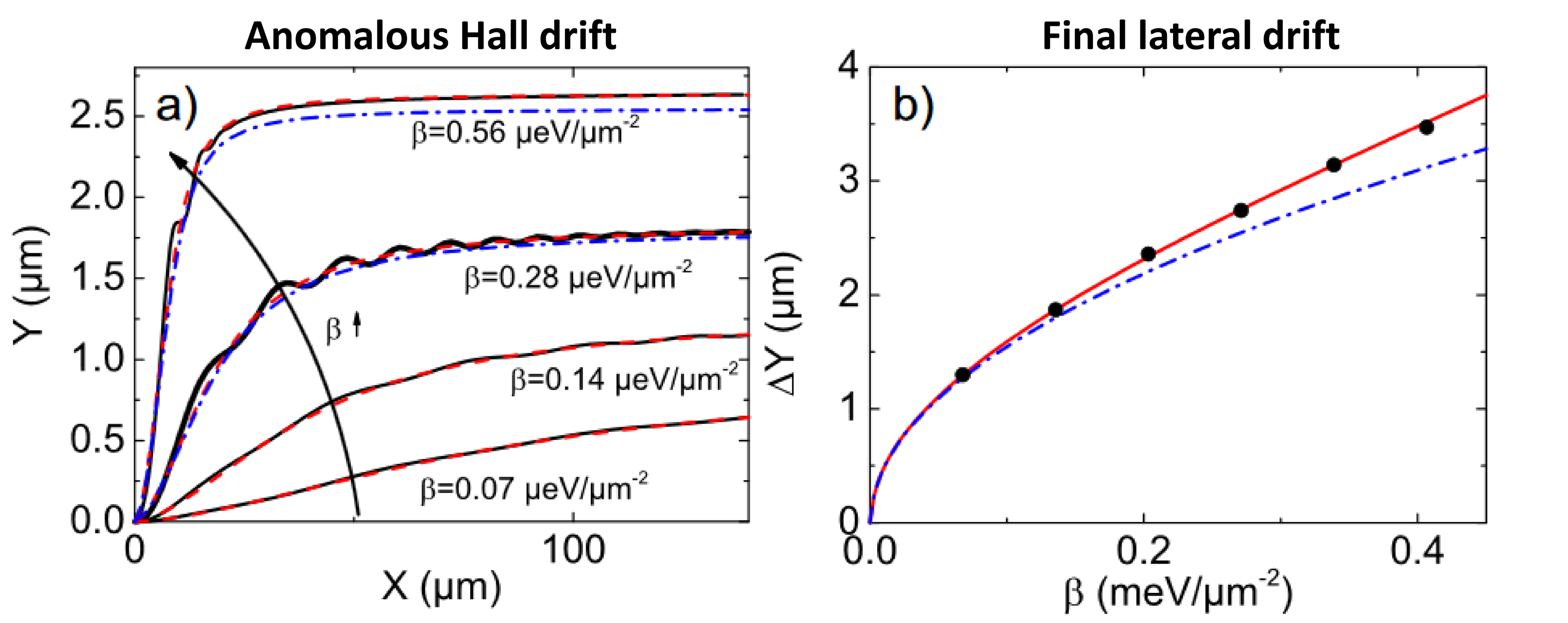


Time evolution of a polariton wavepacket: Numerics/Analytics

Full numerical simulation using spinor Schrödinger eq.

Wedge potential $U(x)=-Fx$

$$i\hbar\frac{\partial}{\partial t}\psi_\pm=-\frac{\hbar^2}{2m}\Delta\psi_\pm-\frac{i\hbar}{2\tau}\psi_\pm\pm\Delta\psi_\pm+\beta\left(\frac{\partial}{\partial x}\mp i\frac{\partial}{\partial y}\right)^2\psi_\mp+U(x)\psi_\mp+\hat{P}$$



Analytics: - without non-adiabatic correction (**blue-dashed-eq.(1)**)
- with non-adiabatic correction (**red-eq.(2)**)
Full Schrödinger numeric (**black**)

Conclusions

- Berry curvature + quantum metric = QGT
- Directly measurable in planar cavities and cavity lattices (see [6])
- Both components important for realistic (nonadiabatic) AHE trajectory

References

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