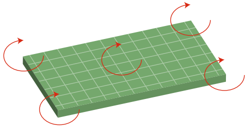
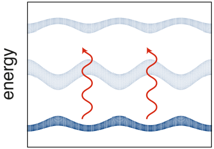
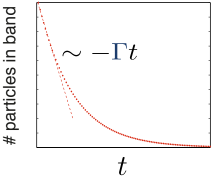
Probing Quantum Geometry by “Heating”: From Quantized Circular Dichroism to Topological Order

Nathan Goldman







Aussois, 27th November 2018

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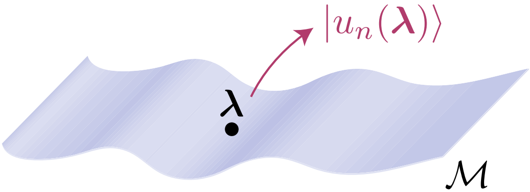
Collaborations :

- Theory
  - N. R. Cooper (Cambridge)
  - A. Dauphin (ICFO)
  - A. G. Grushin (Grenoble)
  - T. Ozawa (Riken)
  - C. Repellin (MIT)
  - G. Palumbo and D. T. Tran (ULB)
  - P. Zoller (IQOQI)
- Experiments (group of K. Sengstock in Hamburg)
  - L. Asteria
  - N. Fläschner
  - B. S. Rem
  - K. Sengstock
  - M. Tarnowski
  - C. Weitenberg

- System described by Hamiltonian $\hat{H}(\boldsymbol{\lambda})$ with $\boldsymbol{\lambda} \in \mathcal{M}$

- Local eigenstates : $\hat{H}(\boldsymbol{\lambda})|u_n(\boldsymbol{\lambda})\rangle = \varepsilon_n(\boldsymbol{\lambda})|u_n(\boldsymbol{\lambda})\rangle$



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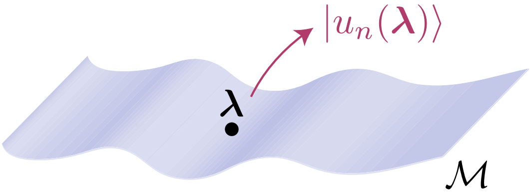
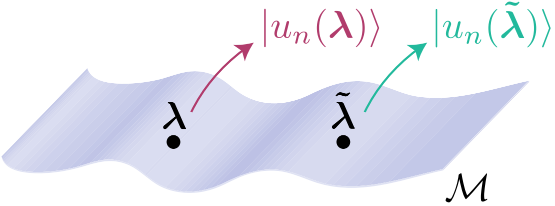


- Projection on nth band ε_n (adiabatic evolution) :

$$|\psi(t)\rangle = e^{i\theta(t)}|u_n[\boldsymbol{\lambda}(t)]\rangle, \qquad \forall \text{ time } t$$

$\longrightarrow$ geometric structure related to the Hilbert space !

- **Geometry** : (a) distance between states



- The quantum (Fubini-Study) metric :

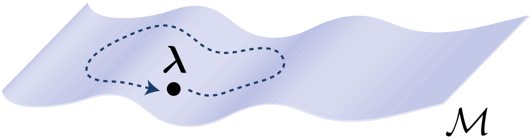
$$g_{\mu\nu}\ \mathrm{d}\lambda^{\mu}\mathrm{d}\lambda^{\nu} = 1 - |\langle u_n(\boldsymbol{\lambda})|u_n(\boldsymbol{\lambda} + \mathrm{d}\boldsymbol{\lambda})\rangle|^2$$

[Manifestations in condensed-matter, quantum information, . . .]

[see review : Kolodrubetz, Polkovnikov et al. Phys. Rep. 697 (2017)]

- **Geometry** : (b) the curvature (geometric phase)

$$|u_n(\boldsymbol{\lambda})\rangle \longrightarrow |u_n(\boldsymbol{\lambda})\rangle \exp\left(i\int_\Sigma \boldsymbol{\Omega}^{(n)}\cdot \mathrm{d}\boldsymbol{S}\right)$$



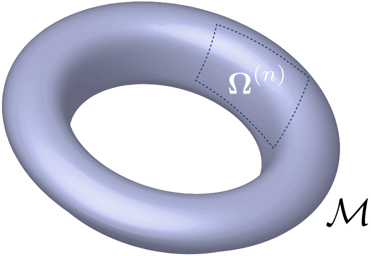
- The Berry curvature (in band ε_n) :

$$\boldsymbol{\Omega}^{(n)} = \boldsymbol{\nabla}_{\boldsymbol{\lambda}} \times \left(i\langle u_n|\boldsymbol{\nabla}_{\boldsymbol{\lambda}} u_n\rangle\right)$$

- *"Aharonov-Bohm"* phase in parameter space :

$\rightarrow$ Berry curvature $\equiv$ "magnetic field" in $\boldsymbol{\lambda}$ space

- **Topology** : Gauss-Bonnet in quantum physics



- The Chern number of the nth band ε_n :

$$\nu^{(n)} = \frac{1}{2\pi} \int_{\mathcal{M}} \mathbf{\Omega}^{(n)} \cdot \mathrm{d}\boldsymbol{S} \quad \in \mathbb{Z}$$

- Total *"flux"* of the field $\mathbf{\Omega}^{(n)}$ through $\mathcal{M}$

$\rightarrow \nu^{(n)} \equiv$ "monopole" charge in $\boldsymbol{\lambda}$ space

- **Metric and curvature united** : the geometric tensor

$$G_{\mu\nu}^{(n)} = \sum_{m \neq n} \frac{\langle u_n | \partial_\mu \hat{H} | u_m \rangle \langle u_m | \partial_\nu \hat{H} | u_n \rangle}{\left(\varepsilon_n - \varepsilon_m\right)^2} \qquad \text{(in band } \varepsilon_n\text{)}$$

- Quantum metric : $g_{\mu\nu}^{(n)} = \mathcal{R}\left(G_{\mu\nu}^{(n)}\right)$
- Berry curvature : $\Omega_{\mu\nu}^{(n)} = -2\,\mathcal{I}\left(G_{\mu\nu}^{(n)}\right)$ [note : $(\mathbf{\Omega})^\alpha = \epsilon^{\alpha\mu\nu}\Omega_{\mu\nu}$]

Universal method to extract $G_{\mu\nu}$?

[beyond state tomography ; see Fläschner et al. Science ’16]

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- Measure the excited fraction over time

$$n_{\mathsf{ex}}(t) = \frac{2\pi t}{\hbar}\sum_{n\neq 0}\left|\frac{\mathcal{E}}{\hbar\omega}\langle n|\partial_\mu \hat{H}(\boldsymbol{\lambda}^0)|0\rangle\right|^2 \delta(\varepsilon_n - \varepsilon_0 - \hbar\omega)$$

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- Integrate the excited rate $\Gamma = n_{\text{ex}}(t)/t$ over frequency

$$\Gamma^{\text{int}} = \int_0^\infty \Gamma(\omega)\mathrm{d}\omega = \frac{2\pi\mathcal{E}^2}{\hbar^2} g_{\mu\mu}(\boldsymbol{\lambda}^0)$$

[Ozawa and Goldman, PRB **97**, 201117(R) (2018)]

A general protocol (2)

- If one modulates two parameters (say λ_μ and λ_ν)

$$\lambda_\mu(t) = \lambda_\mu^0 + 2(\mathcal{E}/\hbar\omega)\cos(\omega t)$$
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- Measure the differential integrated rate

$$\Delta\Gamma^{\text{int}} = \int_0^\infty \Gamma^+(\omega) - \Gamma^-(\omega)\,\mathrm{d}\omega = \frac{8\pi\mathcal{E}^2}{\hbar^2}\, g_{\mu\nu}(\boldsymbol{\lambda}^0)$$

$\rightarrow$ one reconstructs the full quantum metric !

[Ozawa and Goldman, PRB **97**, 201117(R) (2018)]

A general protocol (3)

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$$\lambda_\mu(t) = \lambda_\mu^0 + 2(\mathcal{E}/\hbar\omega)\cos(\omega t)$$
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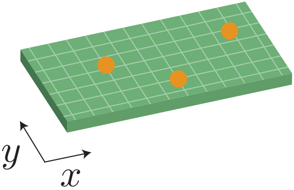
- Measure the differential integrated rate

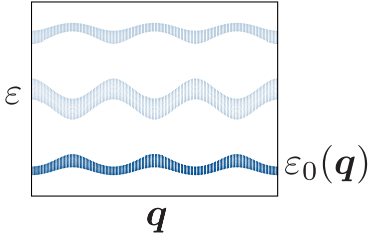
$$\Delta\Gamma^{\text{int}} = \int_0^\infty \Gamma^+(\omega) - \Gamma^-(\omega)\,\mathrm{d}\omega = \frac{4\pi\mathcal{E}^2}{\hbar^2}\,\Omega_{\mu\nu}(\boldsymbol{\lambda}^0)$$

$\rightarrow$ one reconstructs the full Berry curvature !

[Tran et al. Science Advances ’17 + Ozawa-Goldman PRB(R) ’18]

Application : Particles moving on a 2D lattice





- Parameters ($\boldsymbol{\lambda}$) are the quasi-momenta $\boldsymbol{q} = (q_x, q_y)$
- Accessing $\Omega_{xy}(\boldsymbol{q}^0)$ requires modulating

$$q_x(t) = q_x^0 + A\cos(\omega t)$$
$$q_y^{\pm}(t) = q_y^0 \pm A\sin(\omega t)$$

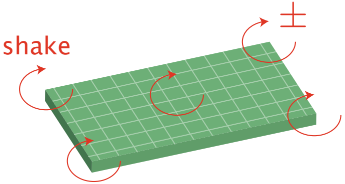
In practice ?

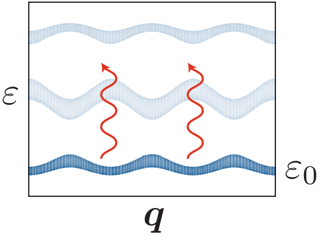
Application : Particles moving on a 2D lattice

- Modulating the quasi-momenta ?

$$q_x(t) = q_x^0 + A\cos(\omega t)$$
$$q_y^{\pm}(t) = q_y^0 \pm A\sin(\omega t)$$

- Practice : **shake** the lattice circularly ($\equiv$ circ. pol. light)

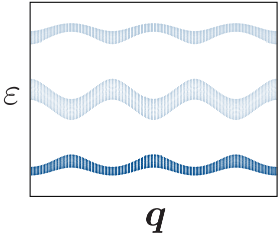




Geometry revealed through repopulation of the bands upon shaking !

Application : Chern insulator

- **Filled** Bloch band with non-zero Chern number

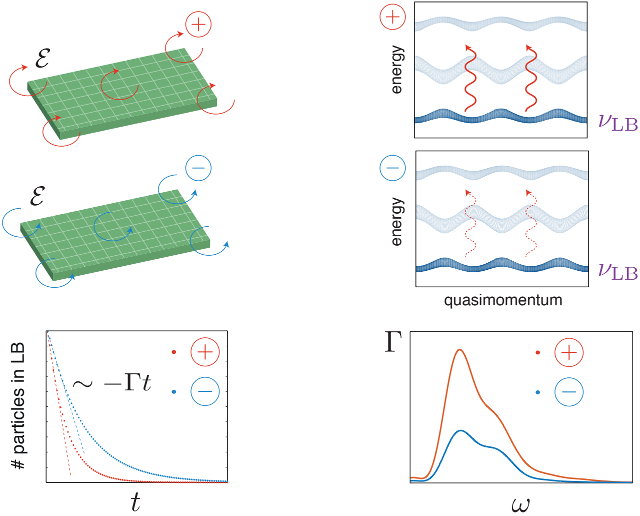


$$\nu = \frac{1}{2\pi}\int_{\mathrm{FBZ}} \Omega \neq 0$$

- The protocol probes the **averaged** Berry curvature

$$\Delta\Gamma^{\mathrm{int}} \sim \frac{1}{2\pi}\int_{\mathsf{FBZ}} \Omega_{xy}\, \mathrm{d}q_x \mathrm{d}q_y = \nu \in \mathbb{Z}$$

$\rightarrow$ a quantized dissipative response ?



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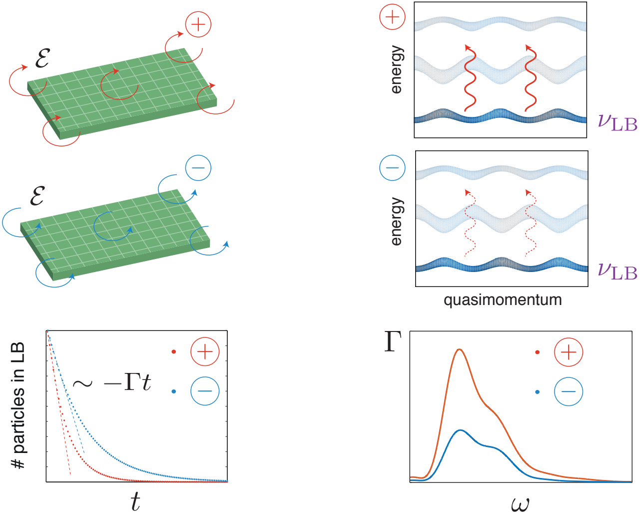

Quantization law for the rates

$$\Delta\Gamma^{\text{int}}/A_{\text{syst}} \equiv \frac{1}{2A_{\text{syst}}}\int_0^\infty \left(\Gamma^+ - \Gamma^-\right)\, \mathrm{d}\omega = (\mathcal{E}/\hbar)^2 \times \nu_{\text{LB}}$$

[Tran et al. Science Advances ’17]

Quantized circular dichroism and relation to conductivity

- Circular dichroism and dissipative optical conductivity

$$\hbar\omega\left(\Gamma^{+}(\omega)-\Gamma^{-}(\omega)\right)=\mathcal{E}^{2}\left(4A_{\mathsf{syst}}\right)\times\sigma_{\mathsf{I}}^{xy}(\omega)$$

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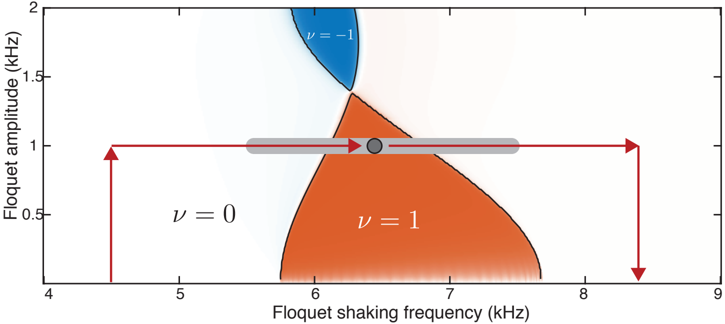
- Chern insulator case : quantized circular dichroism

$$\sigma_{xy}=(1/h)\nu_{\mathsf{LB}}\longrightarrow\Delta\Gamma^{\mathsf{int}}/A_{\mathsf{syst}}=\left(E^{2}/\hbar^{2}\right)\nu_{\mathsf{LB}}$$

[Bennett & Stern ’65 , Tran et al. Science Advances ’17]

Experimental demonstration of quantized circular dichroism

- ^{40}K atoms (fermions) in a honeycomb optical lattice
- Chern insulator is realized by Floquet engineering

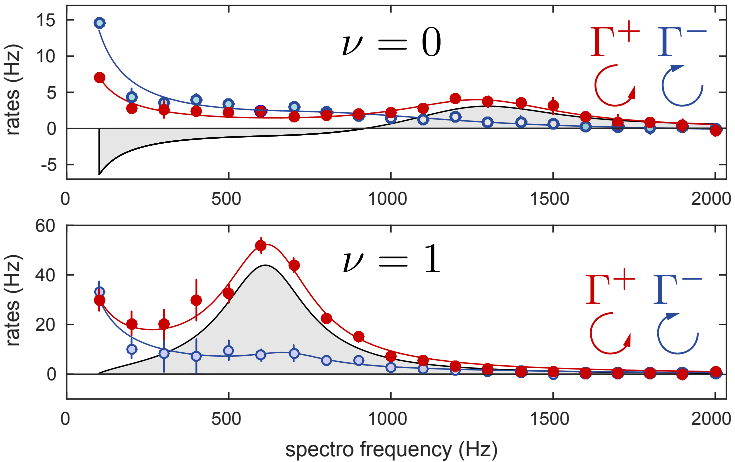


- Excitation rates $\Gamma^{\pm}$ are measured upon applying an additional circular shaking (orientation $\pm$)

[Asteria et al. arXiv :1805.11077]

Experimental demonstration of quantized circular dichroism

- Compare the rates in trivial and non-trivial regimes



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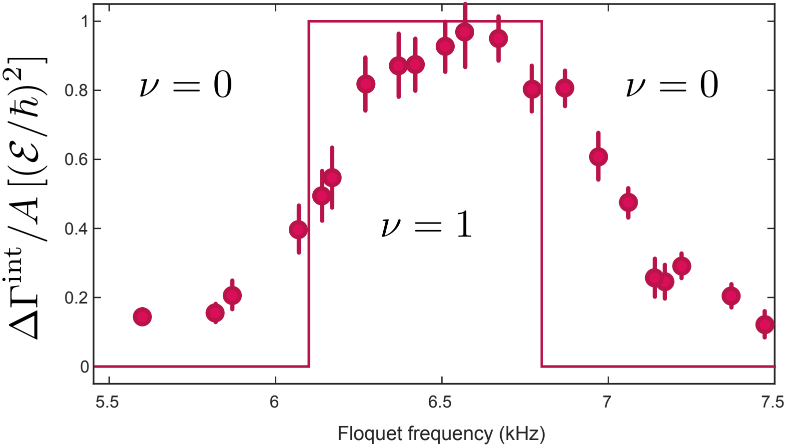
Experimental demonstration of quantized circular dichroism

- Quantized value reached deep inside topolog. regime

$$\Delta\Gamma^{\text{int}}/A_{\text{syst}} = (E^2/\hbar^2)\,\nu_{\text{LB}}^{\text{exp}}, \quad \text{with} \quad \nu_{\text{LB}}^{\text{exp}} = 0.9(1)$$



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Other experimental results

- The experiment probes the dissipative optical conductivity

$$\Gamma^{+}(\omega) - \Gamma^{-}(\omega) = \mathcal{E}^2 \left(\frac{4A_{\mathsf{syst}}}{\hbar\omega} \right) \times \sigma_{\mathsf{I}}^{xy}(\omega)$$

$\rightarrow$ first observation in topological quantum gases !

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- By shaking linearly, one probes the quantum metric

$$(\hbar/\mathcal{E})^2 (1/2\pi) \left\{ \Gamma_x^{\text{int}} + \Gamma_y^{\text{int}} \right\} = \overline{\text{Tr}[g_{\mu\nu}(\boldsymbol{k})]} \equiv \Omega_I,$$

$\rightarrow$ exp. access to the Wannier spread functional

[Marzari & Vanderbilt '97, Ozawa & Goldman '18, Asteria et al. arXiv :1805.11077]

Application : Fractional Chern insulators

- Universal relation for the differential integrated rate

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- Fractional Chern insulator : $\sigma_{xy} = (1/h)\,\nu^{\text{MB}}$, $\nu^{\text{MB}} \in \mathbb{Q}$

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- Hardcore bosons on square lattice with flux $n_\Phi = 1/4$

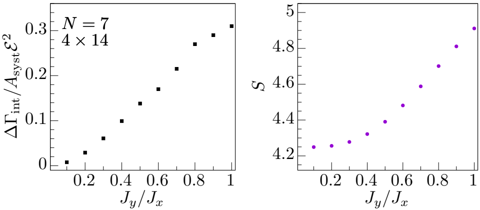
N_{part}	$N_{\text{supercell}}$	ν^{MB}	$\Delta\Gamma^{\text{int}}/(A_{\text{syst}}\mathcal{E}^2)$
4	8	0.293	0.289
5	10	0.311	0.307
6	12	0.453	0.448
7	14	0.4638	0.45

$\rightarrow$ converges towards $\nu^{\text{MB}} = 0.5$ (**Laughlin state**)

[Repellin and Goldman, arXiv :1811.08523]

Other results

- The dichroic signal distinguishes Laughlin from CDW



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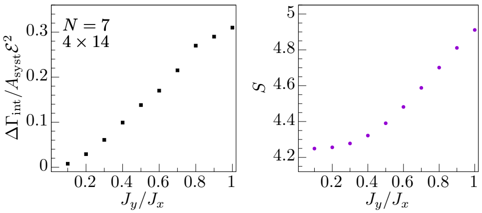
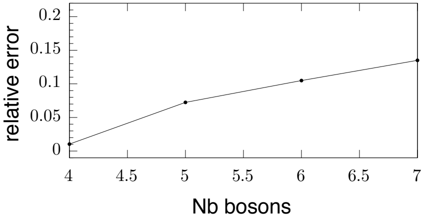


- Contributions from intra-band transitions are very small



$\longrightarrow$ good news for band-mapping !

[Repellin and Goldman, arXiv :1811.08523]

Conclusions

- Quantum geometry can be probed through excitation-rate measurements
- Quantized circular dichroism was revealed in ultracold atoms
- Detection schemes based on excitation-rate measurements are universal (wide range of applications)
- Circular dichroism can unambiguously detect topological order, e.g. fractional Chern insulators

References

- *Probing topology by "heating" : Quantized circular dichroism in ultracold atoms*,
  D. T. Tran, A. Dauphin, A. G. Grushin, P. Zoller and N. Goldman,
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- *Quantized Rabi Oscillations and Circular Dichroism in Quantum Hall Systems*,
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- *Extracting the quantum metric tensor through periodic driving*,
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  Phys. Rev. B 97, 201117(R) (2018)
- *Detecting fractional Chern insulators through circular dichroism*,
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  arXiv :1811.08523

Experimental results :

- *Measuring quantized circular dichroism in ultracold topological matter*,
  L. Asteria, D. T. Tran, T. Ozawa, M. Tarnowski, B. S. Rem, N. Fläschner,
  K. Sengstock, N. Goldman and C. Weitenberg,
  arXiv :1805.11077

Not discussed but related :

- *Revealing tensor monopoles through quantum-metric measurements*,
  G. Palumbo and N. Goldman,
  Phys. Rev. Lett. 121, 170401 (2018) + arXiv :1811.02434