Spin-Orbit Interactions of Light

Konstantin Y. Bliokh

RIKEN Cluster for Pioneering Research, Wako-shi, Japan

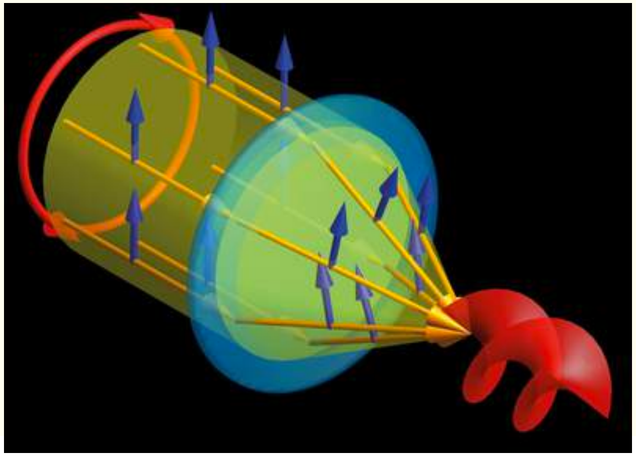
The Australian National University, Canberra, Australia

REVIEW ARTICLE | FOCUS
PUBLISHED ONLINE: 27 NOVEMBER 2015 | DOI: 10.1038/NPHOTON.2010.201
nature photonics

Spin–orbit interactions of light

K. Y. Bliokh[1,2*], F. J. Rodríguez-Fortuño[3], F. Nori[1,4] and A. V. Zayats[3]

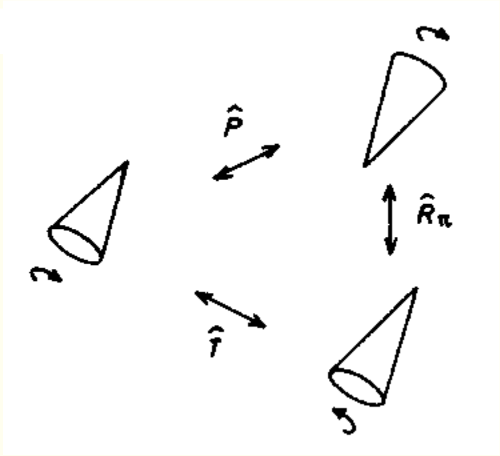
Light carries both spin and orbital angular momentum. These dynamical properties are determined by the polarization and spatial degrees of freedom of light. Nano-optics, photonics and plasmonics tend to explore subwavelength scales and additional degrees of freedom of structured — that is, spatially inhomogeneous — optical fields. In such fields, spin and orbital properties become strongly coupled with each other. In this Review we cover the fundamental origins and important applications of the main spin–orbit interaction phenomena in optics. These include: spin-Hall effects in inhomogeneous media and at optical interfaces, spin-dependent effects in nonparaxial (focused or scattered) fields, spin-controlled shaping of light using anisotropic structured interfaces (metasurfaces) and robust spin-directional coupling via evanescent near fields. We show that spin–orbit interactions are inherent in all basic optical processes, and that they play a crucial role in modern optics.



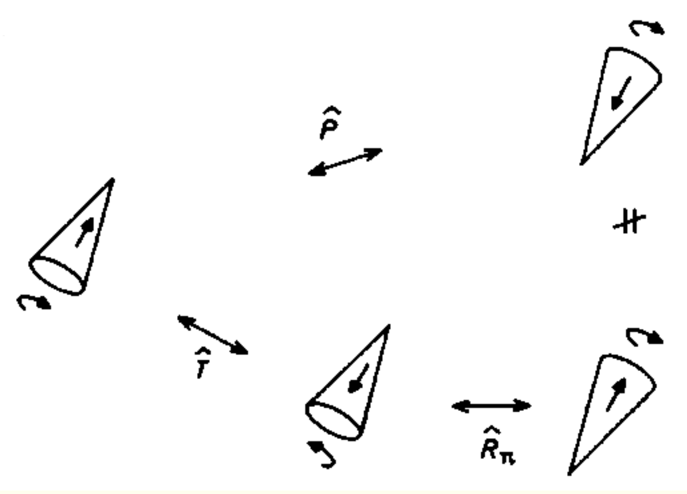
Angular momentum, helicity, chirality

Chirality is a P-odd but T-even property.

Angular momentum is P-even but T-odd:



Only **helicity**, i.e., projection of the angular momentum onto the momentum is P-odd, T-even, and therefore **chiral**:



L.D. Barron, 1986

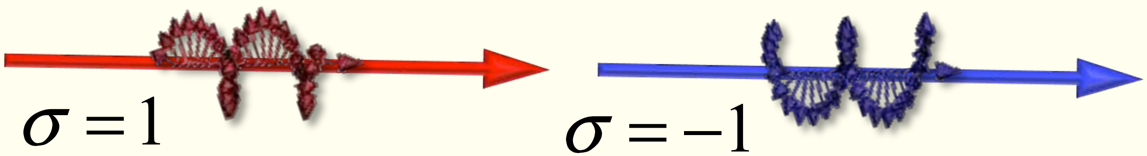
Outline

- **Spin-orbit interaction in paraxial beams**
  - **Propagation in gradient-index media**
  - **Reflection/transmission at an interface**
- **Spin-orbit coupling in nonparaxial fields**
  - **Spin and orbital AM in free space**
  - **Focusing, scattering, imaging**
- **Spin-orbit coupling in inhomogeneous anisotropic structures (metasurfaces)**
- **Transverse spin-momentum locking**

- Spin-orbit interaction in paraxial beams
  - Propagation in gradient-index media
  - Reflection/transmission at an interface
- Spin-orbit coupling in nonparaxial fields
  - Spin and orbital AM in free space
  - Focusing, scattering, imaging
- Spin-orbit coupling in inhomogeneous anisotropic structures (metasurfaces)
- Transverse spin-momentum locking

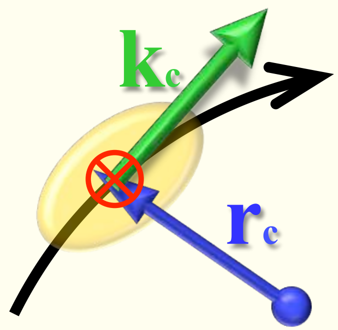
Angular momentum of paraxial light

1. Intrinsic spin AM (polarization)



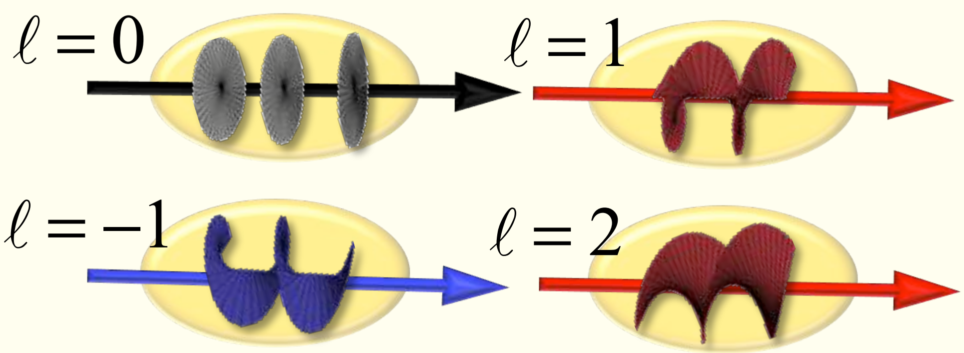
$$\mathbf{S} = \sigma \langle \mathbf{k} \rangle / k$$

2. Extrinsic orbital AM (trajectory)



$$\mathbf{L}_{\text{ext}} = \langle \mathbf{r} \rangle \times \langle \mathbf{k} \rangle$$

3. Intrinsic orbital AM (vortex)



$$\mathbf{L}_{\text{int}} = \langle \left(\mathbf{r} - \langle \mathbf{r} \rangle \right) \times \mathbf{k} \rangle$$

$$= \ell \langle \mathbf{k} \rangle / k$$

Geometric phase

Angular momentum is intimately related to **rotations**.

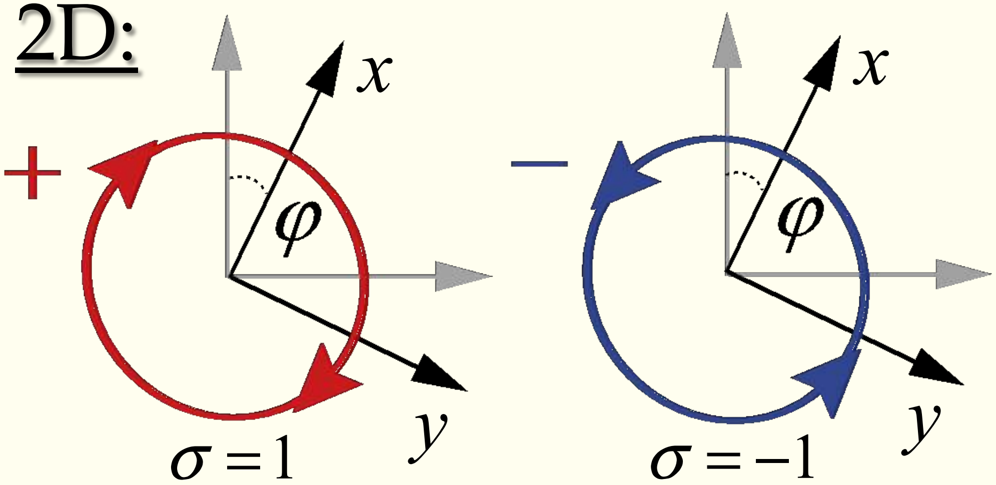
There is a natural **coupling** between the AM and rotations of the system.

It can be described by **geometric (Berry) phases**.

Remarkably, these can be introduced via both geometric and dynamical approaches seemingly unrelated to each other.
However, geometric and dynamical aspects become unified on a deeper level of understanding revealing the **geometrodynamical** nature of the AM physics (cf. general relativity).

Geometric phase

2D:

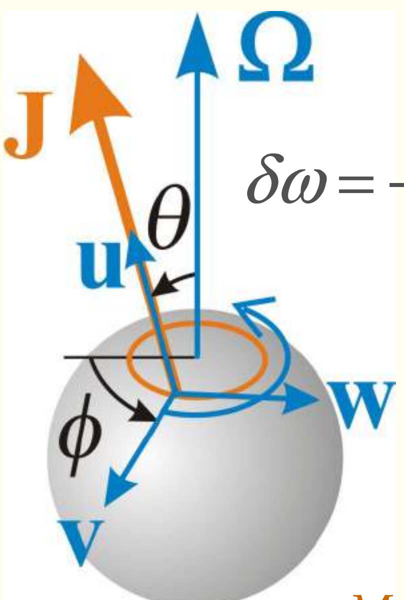


$$\mathbf{e}^{\sigma} = \left(\bar{\mathbf{x}} + i\sigma\bar{\mathbf{y}}\right)$$

$$\mathbf{e}^{\sigma} \to e^{i\Phi}\mathbf{e}^{\sigma},\ E^{\sigma} \to e^{-i\Phi}E^{\sigma}$$

$$\boxed{\Phi = -\sigma\varphi}$$

3D:



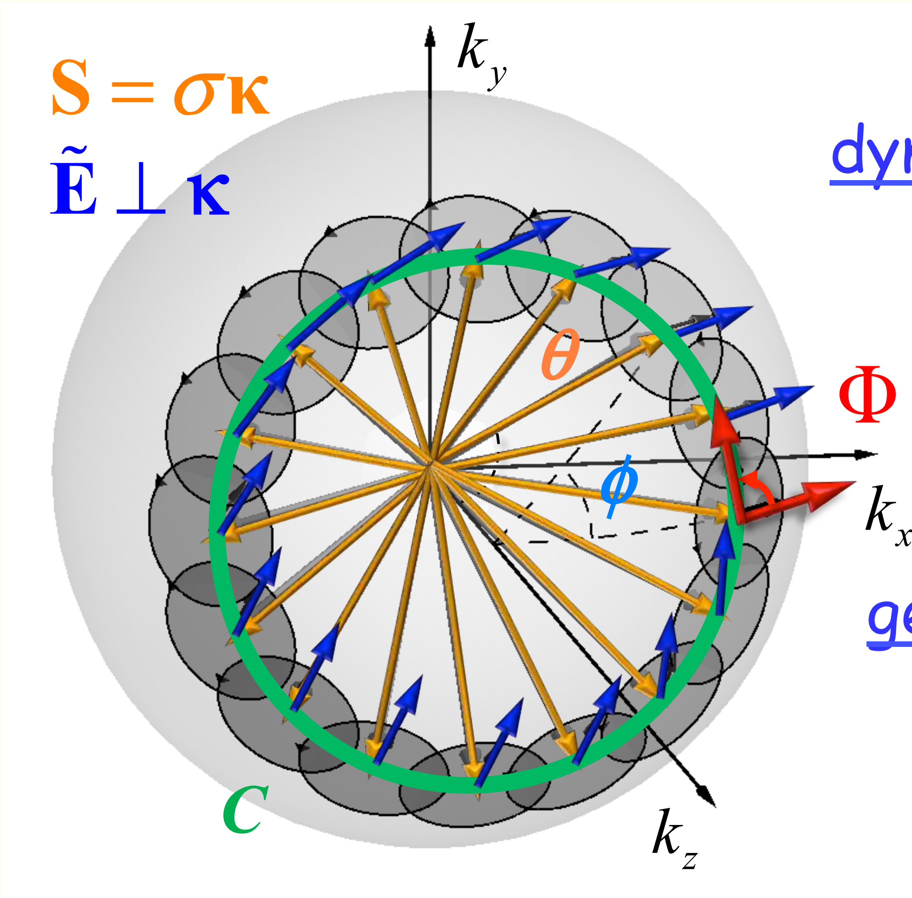
$$\delta\omega = -\mathbf{J}\cdot\mathbf{\Omega}$$

$$\boxed{\Phi = -\int \mathbf{J}\cdot\mathbf{\Omega}_{\zeta}\, d\zeta}$$

AM-rotation coupling:
Coriolis / angular-Doppler effect

Mashhoon, 1988; Garetz, 1979; Bialynicki-Birula, 1997; Hannay, 1998

Geometric phase



$$\mathbf{e}^{\sigma}(\mathbf{k}) = \left(\bar{\boldsymbol{\theta}} + i\sigma\bar{\boldsymbol{\phi}}\right)e^{i\sigma\phi}$$

dynamics, **S**:

$$\Phi = 2\pi\sigma - \int d\phi\, S_z$$
$$= 2\pi\sigma\left(1 - \cos\theta\right)$$

geometry, **E**:

$$\Phi = \sigma \int_C \boldsymbol{\mathcal{A}}(\mathbf{k}) \cdot d\mathbf{k}$$

$$\boldsymbol{\mathcal{A}} = \frac{1-\cos\theta}{k\sin\theta}\bar{\boldsymbol{\phi}}, \quad \boldsymbol{\mathcal{F}} = \partial_{\mathbf{k}} \times \boldsymbol{\mathcal{A}} = \frac{\mathbf{k}}{k^3}$$

– Berry connection, curvature (parallel transport)

M. V. Berry, 1984

Geometric phase: Gauge fields

The geometric phase appears in all situations with variations **k**-vectors (directions of propagation) of light.

In this manner, the Berry connection plays the role of an effective **gauge field (vector-potential)** in the **k**-space.

$\boldsymbol{\mathcal{A}}$ - "vector-potential"

$$\boldsymbol{\mathcal{F}} = \partial_{\mathbf{k}} \times \boldsymbol{\mathcal{A}} = \frac{\mathbf{k}}{k^3}$$ - "magnetic field"

$$\boxed{\Phi = \sigma \int_C \boldsymbol{\mathcal{A}}(\mathbf{k}) \cdot d\mathbf{k}}$$ - "Aharonov-Bohm (Dirac) phase"

Spin-orbit Lagrangian and phase

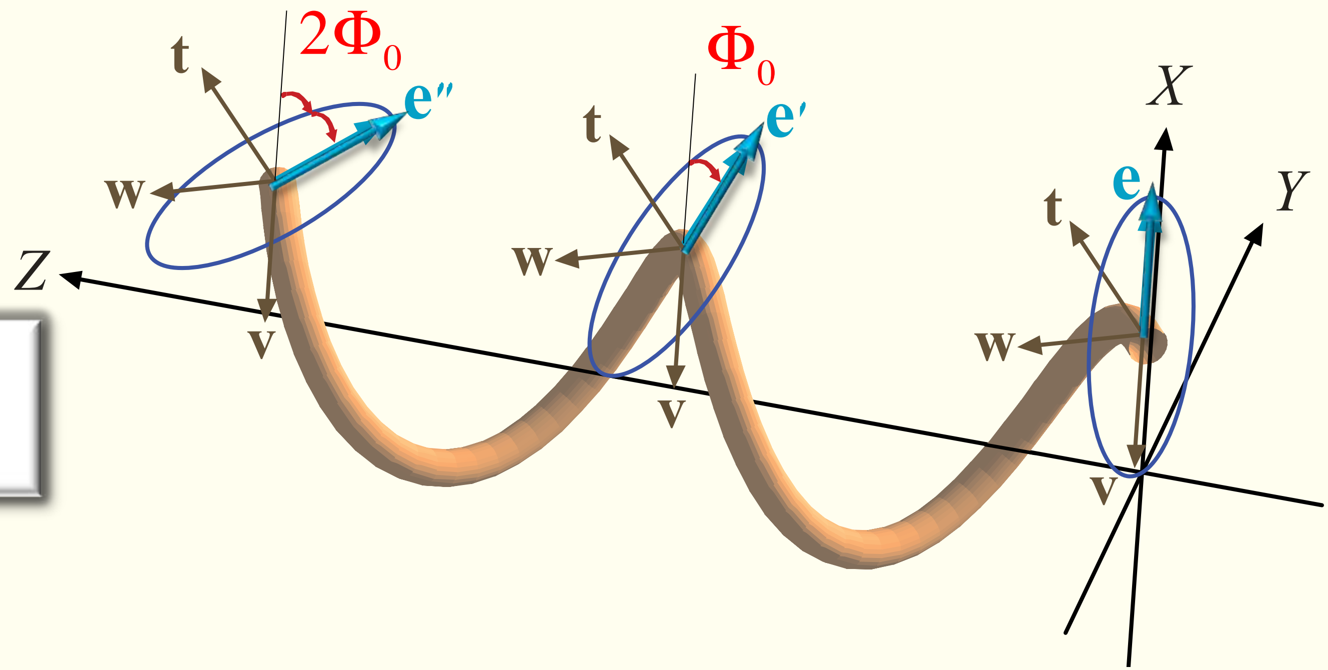
$$\mathcal{L}_{\mathrm{SOI}} = \sigma \boldsymbol{\mathcal{A}} \cdot \dot{\mathbf{k}} = \mathbf{S} \cdot \boldsymbol{\Omega}$$

– SOI Lagrangian from Maxwell equations

Bliokh, 2008

Berry phase:

$$\Phi = \int \mathcal{L}_{\mathrm{SOI}}\, d\tau$$



Rytov, 1938; Vladimirskiy, 1941; Ross, 1984; Tomita, Chiao, Wu, PRL 1986

$$\hat{\boldsymbol{\mathcal{A}}} = \frac{\hat{\mathbf{S}} \times \mathbf{p}}{2p^2}\left(1 - \frac{m}{E}\right), \quad \hat{\mathcal{L}}_{\mathrm{SOI}} = \hat{\boldsymbol{\mathcal{A}}} \cdot \dot{\mathbf{p}} \approx \frac{e}{4m^2}\hat{\boldsymbol{\sigma}}\left(\boldsymbol{\mathcal{E}} \times \mathbf{p}\right)$$

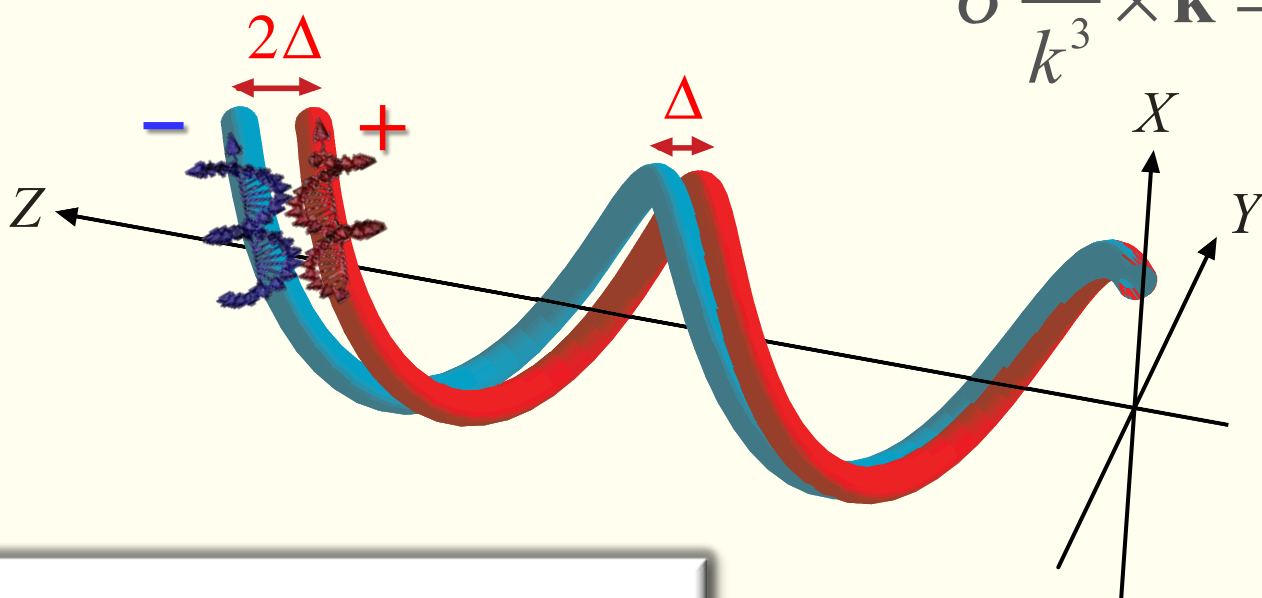
– for Dirac equation

Spin-Hall effect of light

$$\dot{\mathbf{k}}_c = k\nabla \ln n\,,\ \ \dot{\mathbf{r}}_c = \boldsymbol{\kappa}_c - \sigma\mathcal{F}\times\dot{\mathbf{k}}_c$$

– ray equations (equations of motion)

$$\sigma\frac{\mathbf{k}}{k^3}\times\dot{\mathbf{k}} = \lambda\mathbf{S}\times\dot{\boldsymbol{\kappa}}$$

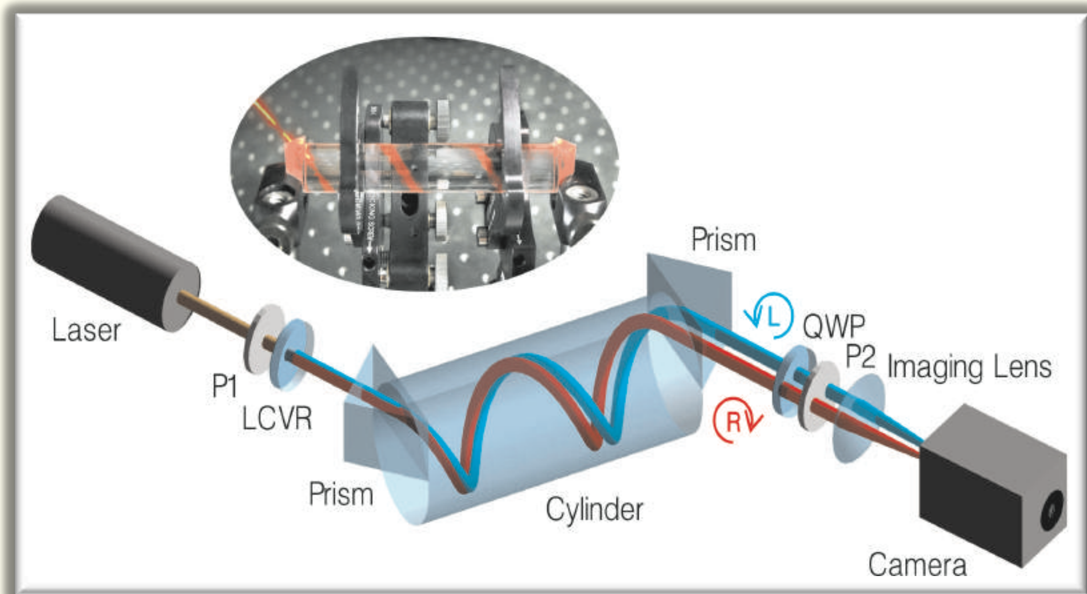


$$\mathbf{J} = \mathbf{r}_c \times \mathbf{k}_c + \sigma\boldsymbol{\kappa}_c = const$$

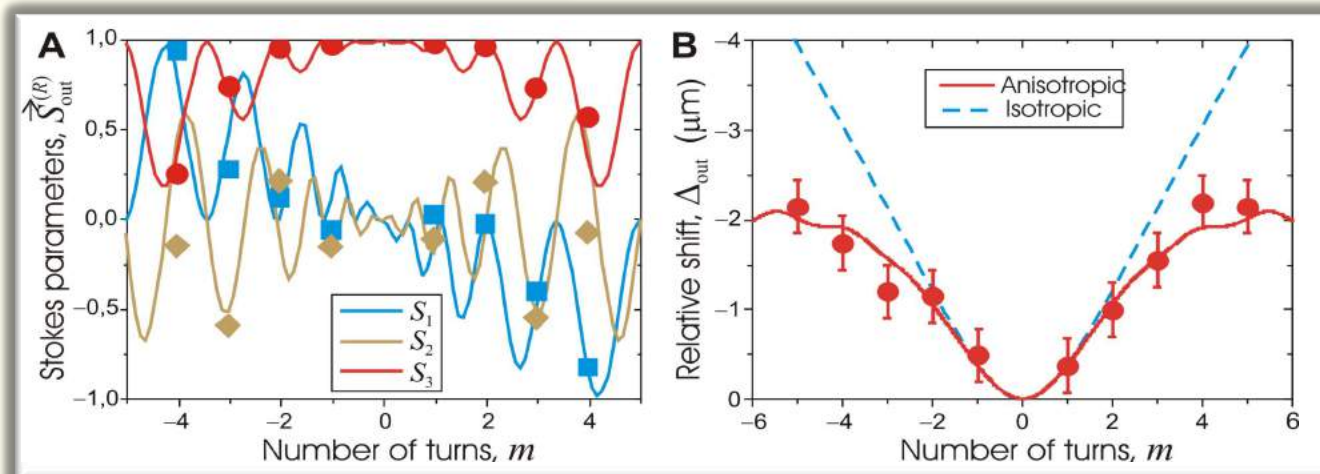
– AM integral

Liberman & Zeldovich, PRA 1992;
Bliokh & Bliokh, PLA 2004, PRE 2004; Onoda et al., PRL 2004

Spin-Hall effect of light



- Berry phase
- Spin-Hall effect
- Anisotropy



Bliokh et al., Nature Photon. 2008

Origin of the spin-Hall shift

The spin-Hall-effect equations of motion can be derived *ab initio* from Maxwell equations.

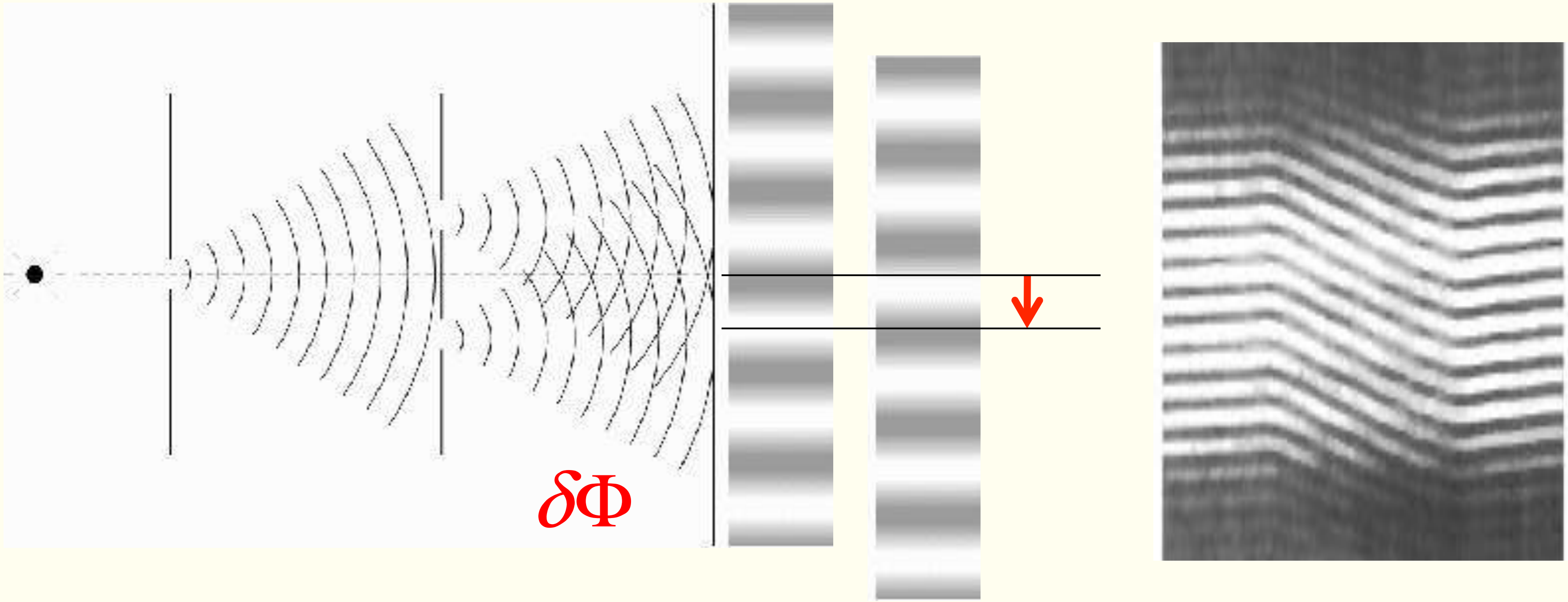
Remarkably, the geometric Berry curvature acts as a real physical field and provides real "force". This is the essence of **geometrodynamics** in the **particle** description.

But what causes the Hall-effect shift in the **wave** description? It turns out that the shift is caused by the **transverse gradient of the Berry phase for different plane waves forming a transversely confined beam**.

Indeed, in wave physics, any shift can be attributed to the relative phase between the interfering waves:

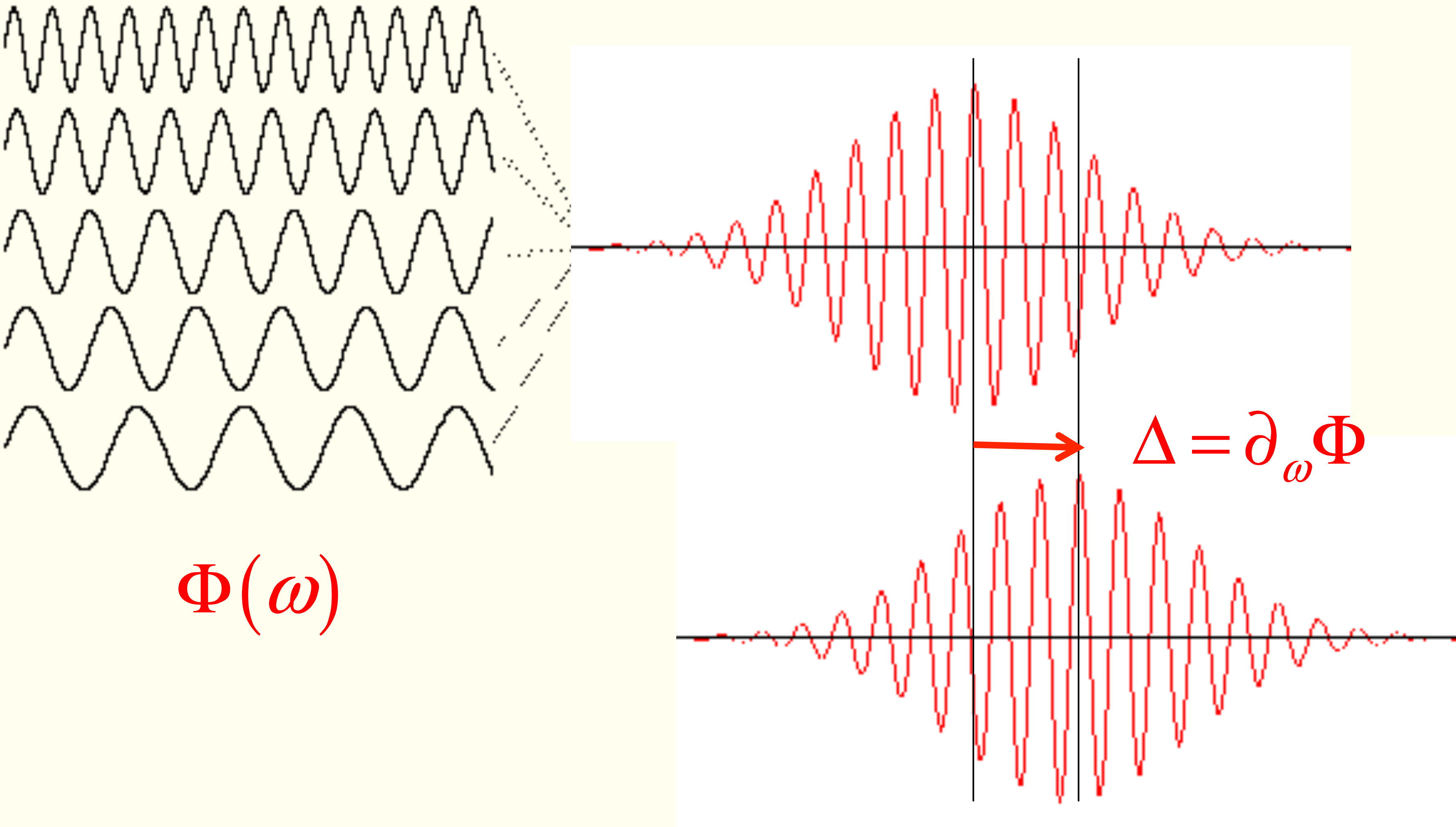
Origin of the Hall-effect shift

Relative phase between two waves induces shift of the interference pattern (e.g., **Aharonov-Bohm effect**):



Origin of the Hall-effect shift

Relative phase gradient between many waves induces shift of the wave packet (e.g., **Wigner time delay**):

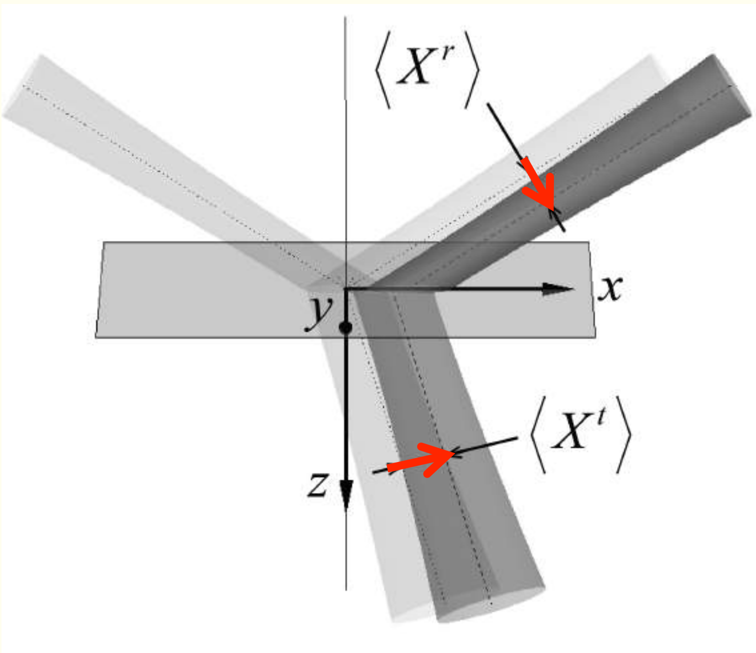


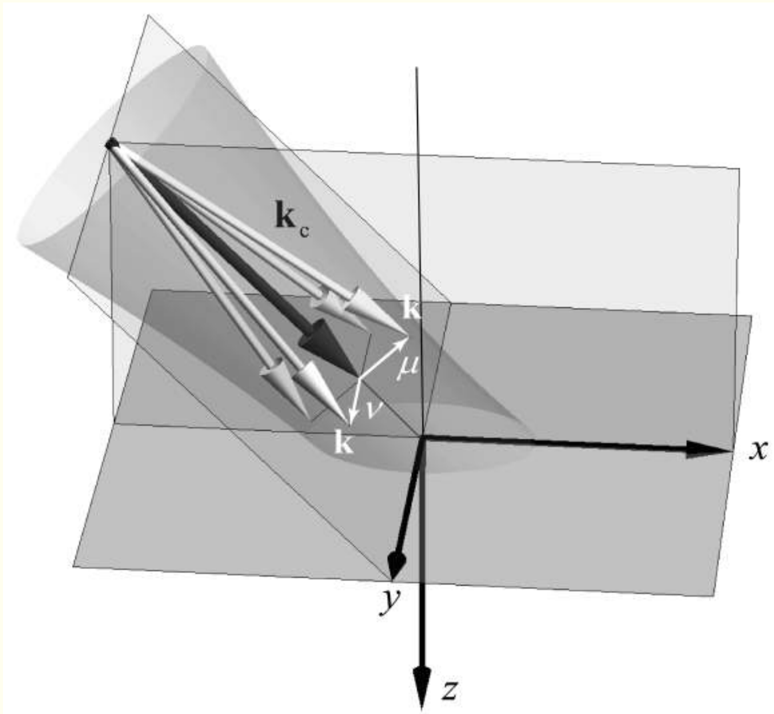
$$\Phi(\omega)$$

$$\Delta = \partial_\omega \Phi$$

Origin of the Hall-effect shift

Phase gradient from spatial dispersion induces
The real-space shift (e.g., **Goos-Hänchen effect**):

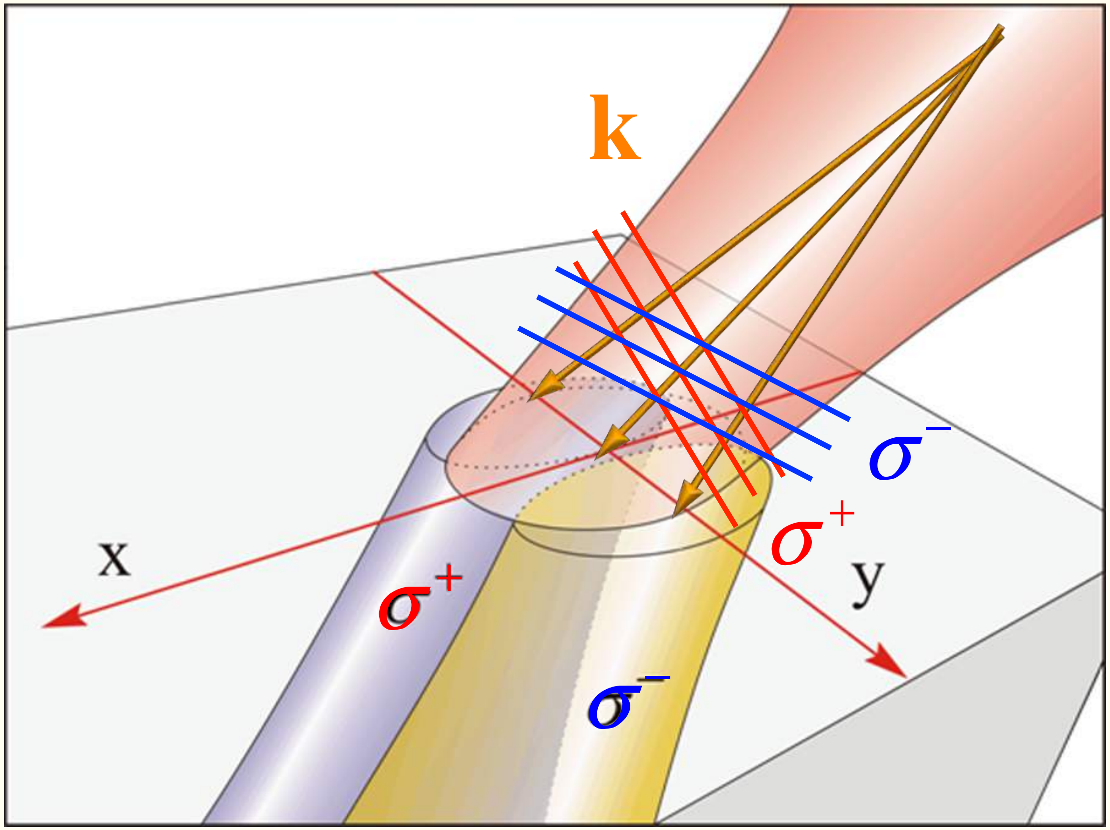




$$X = \partial_{k_X} \Phi \qquad \Phi(\theta) = \Phi(k_X / k)$$

Origin of the spin-Hall shift

In a similar way, transverse Berry-phase gradient induces the spin-Hall-effect shift:

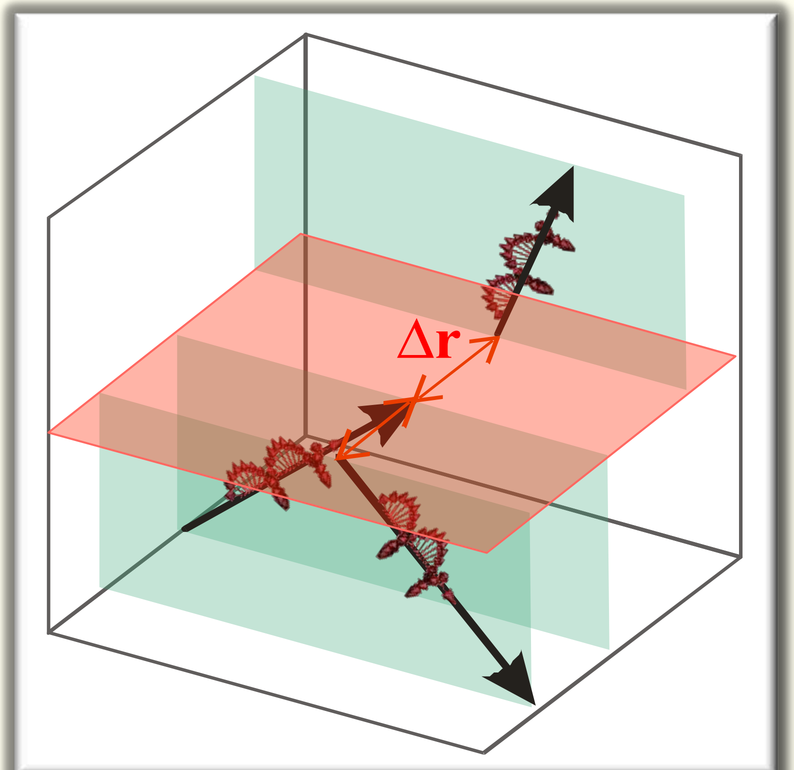


$$\Phi(\varphi) \propto \sigma k_y / k$$

$$\boxed{Y = -\partial_{k_y} \Phi}$$

Reflection/refraction at an interface

This can be illustrated by the simplest example of the spin Hall effect of light that arises upon the beam reflection/refraction at a plane interface:

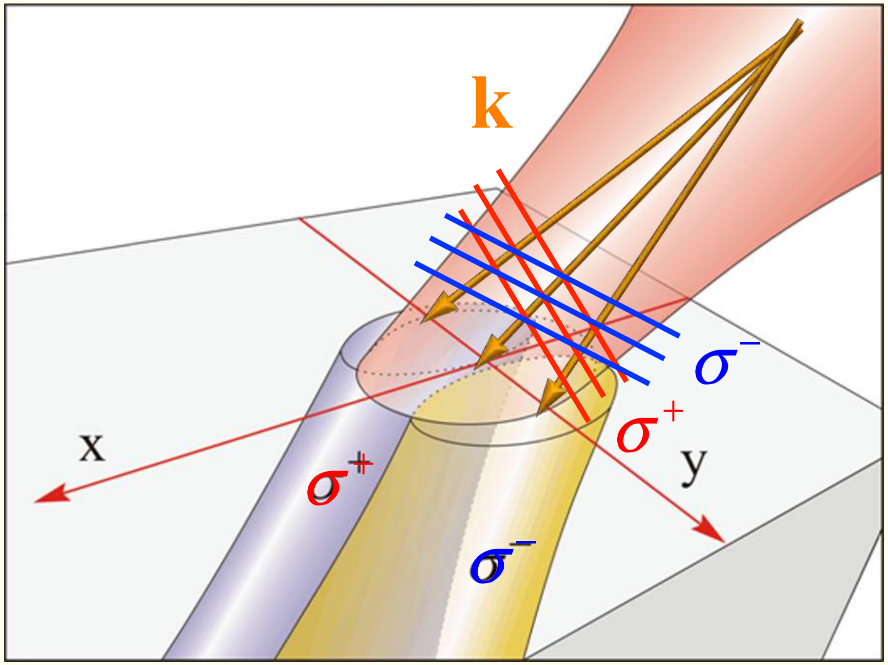


$$\Delta \mathbf{r} \propto \sigma \lambda\!\!\!^{-}$$

Imbert–Fedorov transverse shift

Fedorov, 1955; Schilling, 1965;
Imbert, 1972;
……
Onoda et al., PRL 2004;
Bliokh & Bliokh, PRL 2006;
Hosten & Kwiat, Science 2008

Spin-Hall effect of light



geometry, phase:

$$\mathbf{k}_c \to \mathbf{k}_c + k_y \bar{\mathbf{y}} :$$

$$\hat{R}_z\left(\kappa_y / \sin\theta\right), \quad S_z = \sigma \cos\theta$$

$\Rightarrow$

$$\boxed{\Phi = -\sigma \kappa_y \cot\theta}$$

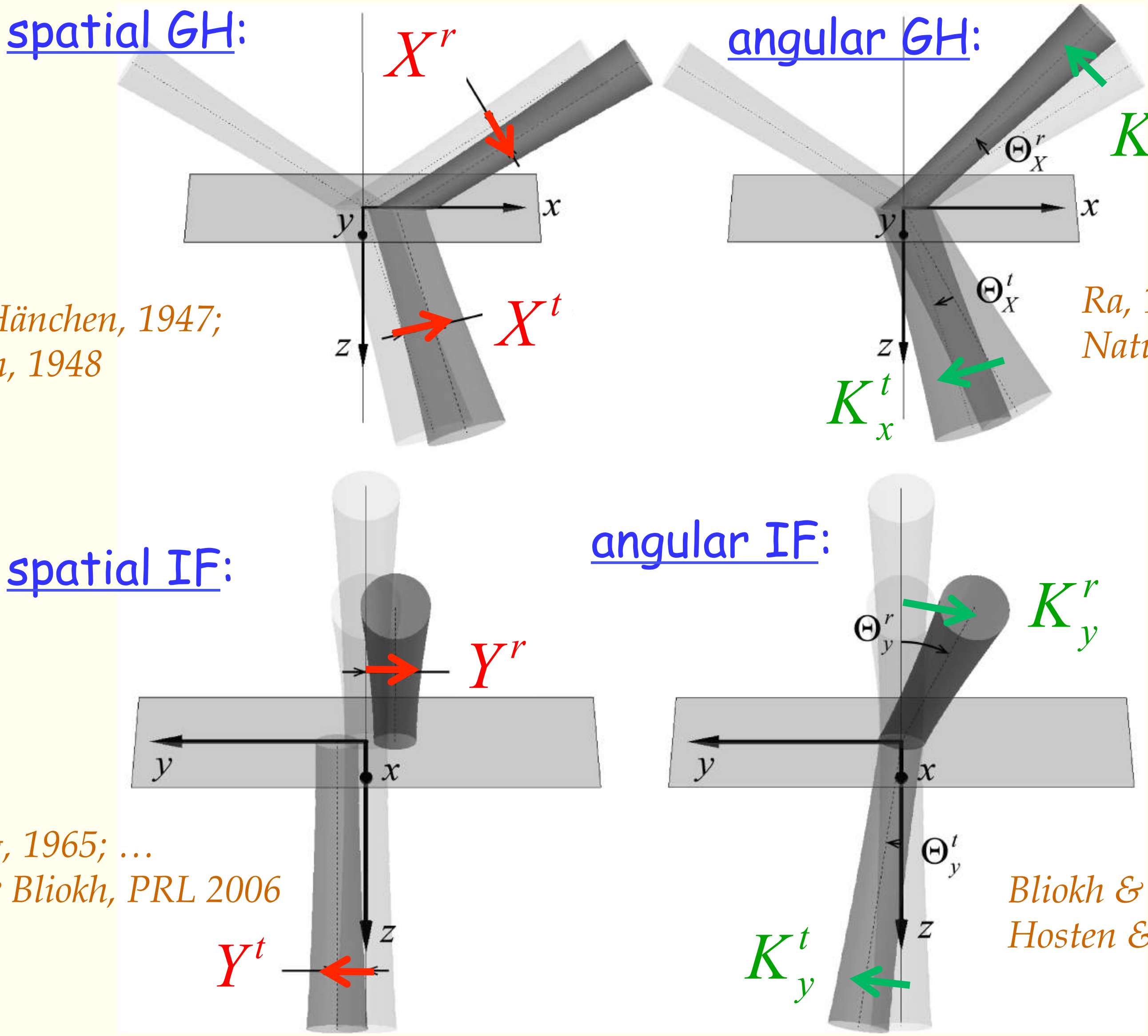
$\Rightarrow$

$$\boxed{Y = -\partial_{k_y} \Phi = \lambda \sigma \cot\theta}$$

dynamics, AM:

$$\mathbf{J} = \mathbf{L}_{\text{ext}} + \mathbf{S} : \quad \delta J_z = -k\, Y \sin\theta + \sigma \cos\theta \;\Rightarrow\; \boxed{\delta J_z = 0}$$

Goos-Hänchen and Imbert-Fedorov shifts



Goos, Hänchen, 1947;
Artman, 1948

Ra, 1973; …; Merano et al.,
Nature Photon. 2009

Schilling, 1965; …
Bliokh & Bliokh, PRL 2006

Bliokh & Bliokh, PRE 2007;
Hosten & Kwiat, Science 2008

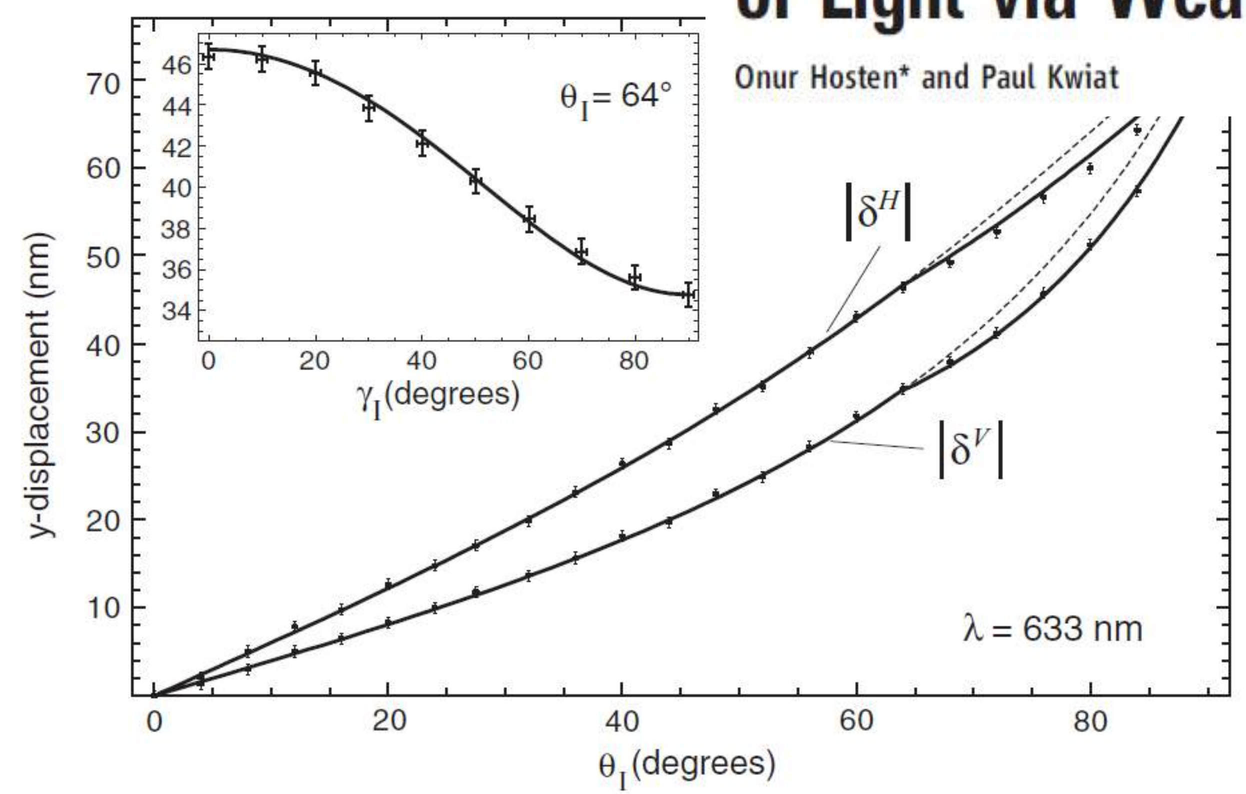
Reflection/refraction at an interface

Remarkably, the beam shift and accuracy of measurements can be enormously increased using the method of **quantum weak measurements**:

www.sciencemag.org SCIENCE VOL 319 8 FEBRUARY 2008

Observation of the Spin Hall Effect of Light via Weak Measurements

Onur Hosten* and Paul Kwiat

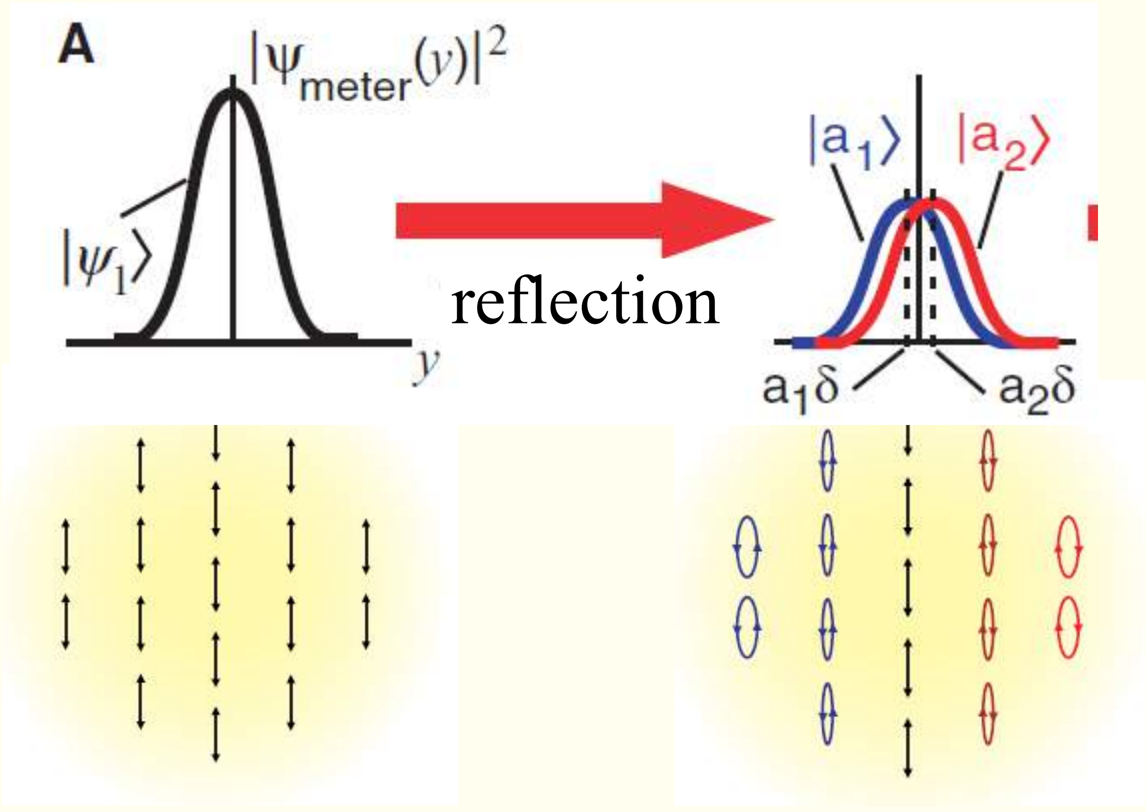


Angstrom accuracy!

Reflection/refraction at an interface

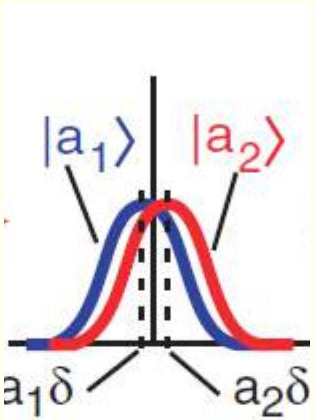
The incident beam is **linearly** polarized, i.e., in the superposition of σ^{+} and σ^{-} states.

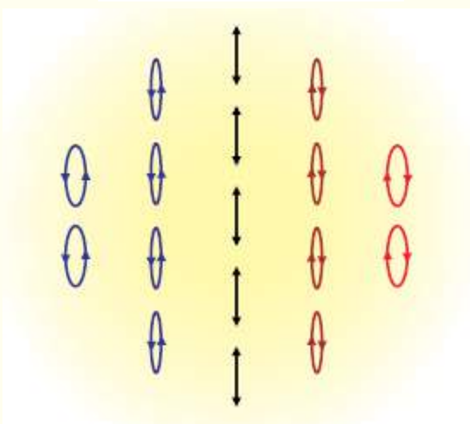
Reflection/refraction shifts these states in the opposite direction on a subwavelength distance:



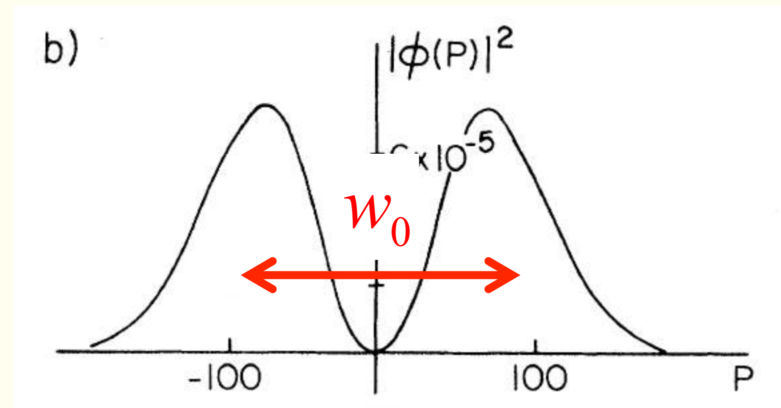
Reflection/refraction at an interface

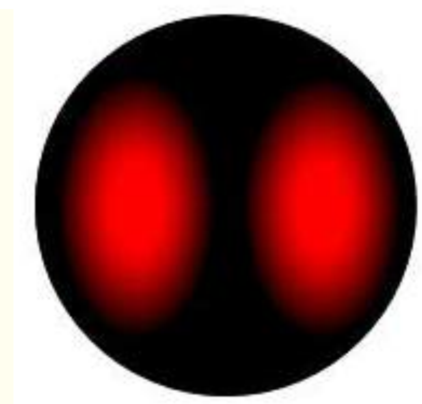
Placing an **orthogonal** linear polarizer at the output (post-selection), we will see double-hump profile with the **beam-width** (instead of wavelength!) **splitting** of maxima:





polarizer

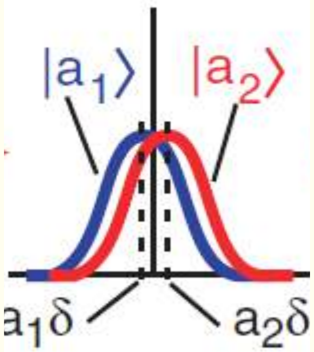


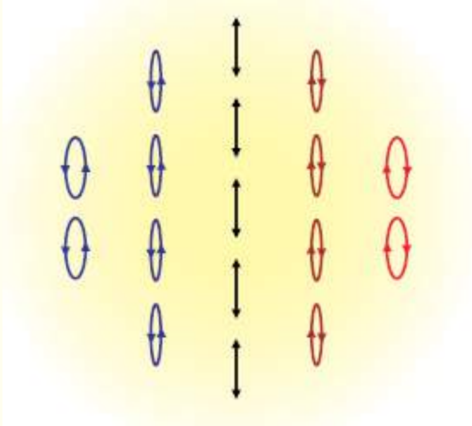


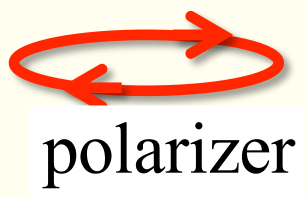
But the shift of the beam centroid is zero.

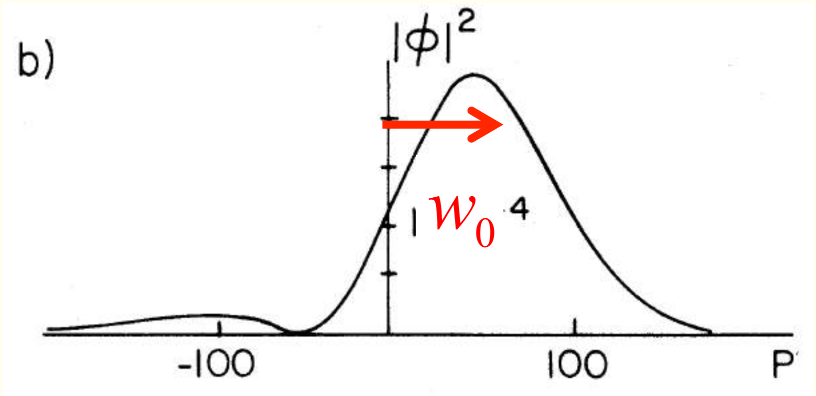
Reflection/refraction at an interface

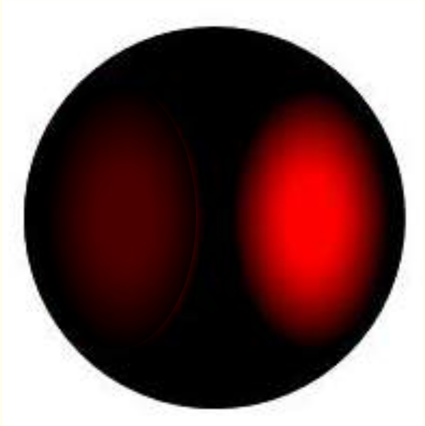
However slight non-orthogonality of the output polarizer (slightly elliptical or linear tilted) drastically deforms the output intensity profile:







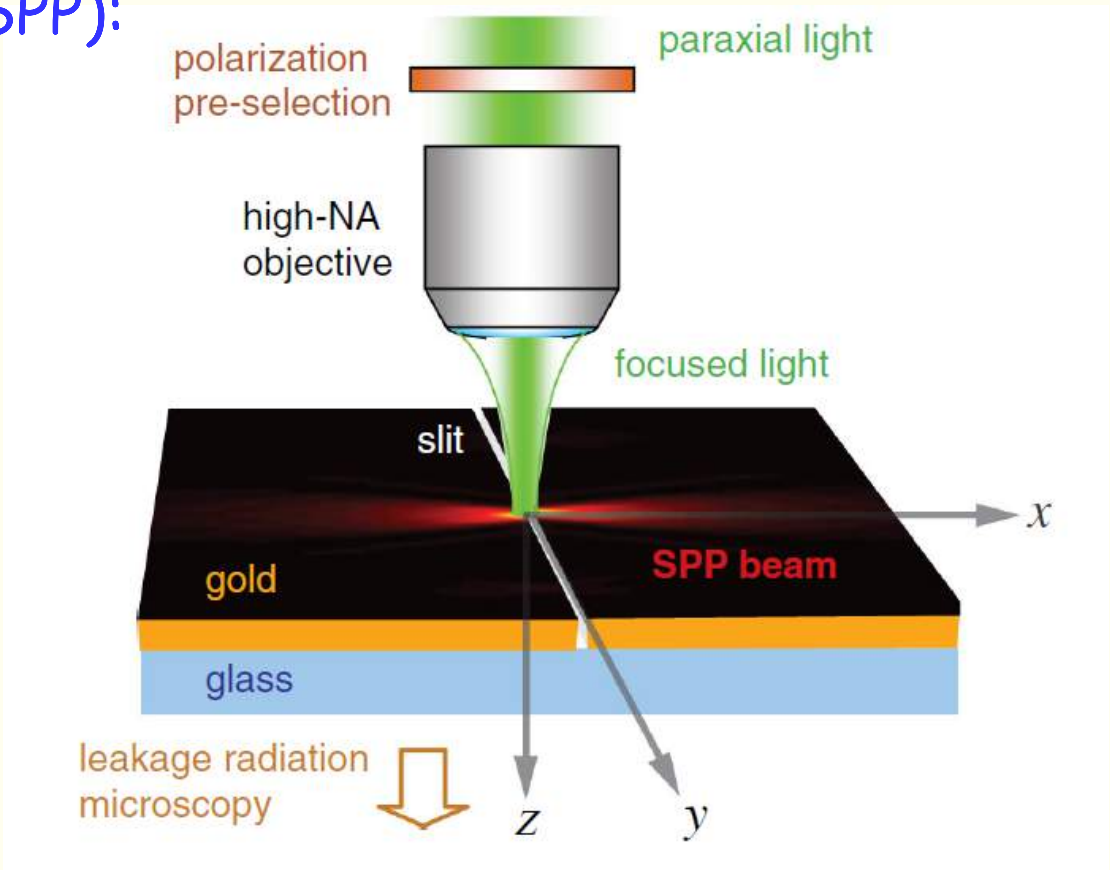




This results in the beam-width centroid shift, which is proportional to the original Hall-effect shift !

Reflection/refraction at an interface

Recently we showed that similar weak measurements of the beam shifts can be realized using **surface plasmon polaritons** (SPP):



An incident optical beam scattered by a single slit produces output SPP beams, akin to refraction at interface.

Reflection/refraction at an interface

But the SPPs are **linearly p-polarized**. This provides the build-in polarization postselection for weak measurements. One has only to take almost orthogonal s-polarization (i.e., along the slit) in the incident light:

$$\Phi_{in} \propto \exp\left[-y^2/w_0^2\right]$$ - input spatial profile

$$\left|\Psi_{in}\right\rangle \simeq \left|Y\right\rangle - \varepsilon\left|X\right\rangle = \frac{\left(-i-\varepsilon\right)\left|R\right\rangle + \left(i-\varepsilon\right)\left|L\right\rangle}{\sqrt{2}}$$

$$\varepsilon \ll 1$$

- input polarization

$$\left|\Psi_{out}\right\rangle = \left|X\right\rangle = \frac{\left|L\right\rangle + \left|R\right\rangle}{\sqrt{2}}$$ - output polarization

$$\sigma_w = \frac{\left\langle \Psi_{out} \middle| \hat{\sigma}_3 \middle| \Psi_{in} \right\rangle}{\left\langle \Psi_{out} \middle| \Psi_{in} \right\rangle} = \frac{i}{\varepsilon}$$ - weak value of helicity

Reflection/refraction at an interface

As a result of the spin-orbit interaction, the output SPP beam profile becomes shifted:

$$\Phi_{out} \propto \exp\left[-\left(y-\Delta\right)^2 / w_0^2\right]$$ - output spatial profile

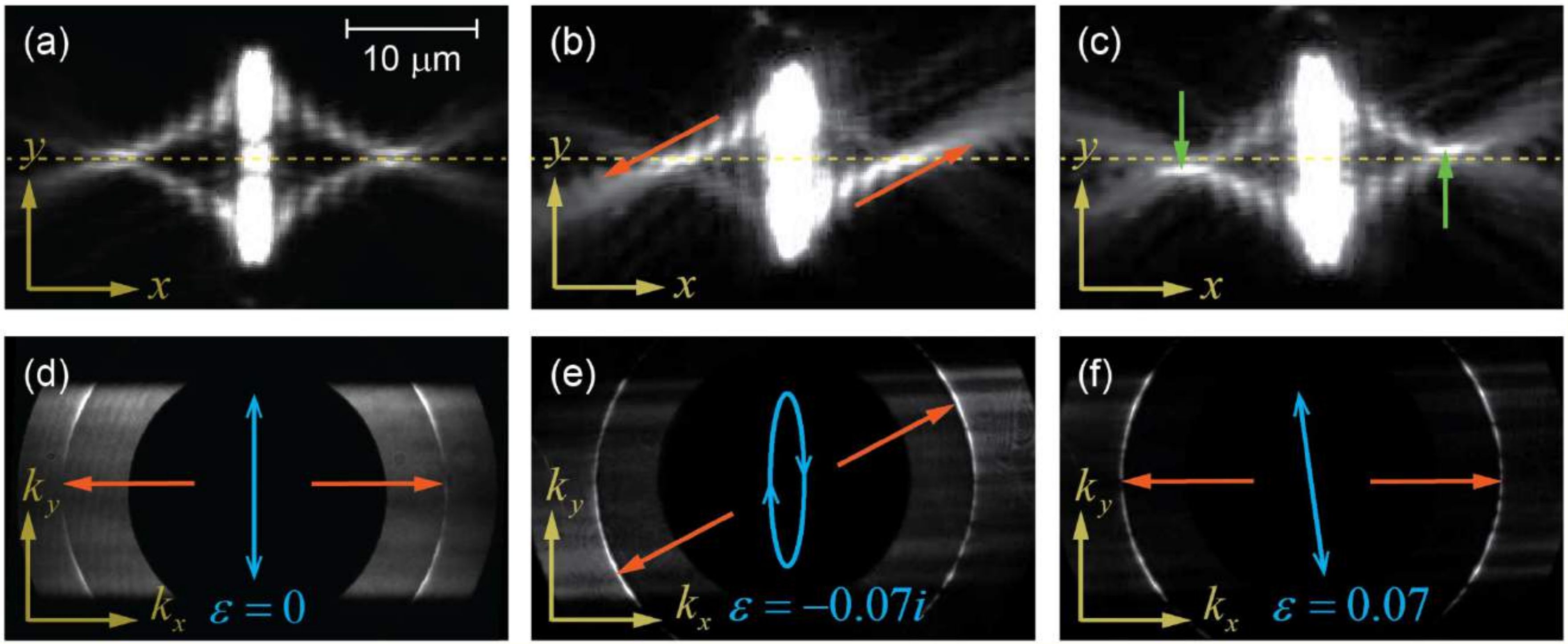
$$\boxed{\Delta \simeq -ƛ\sigma_w = -ƛ\frac{i}{\varepsilon}}$$ - complex beam shift

The real and imaginary part of the complex shift yield the coordinate and momentum spin-Hall shifts:

$$\boxed{\langle y \rangle = \mathrm{Re}\,\Delta} \qquad \boxed{\langle k_y \rangle = 2w_0^{-2}\,\mathrm{Im}\,\Delta}$$

Reflection/refraction at an interface

The results SPP beam fields in the real and momentum (Fourier) spaces at different input polarizations:

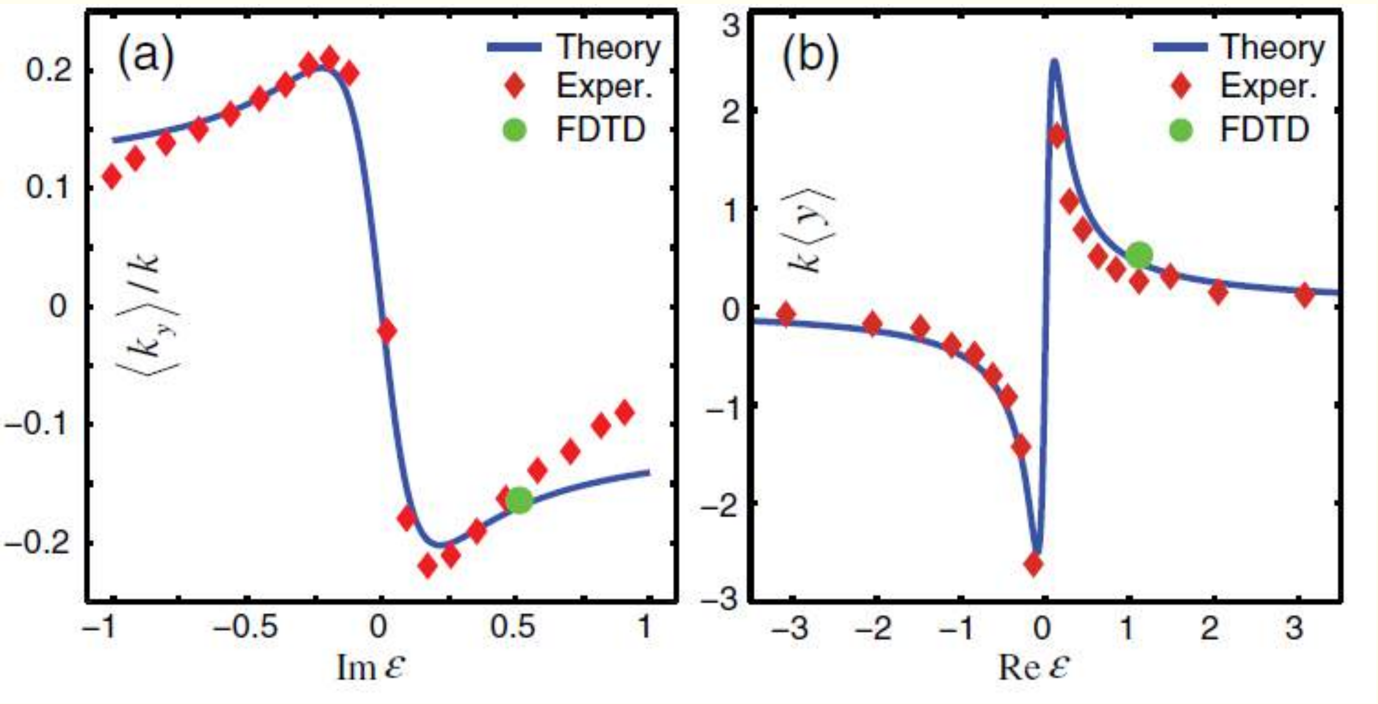


Small real and imaginary ε induces strong deformations and coordinate and momentum spin-Hall shifts in the SPP beams.

Gorodetski et al., PRL 2012

Reflection/refraction at an interface

The results are in perfect agreement with the FDTD and analytical calculations:

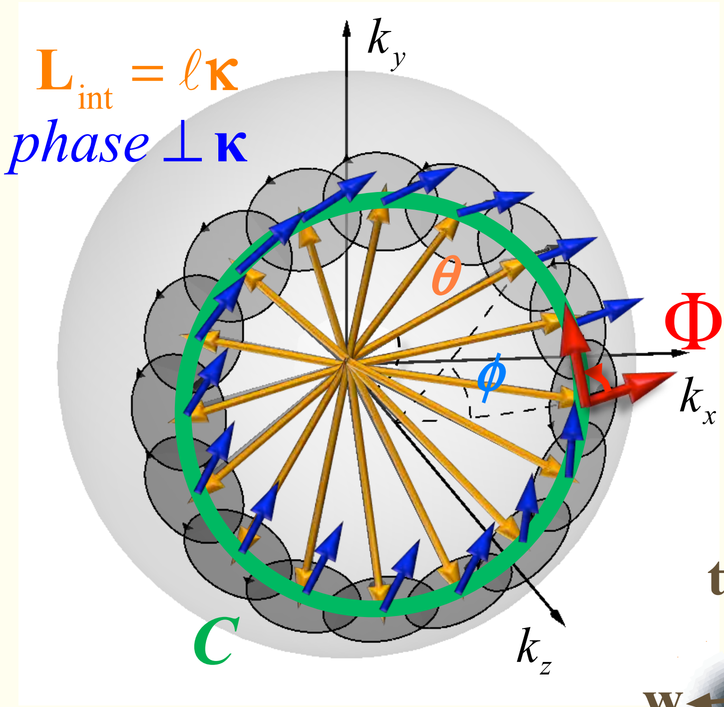


$$\langle y \rangle = \frac{1}{k\theta_c} \frac{2\chi\theta_c \operatorname{Re}\varepsilon}{2\chi^2 |\varepsilon|^2 + \theta_c^2/2} \qquad \langle k_y \rangle = -k\theta_c \frac{\chi\theta_c \operatorname{Im}\varepsilon}{2\chi^2 |\varepsilon|^2 + \theta_c^2/2}$$

Summary of the SOI in paraxial beams

- Geometrodynamics of ligth carrying intrinsic AM; Geometric force from Berry curvature
- Spin and orbital Hall effects; Total AM conservation
- Hall shifts arise from the transverse gradient of the Berry phase
- Beam refraction/reflection at a plane interface
- Weak measurements of the spin Hall effect; Plasmonic spin Hall effect

Orbit-orbit interaction and phase

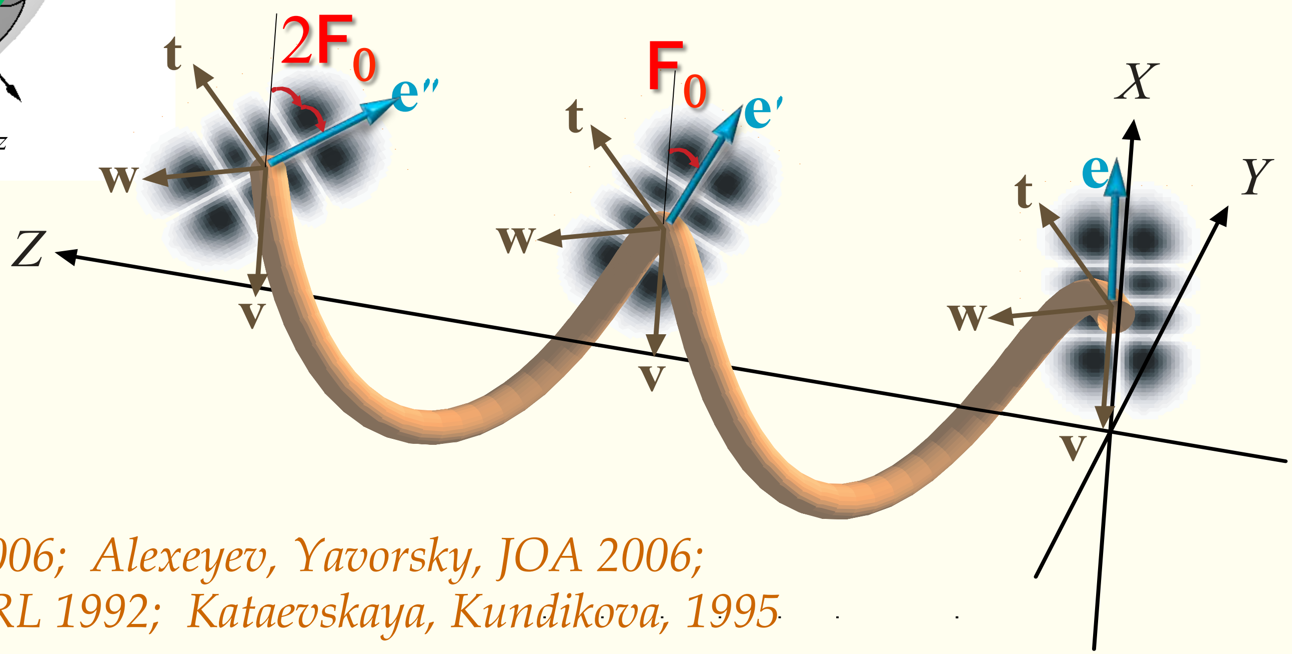


$$\mathcal{L}_{\rm OOI} = \ell \boldsymbol{\mathcal{A}} \cdot \dot{\mathbf{k}} = \mathbf{L}_{\rm int} \cdot \boldsymbol{\Omega}$$

– OOI Lagrangian

$$\Phi = \int \mathcal{L}_{\rm SOI}\, d\tau$$

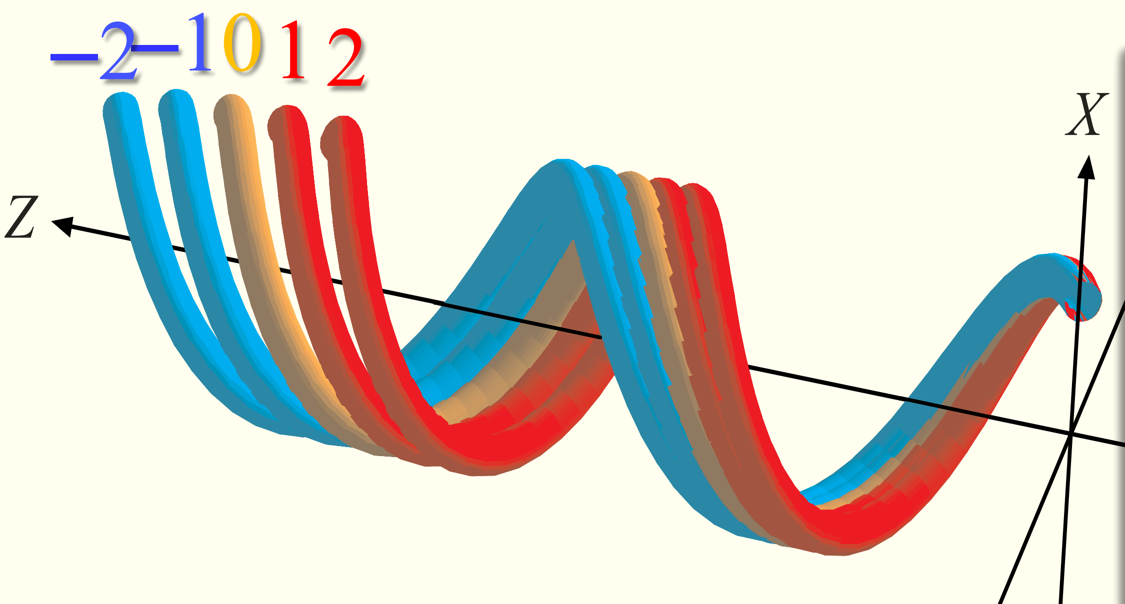
– Berry phase

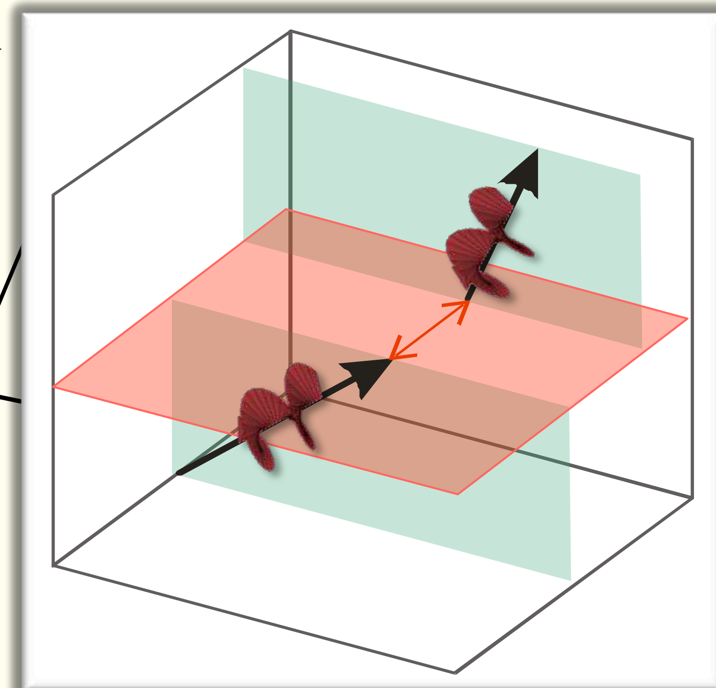


Bliokh, PRL 2006; Alexeyev, Yavorsky, JOA 2006;
Segev et al., PRL 1992; Kataevskaya, Kundikova, 1995

Orbital-Hall effect of light

$$\sigma \to \sigma + \ell$$ – vortex-dependent shifts





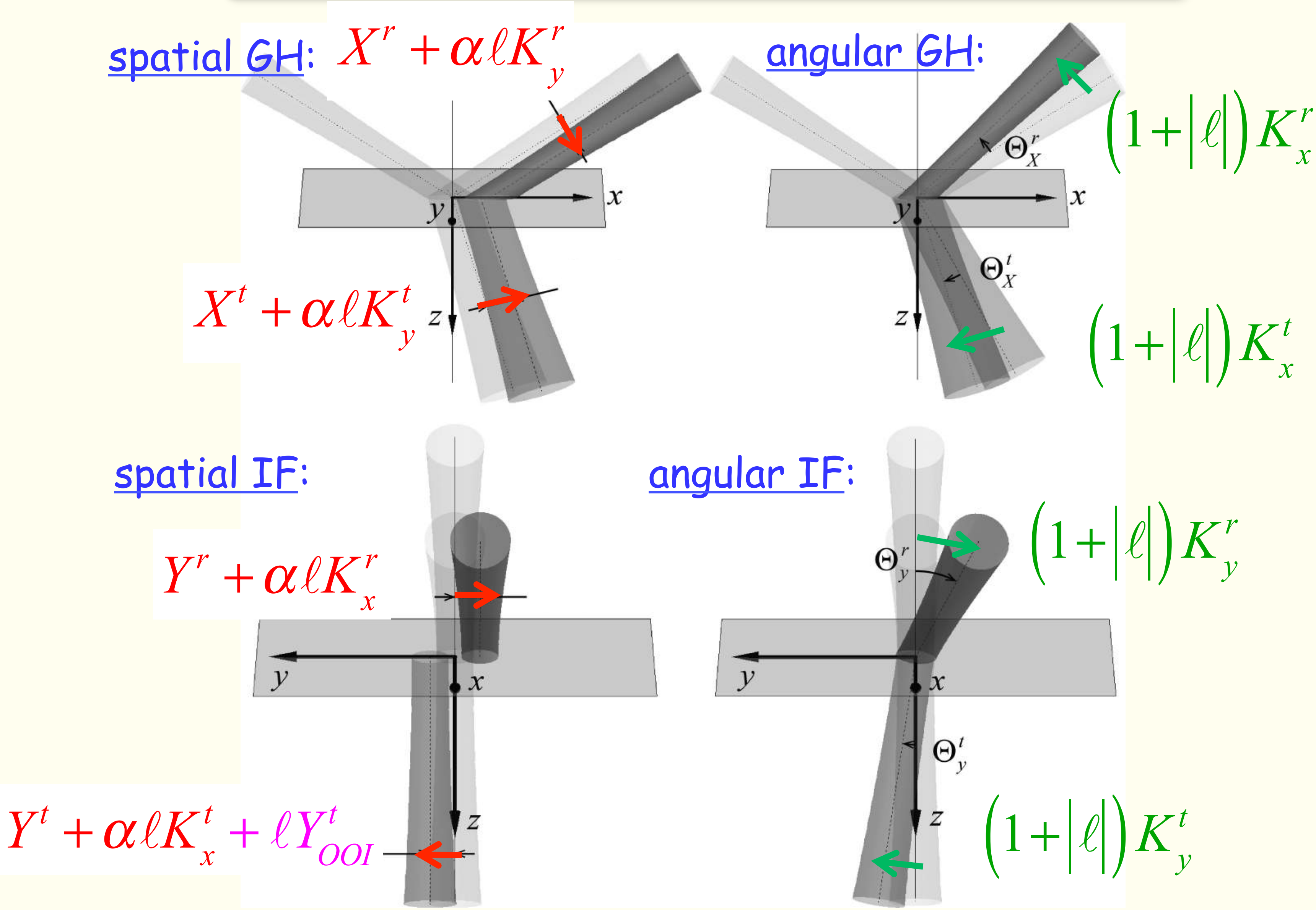
$$\mathbf{J} = \mathbf{r}_c \times \mathbf{k}_c + \left(\sigma + \ell\right)\boldsymbol{\kappa}_c$$

– AM conservation

Bliokh, PRL 2006

Fedoseyev, Opt. Commun. 2001;
Dasgupta & Gupta, ibid. 2006

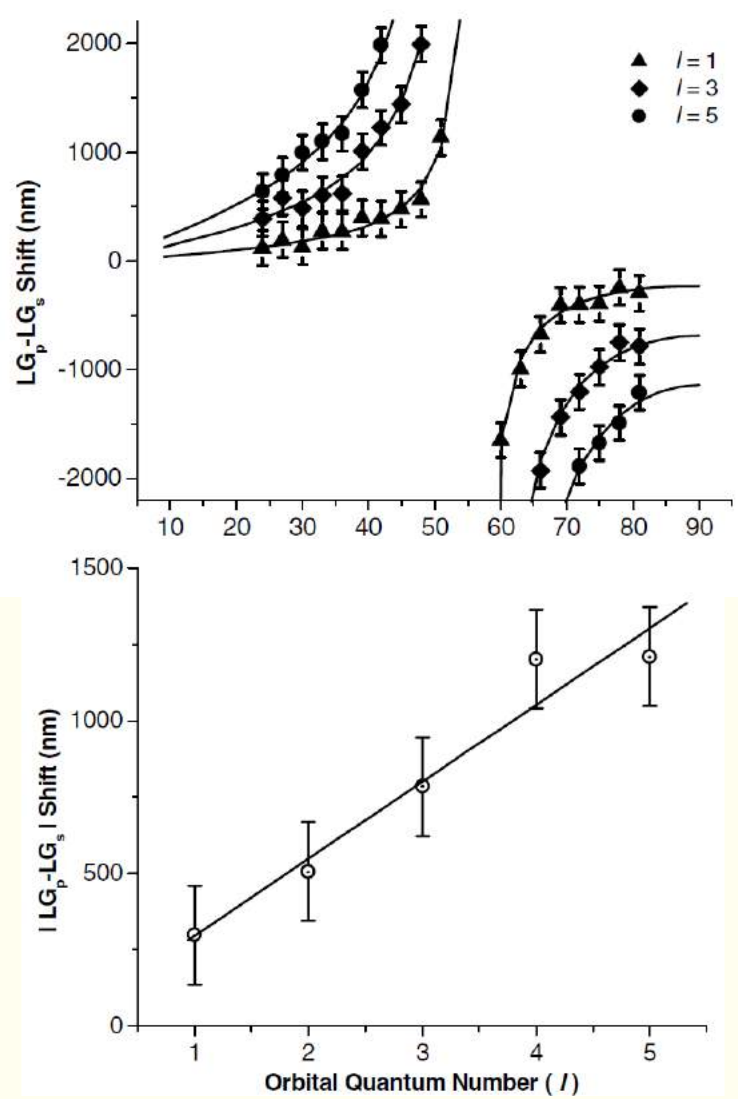
GH and IF shifts for vortex beams

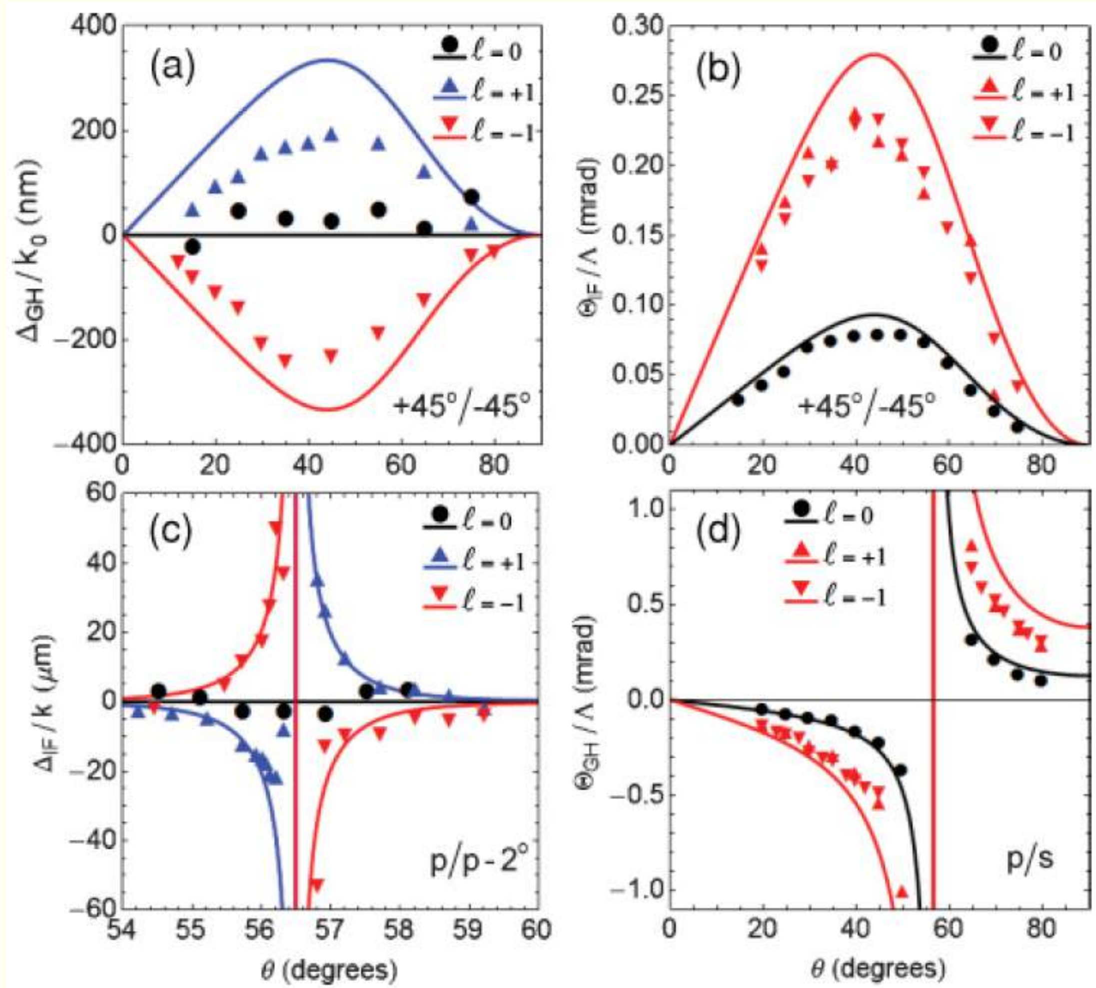


Bliokh et al. Opt. Lett. 2009 (theory); *Merano et al., PRA 2010* (exper.)

Reflection/refraction at an interface

Experimental measurements of the vortex beam shifts:





Fedoseyev, 2001; Dasgupta & Gupta, 2006

Bliokh et al. 2009; Merano et al., 2010

- **Spin-orbit interaction in paraxial beams**
  - **Propagation in gradient-index media**
  - **Reflection/transmission at an interface**
- **Spin-orbit coupling in nonparaxial fields**
  - **Spin and orbital AM in free space**
  - **Focusing, scattering, imaging**
- **Spin-orbit coupling in inhomogeneous anisotropic structures (metasurfaces)**
- **Transverse spin-momentum locking**

Spin and orbital AM in free space

We have shown that the spin Hall effect is related to the transverse Berry-phase gradient of the plane waves forming the beam.

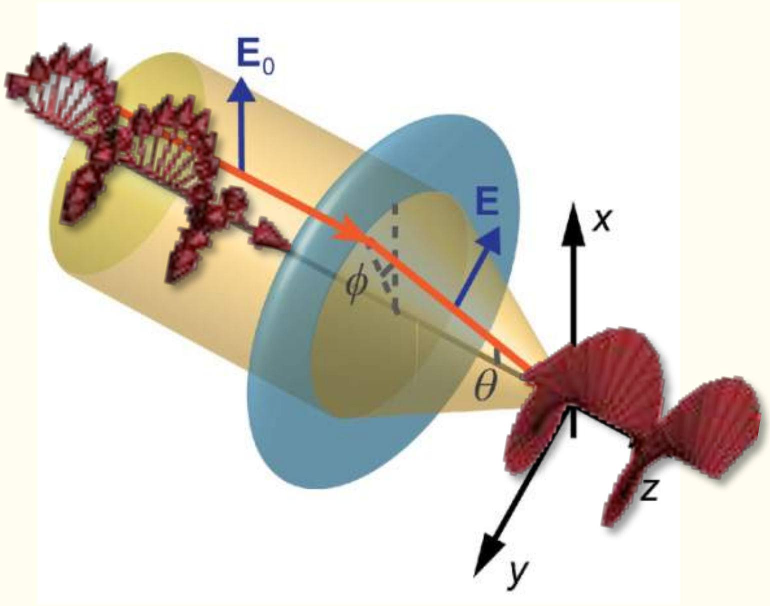
Furthermore, the main scale of the effect is the wavelength.

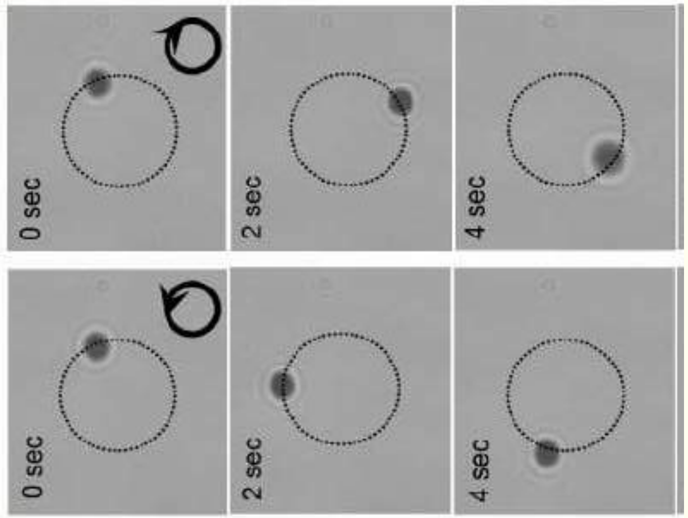
Therefore, it is natural to expect that the SOI phenomena will become more significant in **nonparaxial** (e.g., tightly focused) fields.

And, indeed, some strong spin-dependent AM effect occurs in tightly focused light:

Spin and orbital AM in free space

Focusing of a circularly polarized light with SAM results in vortex component in the 3D nonparaxial field and nonzero OAM proportional to σ. This is **spin-to-orbital AM conversion**:





Y. Zhao et al., PRL 2007

It is related to fundamental peculiarities of the photon AM operators.

Bliokh et al., PRA 2010

Spin and orbital AM in free space

The total AM operator for photon is a sum of the OAM and SAM operators:

$$\boxed{\hat{\mathbf{J}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} + \hat{\mathbf{S}} \equiv \hat{\mathbf{L}} + \hat{\mathbf{S}}}$$

$$\hat{\mathbf{r}} = i\partial_{\mathbf{k}}, \quad \hat{\mathbf{p}} = \mathbf{k},$$

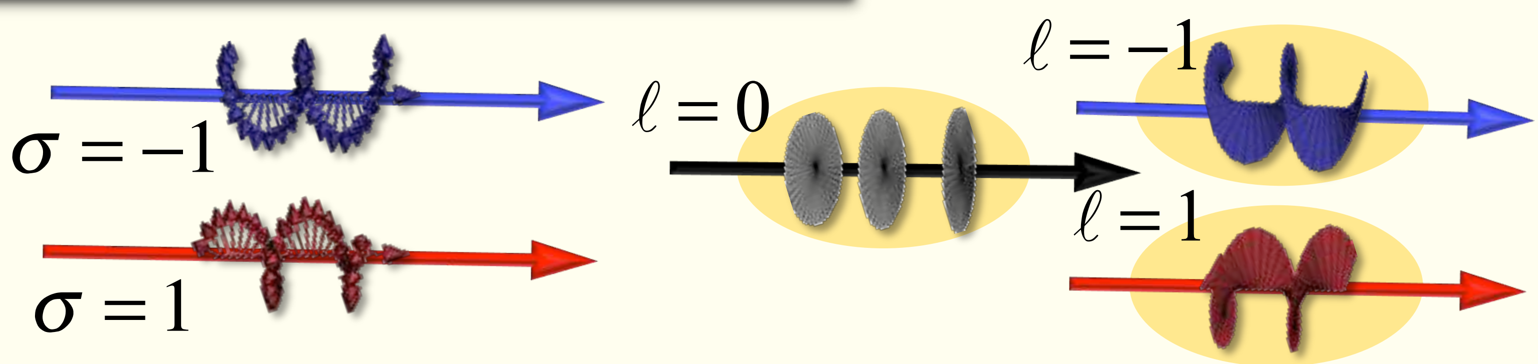
$$\left(\hat{S}_a\right)_{ij} = -i\varepsilon_{aij}$$

$$\boxed{\begin{gathered} \hat{L}_z = -i\partial_\phi, \ \left(\hat{S}_z\right)_{ij} = -i\varepsilon_{zij} \\ \mathbf{E}_{\ell\sigma} \propto \left(\overline{\mathbf{x}} + i\sigma\overline{\mathbf{y}}\right)e^{i\ell\phi} \end{gathered}}$$

– z-components and paraxial eigenmodes:

s- polarization,

ℓ - vortex



Spin and orbital AM in free space

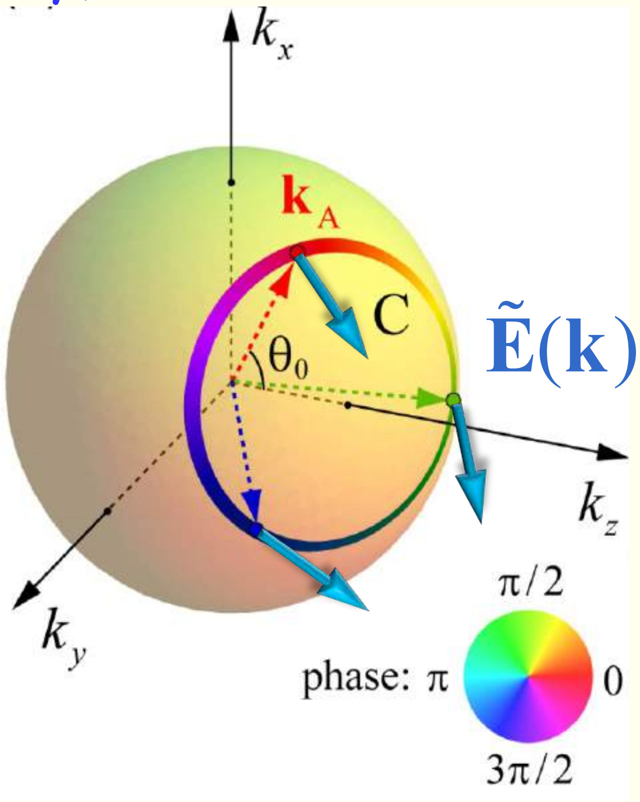
$$\boxed{\tilde{\mathbf{E}}_{\ell\sigma} \propto \left(\bar{\boldsymbol{\theta}} + i\sigma\bar{\boldsymbol{\phi}}\right)e^{i\sigma\phi}e^{i\ell\phi}} \equiv \mathbf{e}^{\sigma}\left(\mathbf{k}\right)e^{i\ell\phi}$$ – nonparaxial vortex beam

Bessel-beams spectrum - a circle in **k**-space (we assume that all the waves have the same helicity):

$$\boxed{\tilde{E}_{\ell}^{\sigma} = A^{\sigma}\delta\left(\theta - \theta_0\right)e^{i\ell\phi}}$$

real space field:

$$\mathbf{E}_{\ell}^{\sigma} \propto A^{\sigma}\begin{pmatrix} \frac{1+\sigma}{2}J_{\ell}(\xi) - \sigma b\, e^{i(\sigma-1)\varphi}J_{\ell+\sigma-1}(\xi) \\ \frac{1-\sigma}{2}J_{\ell}(\xi) + \sigma b\, e^{i(\sigma+1)\varphi}J_{\ell+\sigma+1}(\xi) \\ -i\sigma\sqrt{2ab}\,e^{i\sigma\varphi}J_{\ell+\sigma}(\xi) \end{pmatrix} e^{ik_{\parallel}z + i\ell\varphi},$$



Spin and orbital AM in free space

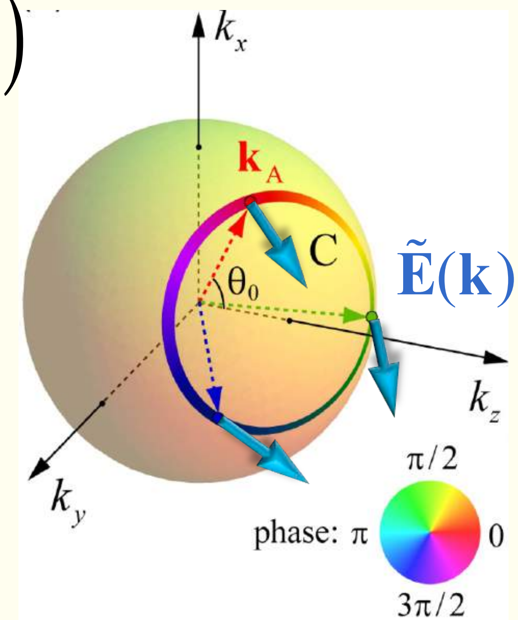
Calculating the SAM and OAM expectation values in the Bessel beam, $\langle \mathbf{O} \rangle = \langle \tilde{E}^{\sigma} | \hat{\mathbf{O}} | \tilde{E}^{\sigma} \rangle$, we arrive at:

$$\boxed{\langle S_z \rangle = \sigma(1 - \Phi_B), \quad \langle L_z \rangle = \ell + \sigma \Phi_B}$$

$$\Phi_B \equiv 2\pi \Phi_B = \oint_C \mathbf{A}_B \cdot d\mathbf{k} = 2\pi(1 - \cos\theta_0)$$

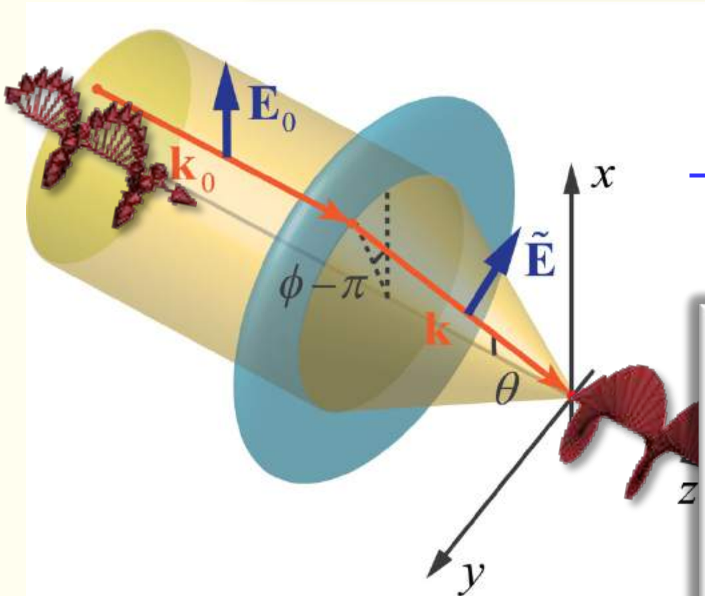
– Berry phase !

Thus, the spin-to-orbit AM conversion in nonparaxial fields originates from the Berry phase associated with the azimuthal distribution of partial waves.



Bliokh et al., PRA 2010

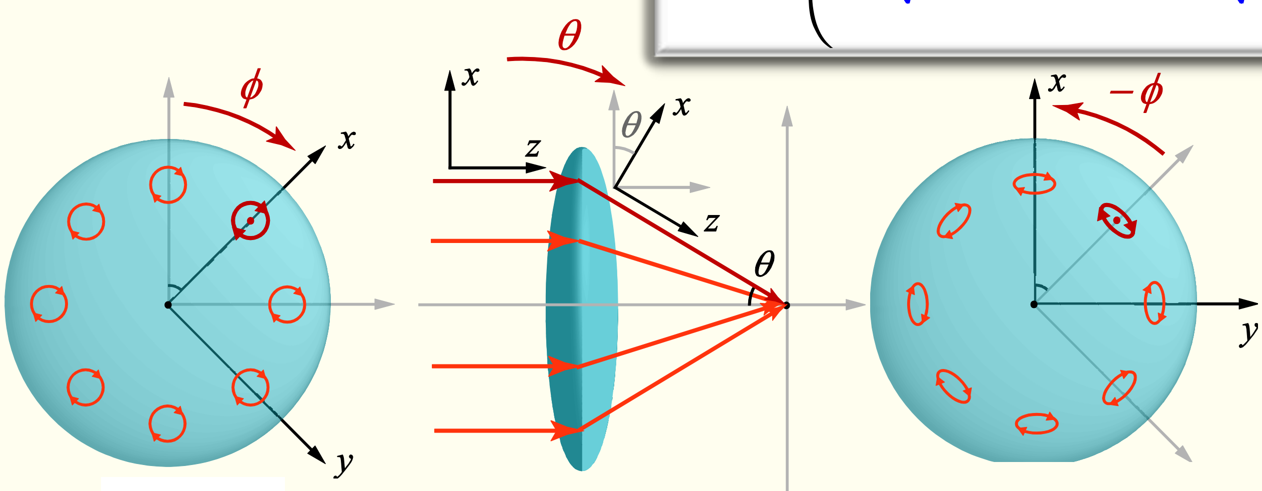
Spin-to-orbit conversion upon focusing



$$\tilde{\mathbf{E}}(\theta,\phi) \propto \sqrt{\cos\theta}\,\hat{U}(\theta,\phi)\mathbf{E}_0$$

– Richards-Wolf theory (1959) = *parallel transport*

$$\hat{U} = \begin{pmatrix} a & -be^{-2i\phi} & \sqrt{2ab}e^{-i\phi} \\ -be^{2i\phi} & a & \sqrt{2ab}e^{i\phi} \\ -\sqrt{2ab}e^{i\phi} & -\sqrt{2ab}e^{-i\phi} & a-b \end{pmatrix}$$



$\hat{R}_z(\phi)$ $\hat{R}_y(\theta)$ $\hat{R}_z(-\phi)$

$$b = \Phi_B(\theta)/2$$

– *Berry phase*

Bliokh et al., OE 2011

Spin and orbital AM in free space

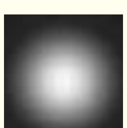
All basic manifestations of the SOI of light can be immediately seen in the spin-dependent real-space intensity distribution of the field.

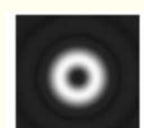
The intensity of the circularly-polarized vector Bessel beam reflects the spin-to-orbital AM conversion:

$$I_\ell^\sigma(\rho) \propto \left|A^\sigma\right|^2 \left[a^2 J_\ell^2(\tilde{\rho}) + b^2 J_{\ell+2\sigma}^2(\tilde{\rho}) + 2ab J_{\ell+\sigma}^2(\tilde{\rho})\right]$$

$$(b = \Phi/2,\ a = 1 - \Phi/2)$$

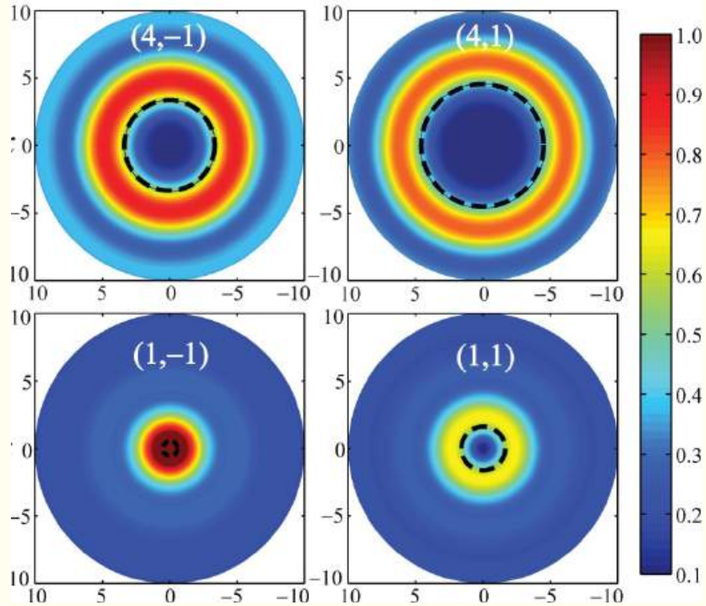
$$\left|\ell,\sigma\right\rangle \to \left[a\left|\ell,\sigma\right\rangle - b\left|\ell+2\sigma,-\sigma\right\rangle - \sqrt{2ab}\left|\ell+\sigma,0\right\rangle\right]$$

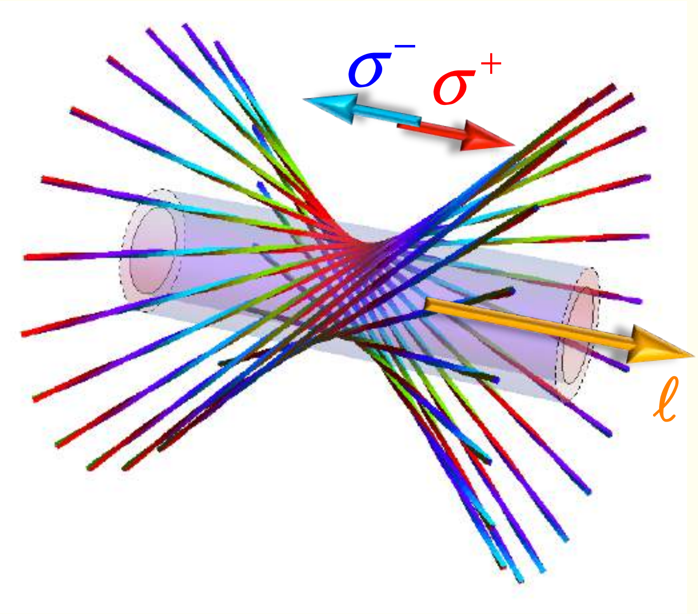


Spin and orbital AM in free space

The transverse intensity distributions show σ-dependent radii: $\sigma = -1$ $\sigma = 1$





The beam radius can be can be obtained from the *quantization (with Berry phase)* and ***fine SOI splitting of the caustic:***

$$\boxed{k_\perp R_\ell^\sigma = \left|\ell + \sigma\Phi\right| = \left|\langle L_z \rangle\right|}$$

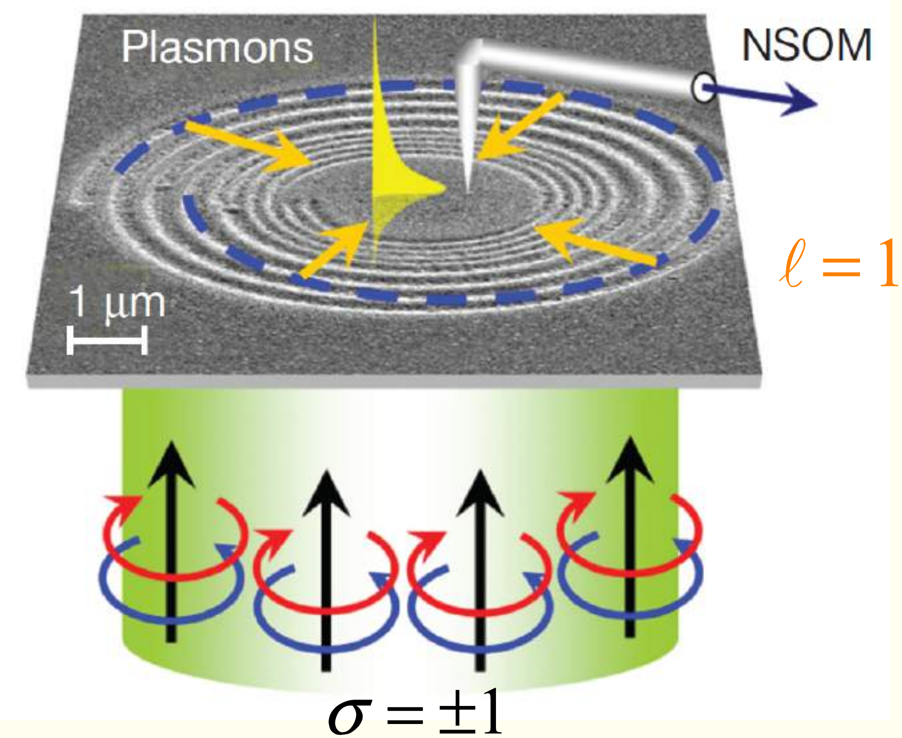
Bliokh et al. PRA 2010

Spin and orbital AM in free space

The spin-dependent radius can be demonstrated in a circular plasmonic cavity generating Bessel modes:

$\underline{\theta_0 = \pi/2}$, $\underline{\Phi_B = 2\pi}$

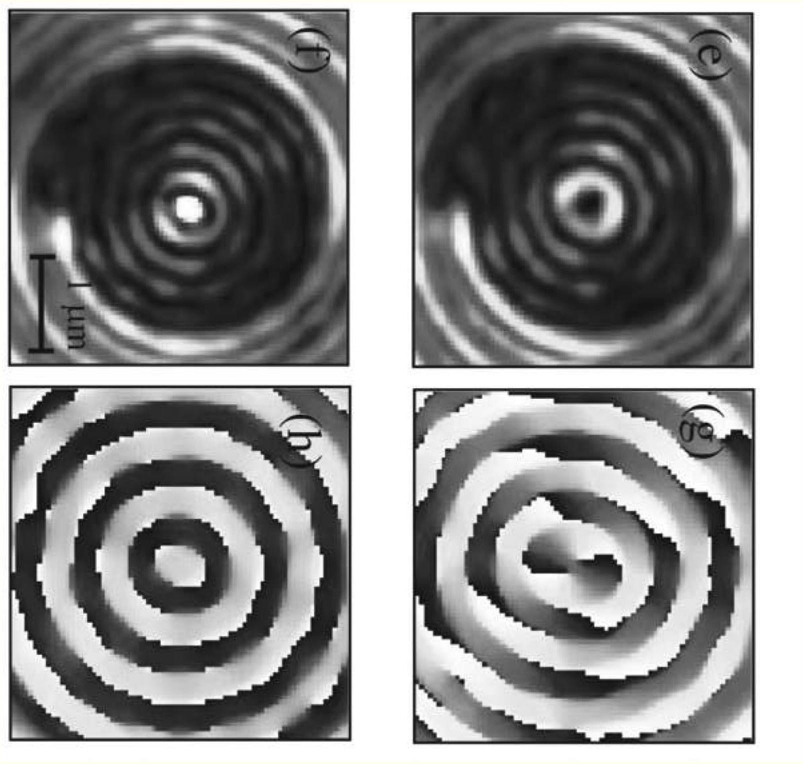
$$\boxed{\ell + \sigma\Phi_B = \ell + \sigma}$$



$\ell = 1$

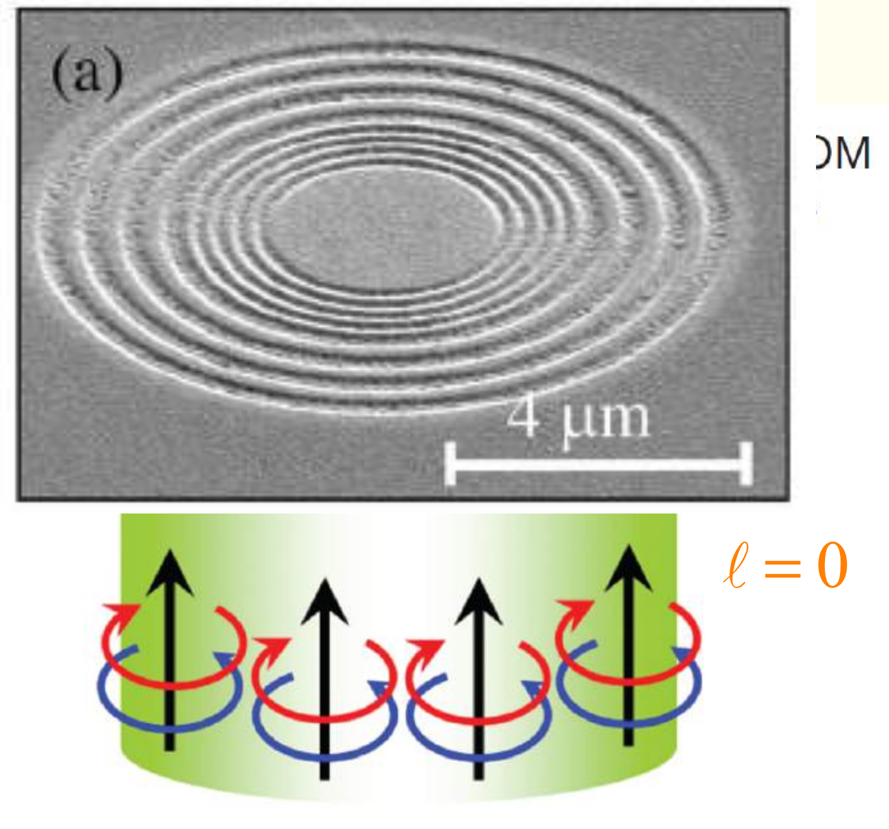
$\sigma = \pm 1$

$\sigma = -1$ $\sigma = 1$



Y. Gorodetski et al., PRL (2008) [cf. QHE in graphene].

Spin and orbital AM in free space

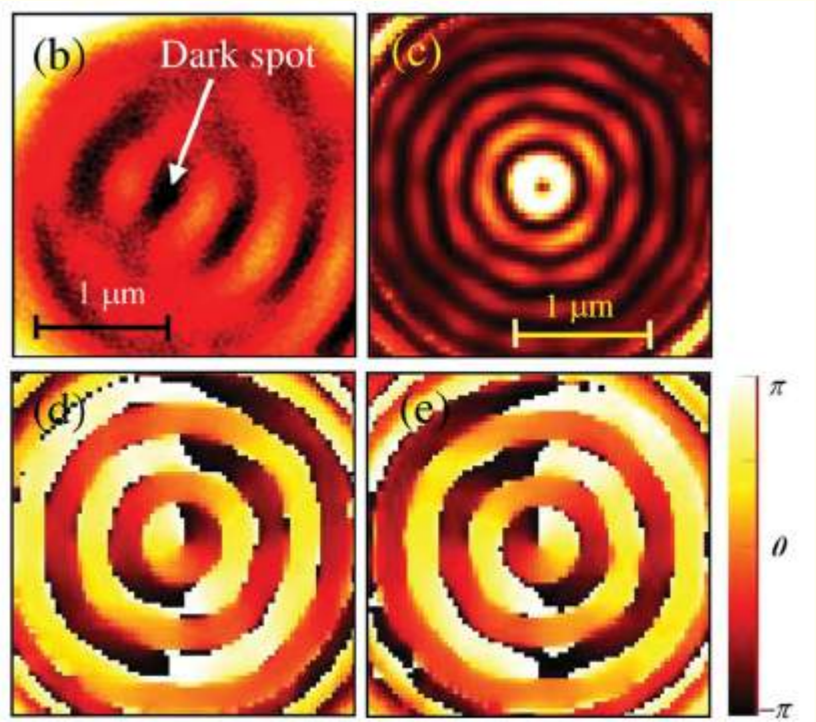


$\ell = 0$

$\sigma = \pm 1$

$$\ell + \sigma \Phi_B = \ell + \sigma$$

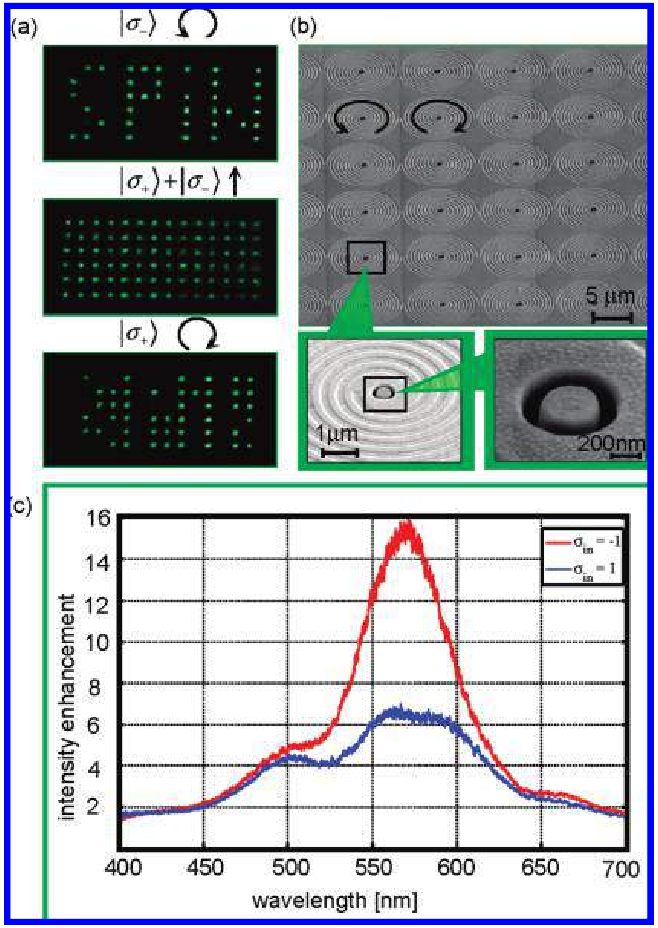
$\sigma = -1$ $\sigma = 1$

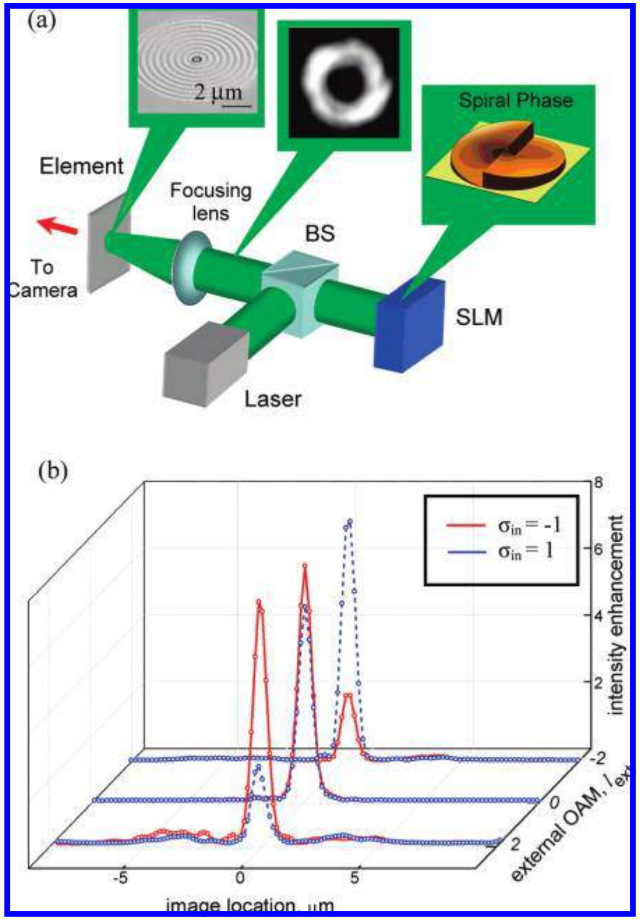


Y. Gorodetski et al., PRL (2008)

Spin and orbital AM in free space

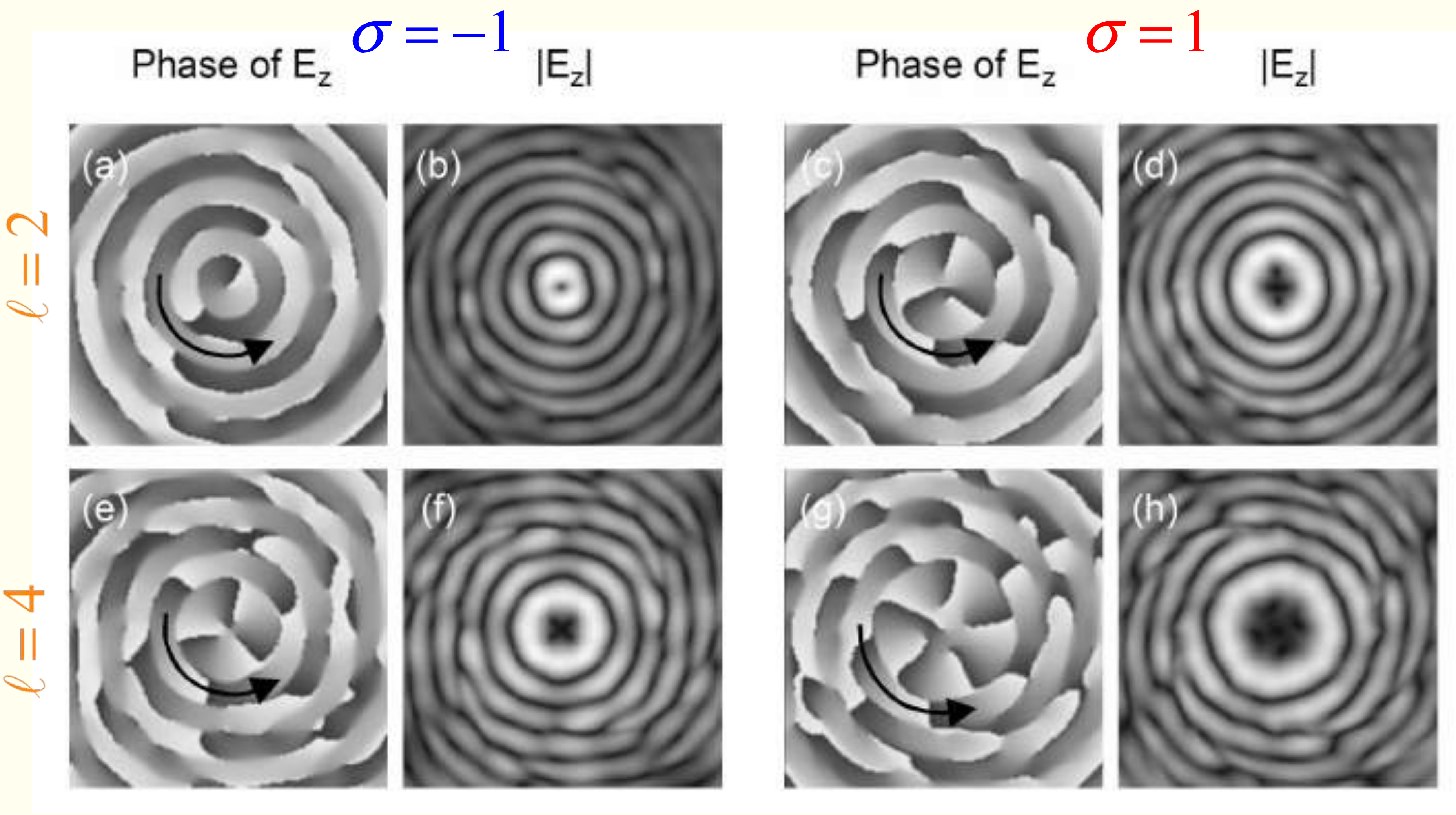
Interesting application to spin-dependent resonant transmission [Ebbesen et al. + Zheludev et al.]:





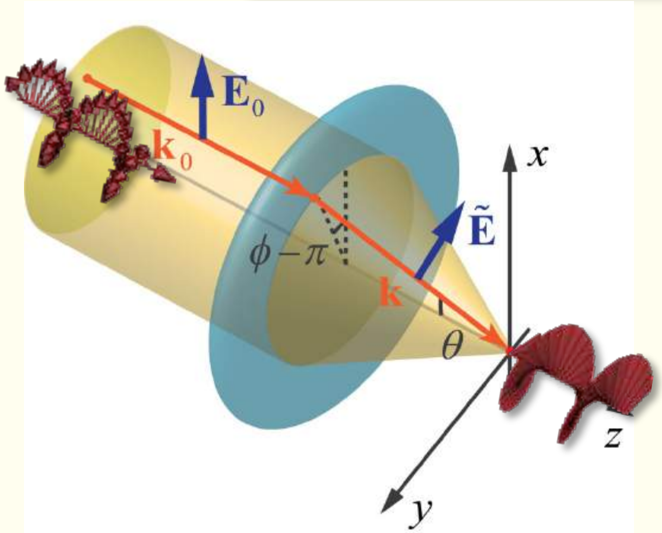
Gorodetski et al. Nano Lett. 2009

Spin and orbital AM in free space



Ohno & Miyanishi, OE 2006

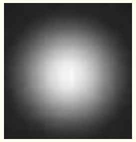
Focusing, scattering, and imaging



The same spin-to-orbital AM conversion occurs in a high-NA focusing, with the same azimuthal Berry phase and spin-dependent intensity.
The only difference is θ-averaging.

$$\langle S_z \rangle = \sigma\left(1 - \bar{\Phi}_B\right), \quad \langle L_z \rangle = \ell + \sigma\bar{\Phi}_B$$

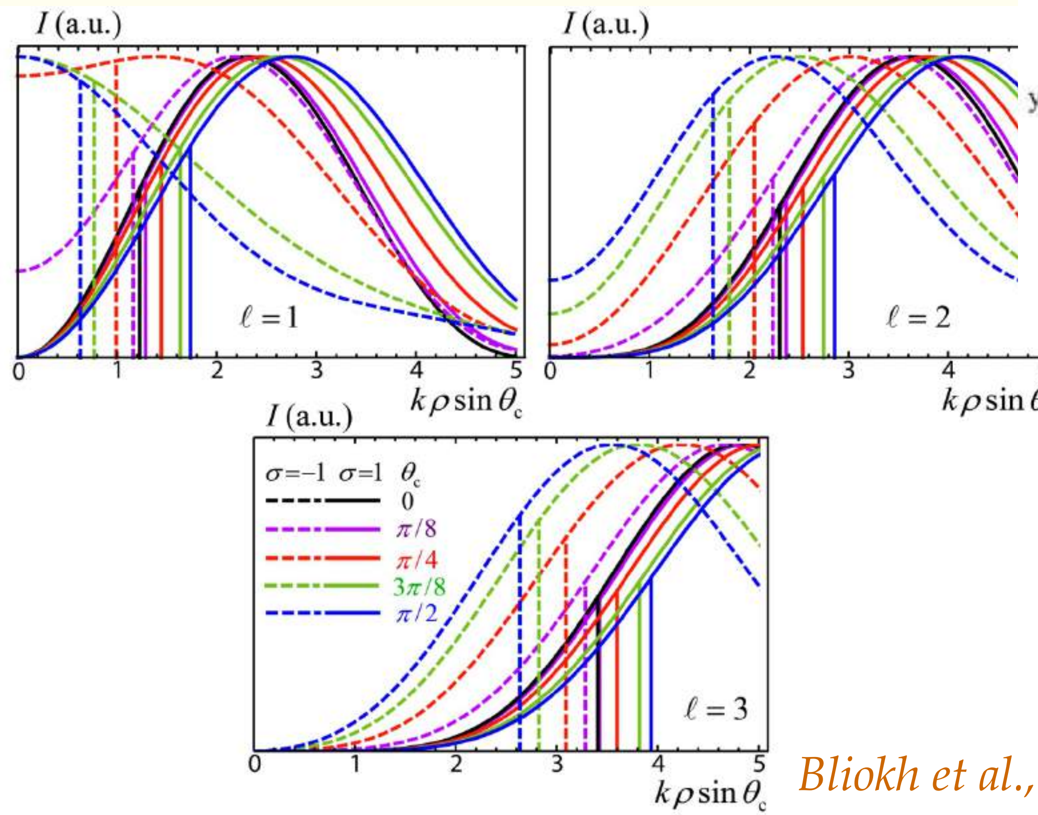
$$I_\ell^\sigma(\rho,z) \propto \left\langle a J_\ell(\tilde{\rho})\right\rangle^2 + \left\langle b J_{\ell+2\sigma}(\tilde{\rho})\right\rangle^2 + 2\left\langle \sqrt{ab}\, J_{\ell+\sigma}(\tilde{\rho})\right\rangle^2$$

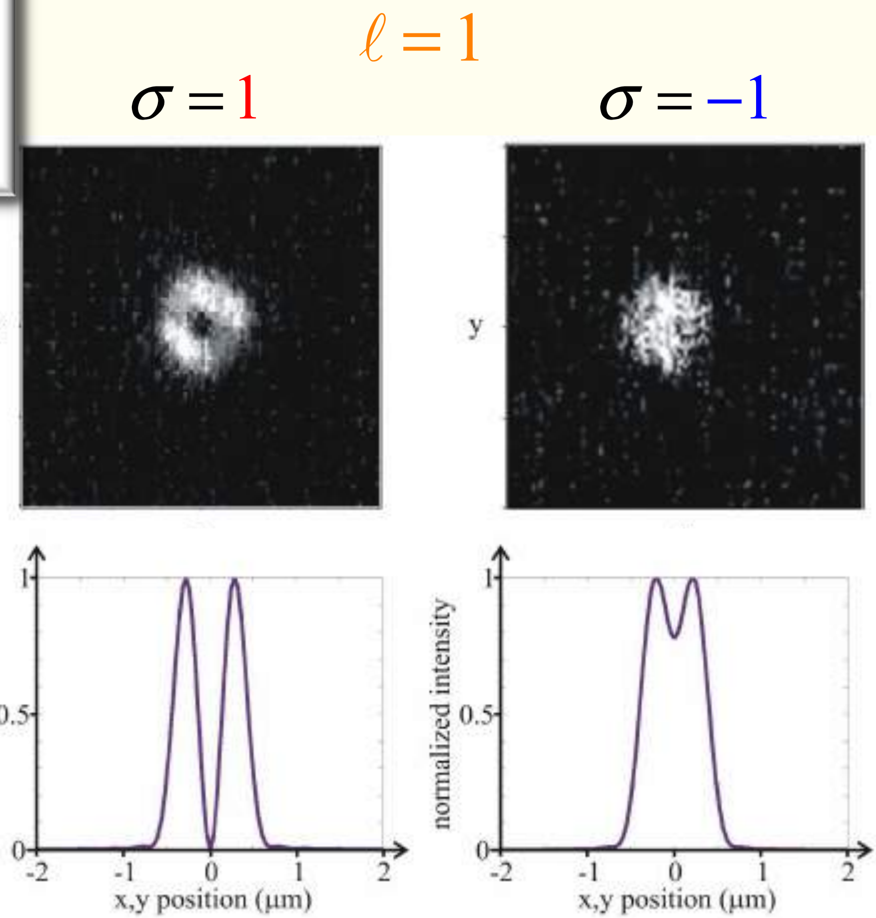
   

Focusing, scattering, and imaging

Spin-dependent intensity and radius of focal spot:

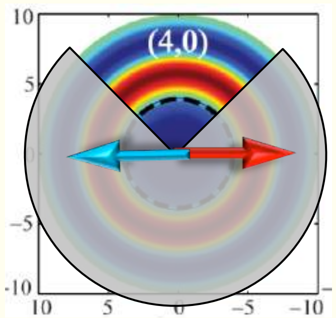
$$R_\ell^\sigma \simeq \frac{\langle L_z \rangle}{k \sin \bar{\theta}} = \frac{\ell + \sigma \left(1 - \cos \bar{\theta}\right)}{k \sin \bar{\theta}}$$





Bliokh et al., OE 2011; Bokor et al., OE 2005

Spin and orbital Hall effects



– azimuthally truncated field (symmetry breaking along x)

$$\phi \in (-\delta, \delta)$$

B. Zel'dovich et al. (1994), K.Y. Bliokh et al. (2008)

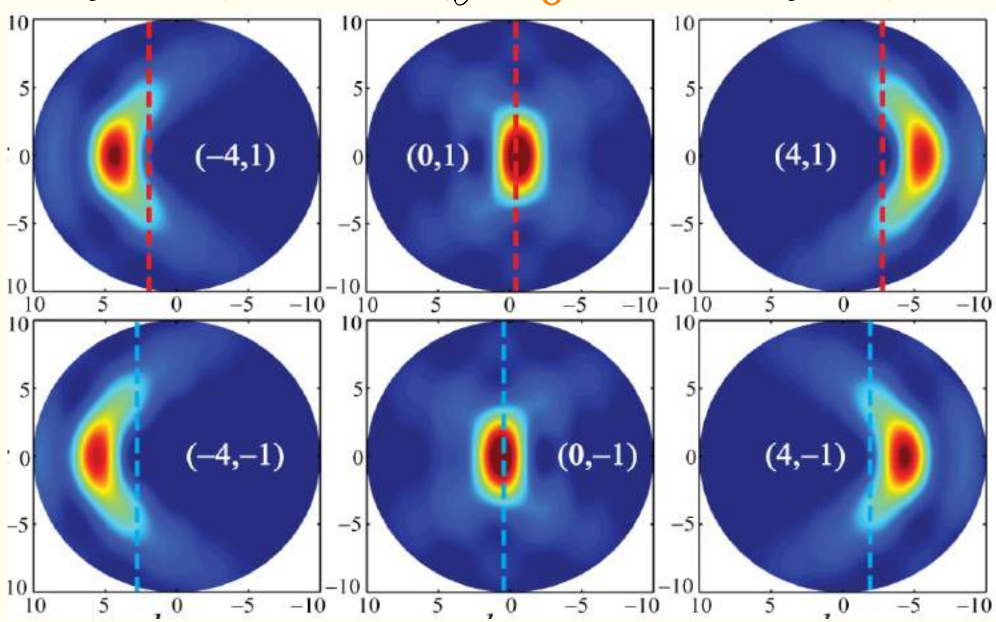
$$\Rightarrow \quad \boxed{k_\perp Y_\ell^\sigma = -\gamma\left(\ell + \sigma\Phi_B\right)}$$

– orbital and spin Hall effects of light

$\ell = -4$ $\quad$ $\ell = 0$ $\quad$ $\ell = 4$

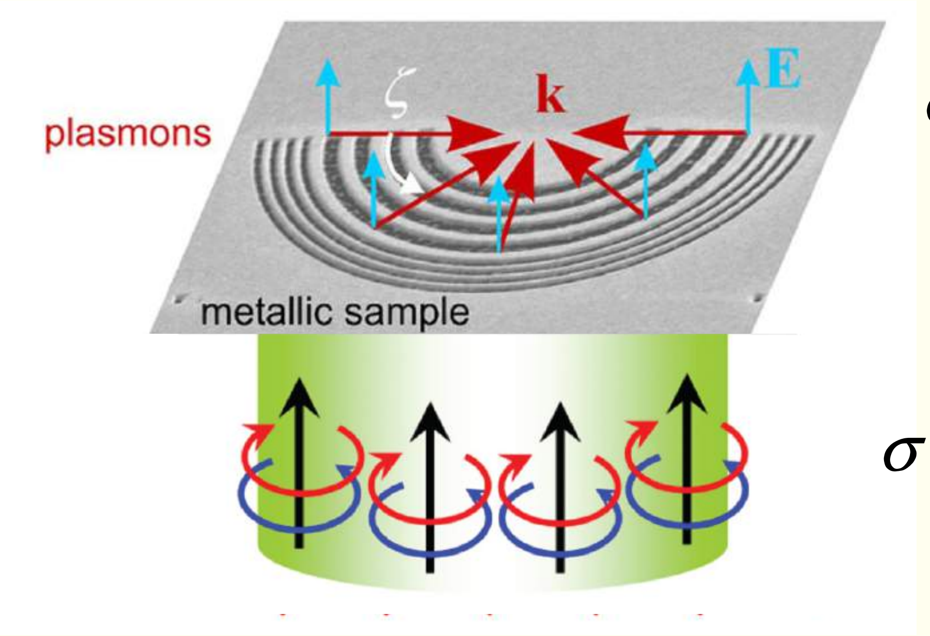
$\sigma = 1$

$\sigma = -1$



Plasmonic experiment

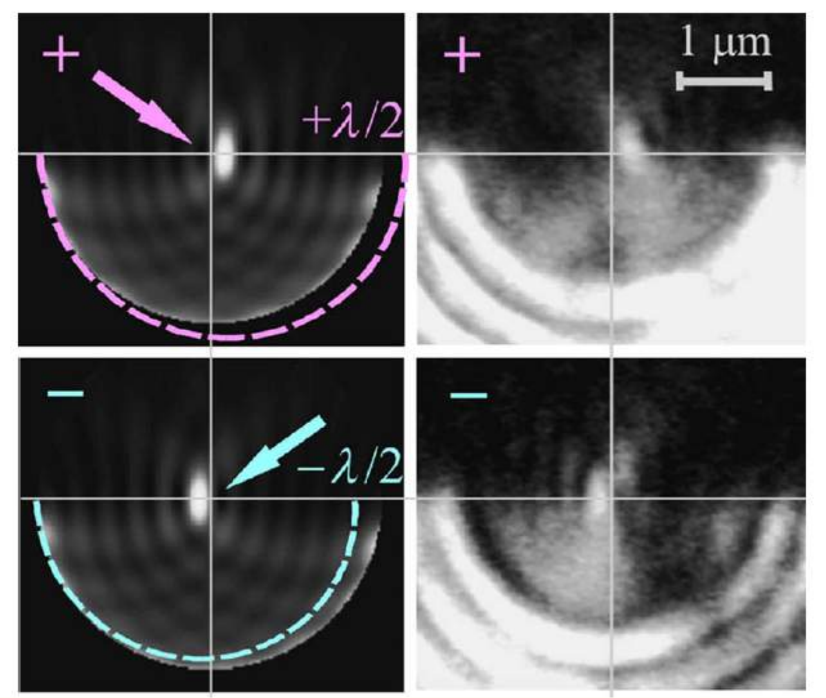
Plasmonic half-lens produces spin Hall effect:



$\ell = 0$

$\sigma = 1$

$\sigma = -1$



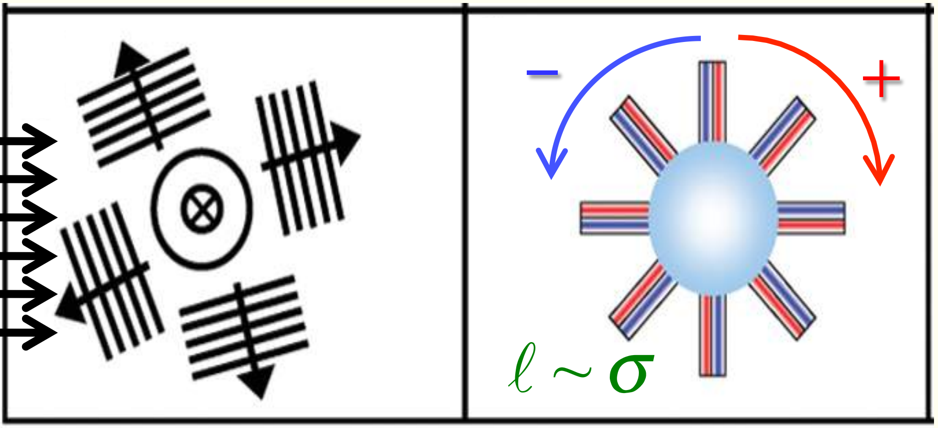
$$Y \sim \sigma ƛ$$

K. Y. Bliokh et al., PRL (2008)

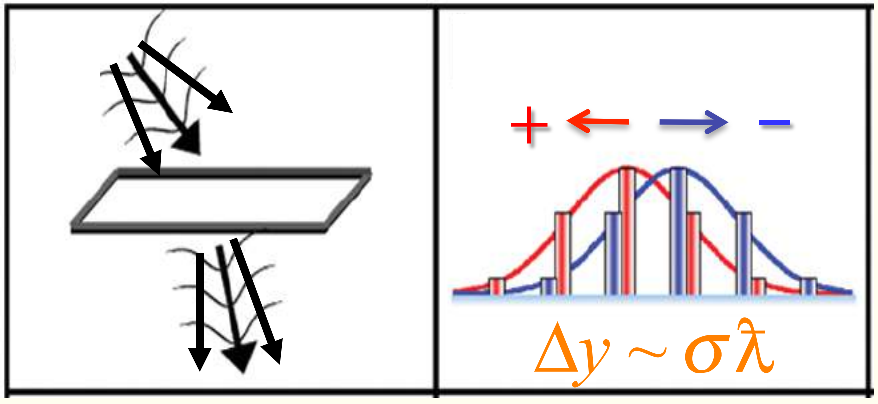
Spin and orbital AM in free space

The SOI of light is a coupling between SAM and OAM. It is caused by the **k**-space distribution and geometric interference of plane waves.

1. In cylindrical geometry polarization → vortex: spin-to-orbit AM conversion



2. In asymmetric fields polarization → position: vortex → position: spin, orbital Hall effects



Summary of the SOI in nonparaxial fields

- Modified SAM, OAM, and coordinate operators compatible with the transversality
- Berry phase terms: Spin-dependent OAM and coordinates
- Spin-to-orbital AM conversion from the Berry phase between interfering plane waves
- Quantization of the beam radius (caustic)
- Spin and orbital Hall effects in asymmetric fields

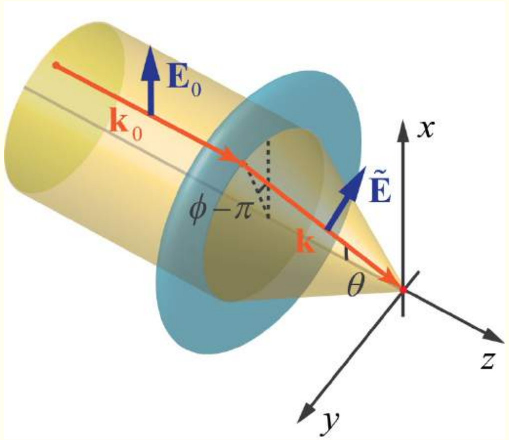
Focusing, scattering, and imaging

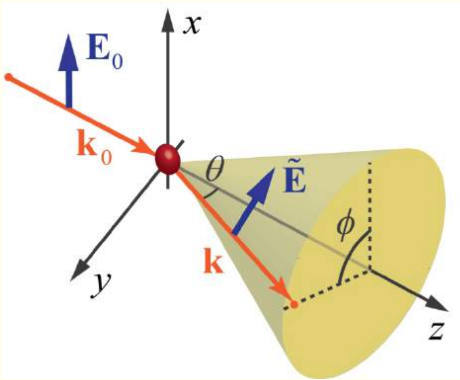
These results can be applied to a number of systems involving nonparaxial fields:

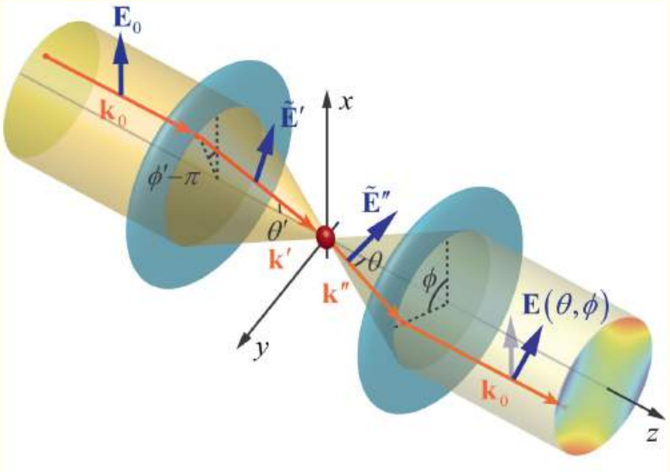
1) Tight focusing by high-NA lens

2) Scattering by small particles

3) High-NA microscopy and imaging

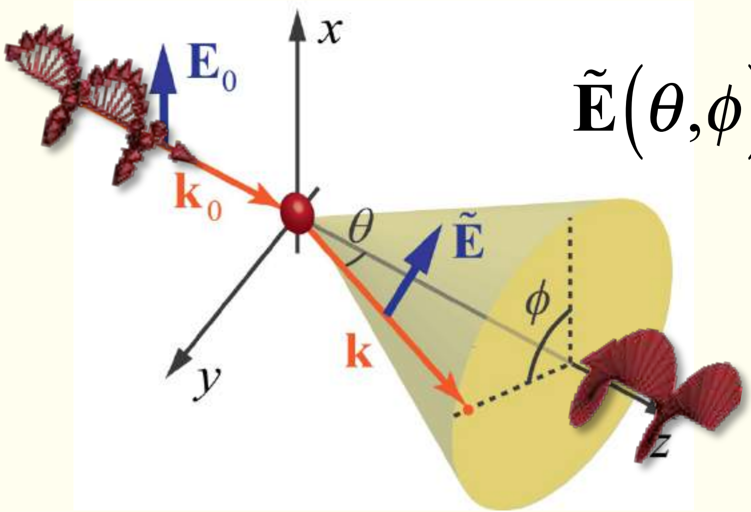






Focusing, scattering, and imaging

Similar redistribution of waves on the sphere in **k**-space occurs upon scattering by a small particle:



$$\tilde{\mathbf{E}}(\theta,\phi) \propto -\hat{\mathbf{r}}\times\left[\hat{\mathbf{r}}\times\mathbf{E}_0(\mathbf{0})\right] = \hat{\Pi}(\theta,\phi)\mathbf{E}_0(\mathbf{0})$$

– dipole (Rayleigh) scattering

= *spherical projection:* $\hat{\Pi} = \hat{U}\hat{\mathrm{P}}_z\hat{U}^\dagger$

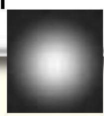
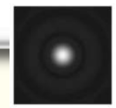
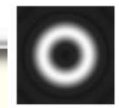
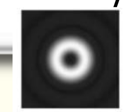
Bliokh et al., 2010, 2011

$$\hat{\Pi} = \frac{1}{2}\begin{pmatrix} 1+a_1 & -b_1 e^{-2i\phi} & -\sqrt{2a_1b_1}\,e^{-i\phi} \\ -b_1 e^{2i\phi} & 1+a_1 & -\sqrt{2a_1b_1}\,e^{i\phi} \\ -\sqrt{2a_1b_1}\,e^{i\phi} & -\sqrt{2a_1b_1}\,e^{-i\phi} & 2b_1^2 \end{pmatrix}$$

Focusing, scattering, and imaging

The same spin-to-orbital AM conversion, but helicity is not conserved:

$$\left|0,\sigma\right\rangle \to \frac{1}{2}\left[(1+a_1)\left|0,\sigma\right\rangle - b_1\left|2\sigma,-\sigma\right\rangle - \sqrt{2a_1b_1}\left|\sigma,0\right\rangle\right]$$

$$l_z = \sigma\frac{1-\cos^2\theta}{1+\cos^2\theta}, \qquad s_z = \sigma\frac{2\cos^2\theta}{1+\cos^2\theta}$$

- OAM and SAM θ-densities:

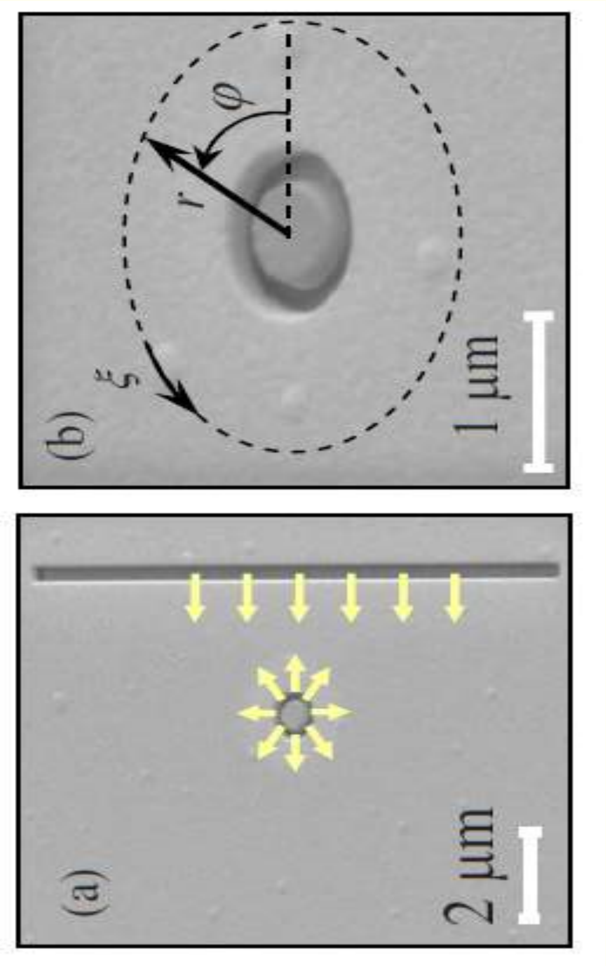
Integrating over all angles θ, we obtain:

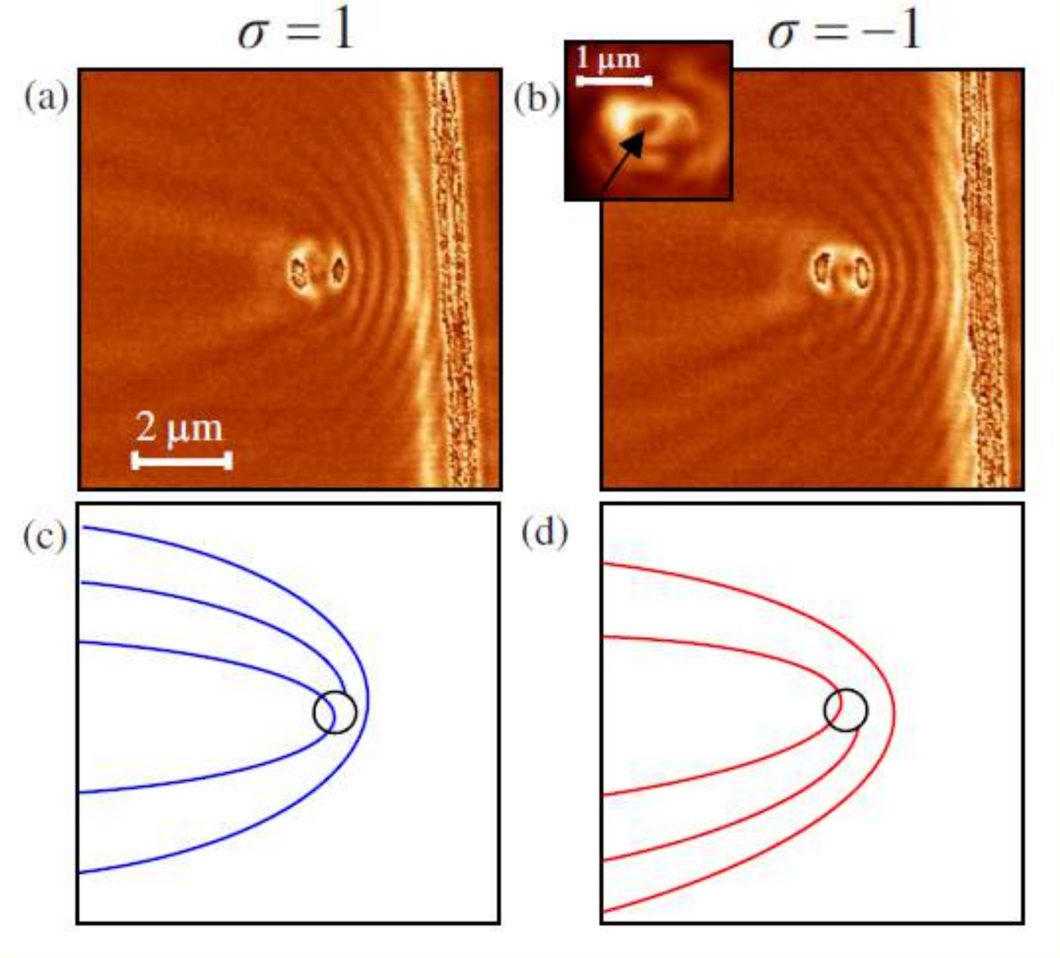
$$\left\langle L_z\right\rangle = \frac{1}{2}\sigma, \quad \left\langle S_z\right\rangle = \frac{1}{2}\sigma$$

Bliokh et al., 2011; Moe & Happer, 1977

Focusing, scattering, and imaging

"Plasmonic Aharonov-Bohm effect" :

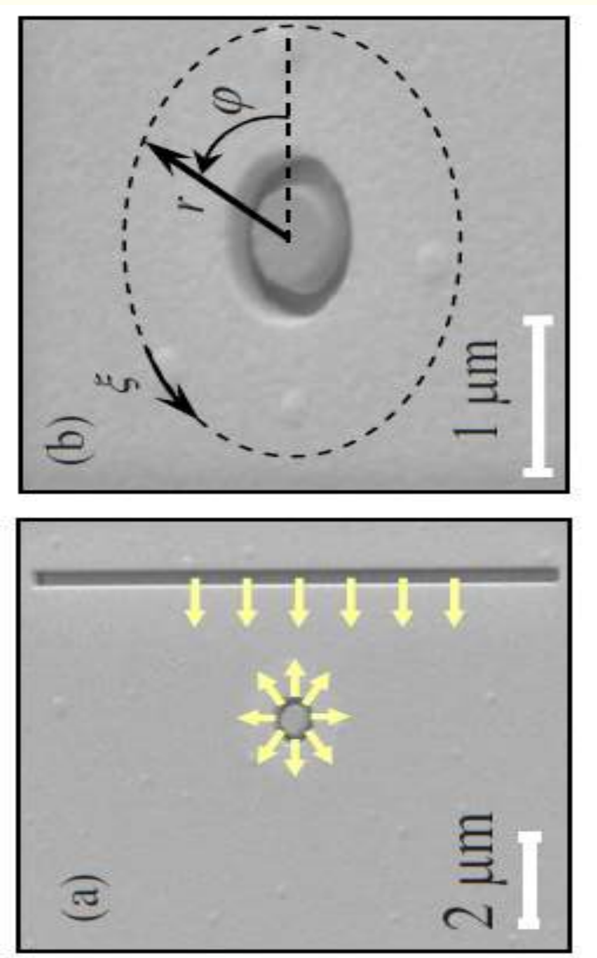


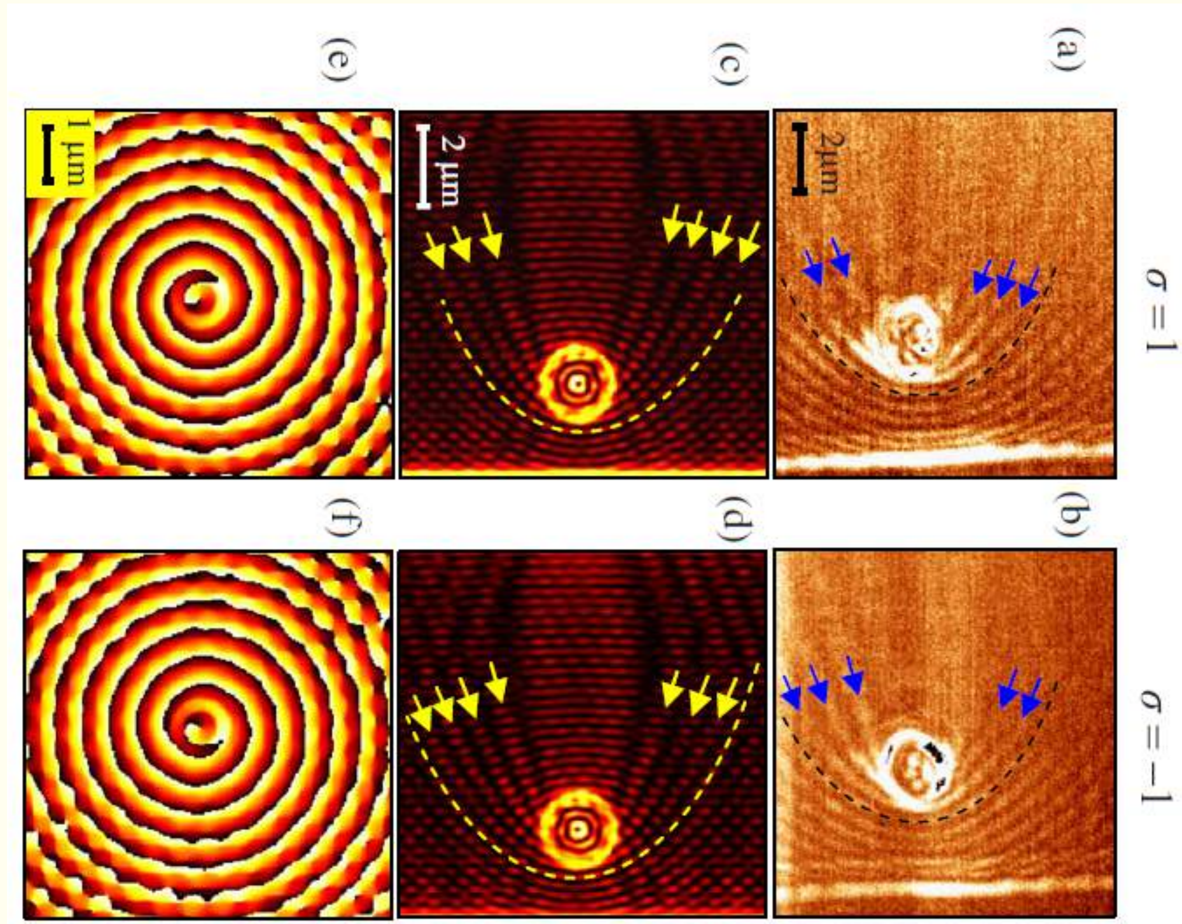


Gorodetski et al., PRB 2010

Focusing, scattering, and imaging

"Plasmonic Aharonov-Bohm effect" :

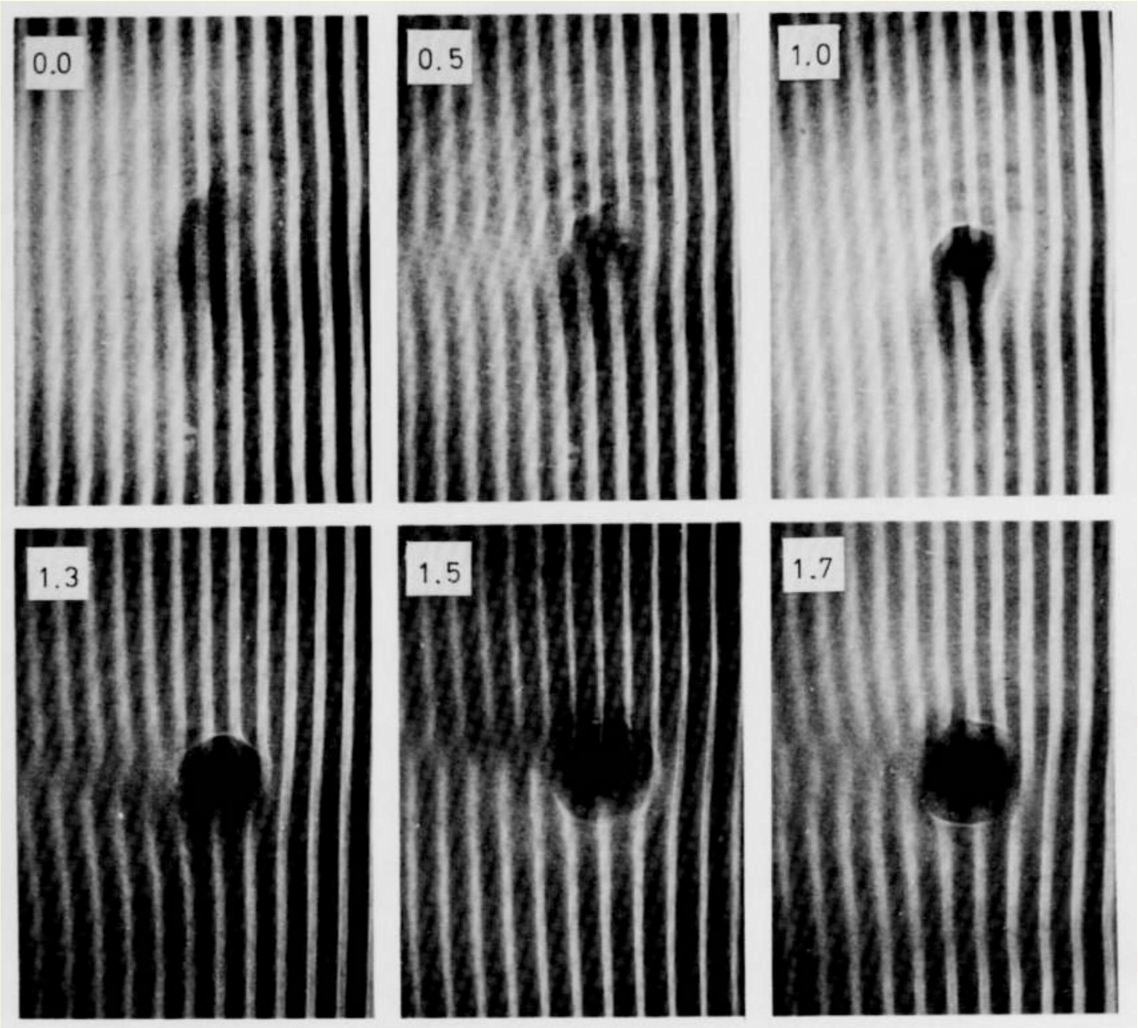


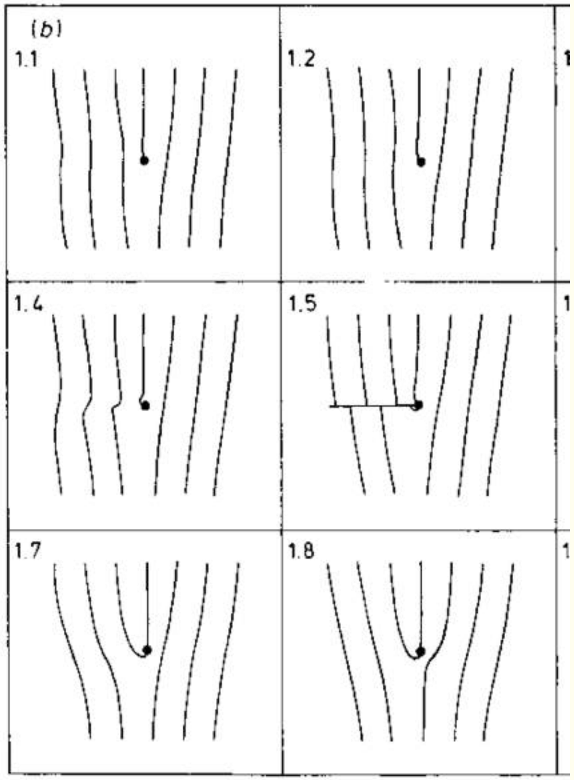


Gorodetski et al., PRB 2010

Focusing, scattering, and imaging

Cf. the water-wave analogue:

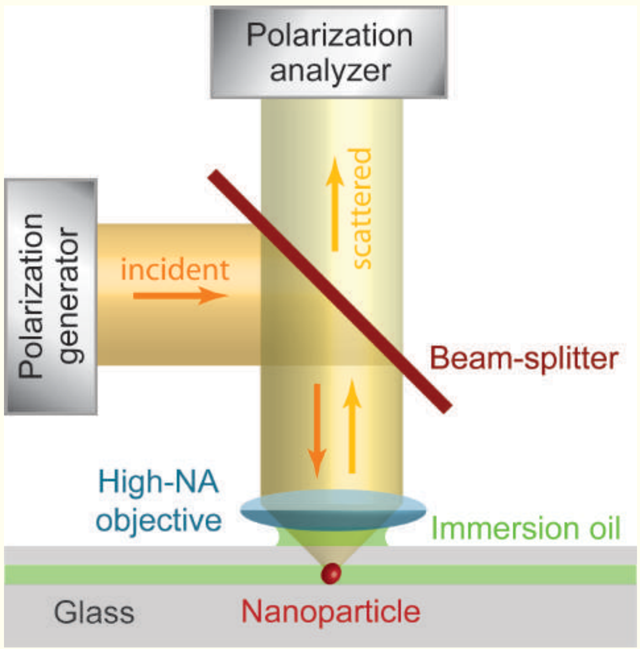


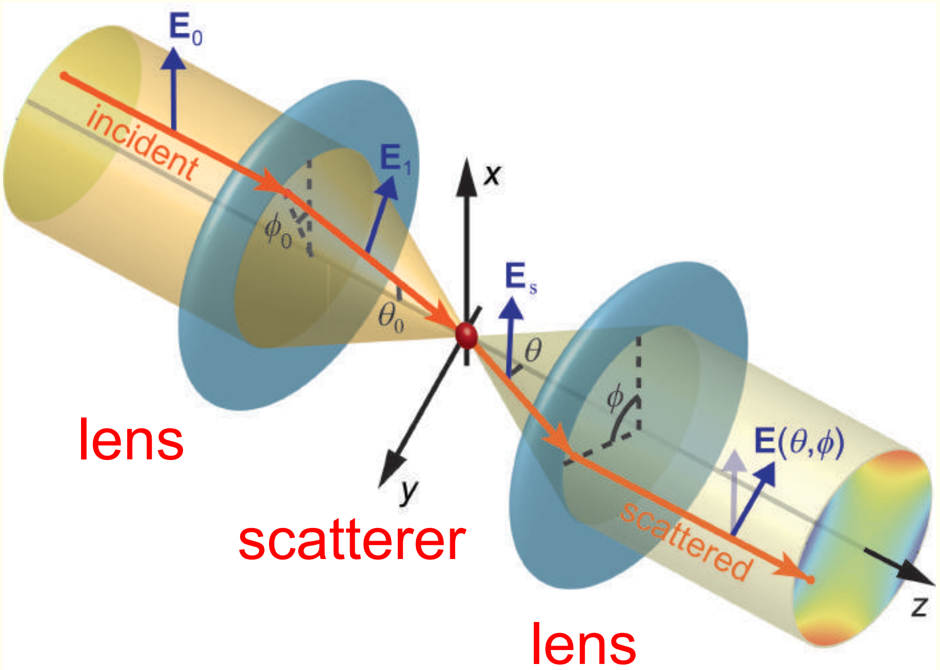


Berry et al., EJP 1980

Focusing, scattering, and imaging

Microscopy involves a series of focusing and scatterings inside, but it can have **paraxial input and output** fields:



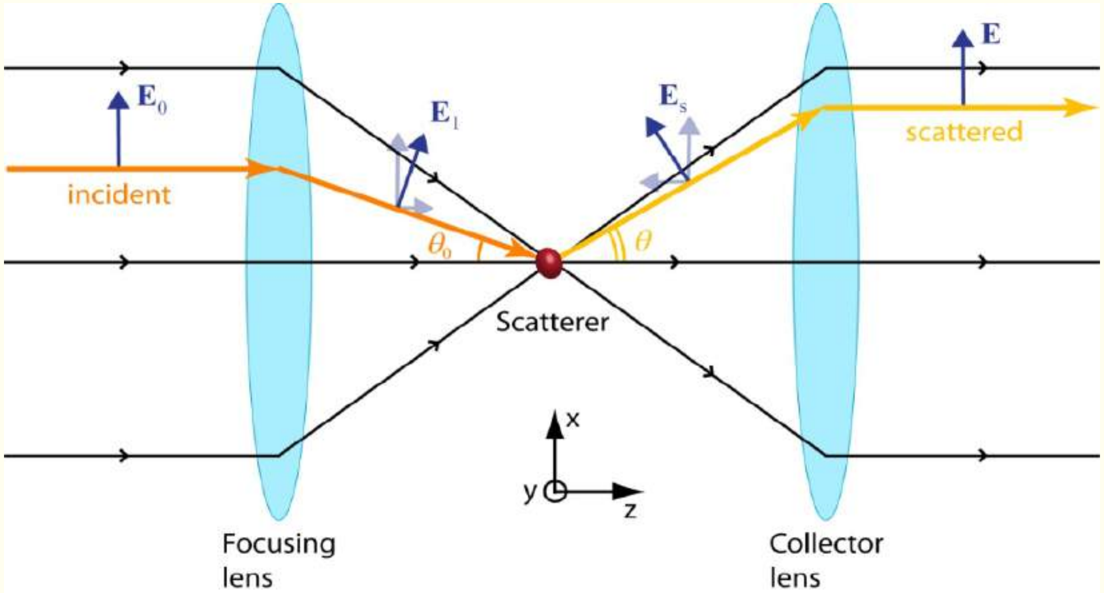


Hence, the SOI phenomena in the output field can be analyzed via usual polarimetry.

Microscopy + Polarimetry = Far-field SOI imaging

Focusing, scattering, and imaging

Using 3D transformations of the field due to the focusing and scattering inside the system, we obtain 2D space-variant Jones matrix connecting incoming and outgoing rays:



$$\hat{J}(\theta,\phi,\mathbf{r}_s) = e^{i\Phi}\,\hat{U}^{-1}_{\text{lens}}(\theta,\phi)\iint d^2\mathbf{r}_0\, e^{i\Phi_0}\,\hat{U}_{\text{lens}}(\theta_0,\phi_0)$$

Rodrigues-Herrera et al., PRL 2010

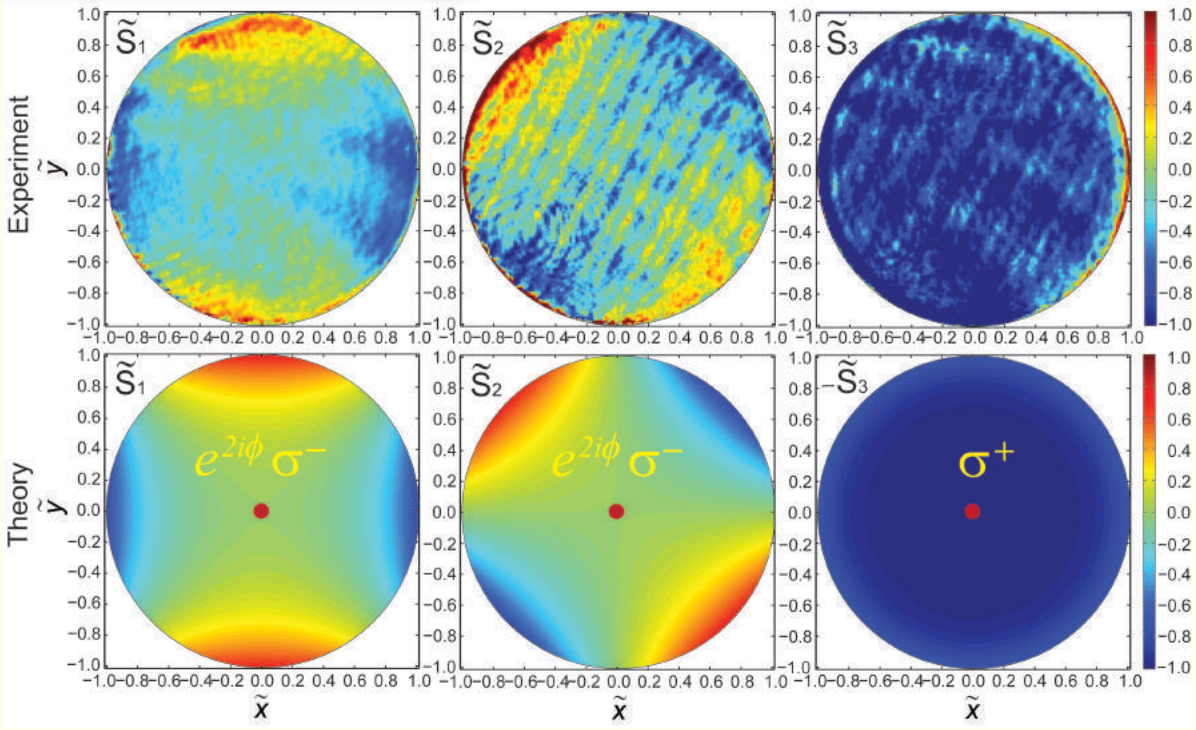
Focusing, scattering, and imaging

If the scatterer is placed precisely **on axis**, the output polarization describes the **spin-to-orbital AM conversion**:

$$\hat{J}^{(0)} \propto \begin{pmatrix} a & -be^{-2i\varphi} \\ -be^{2i\varphi} & a \end{pmatrix}$$

$$|0,\sigma\rangle \to a|0,\sigma\rangle - b|2\sigma,-\sigma\rangle$$

Stokes parameters:



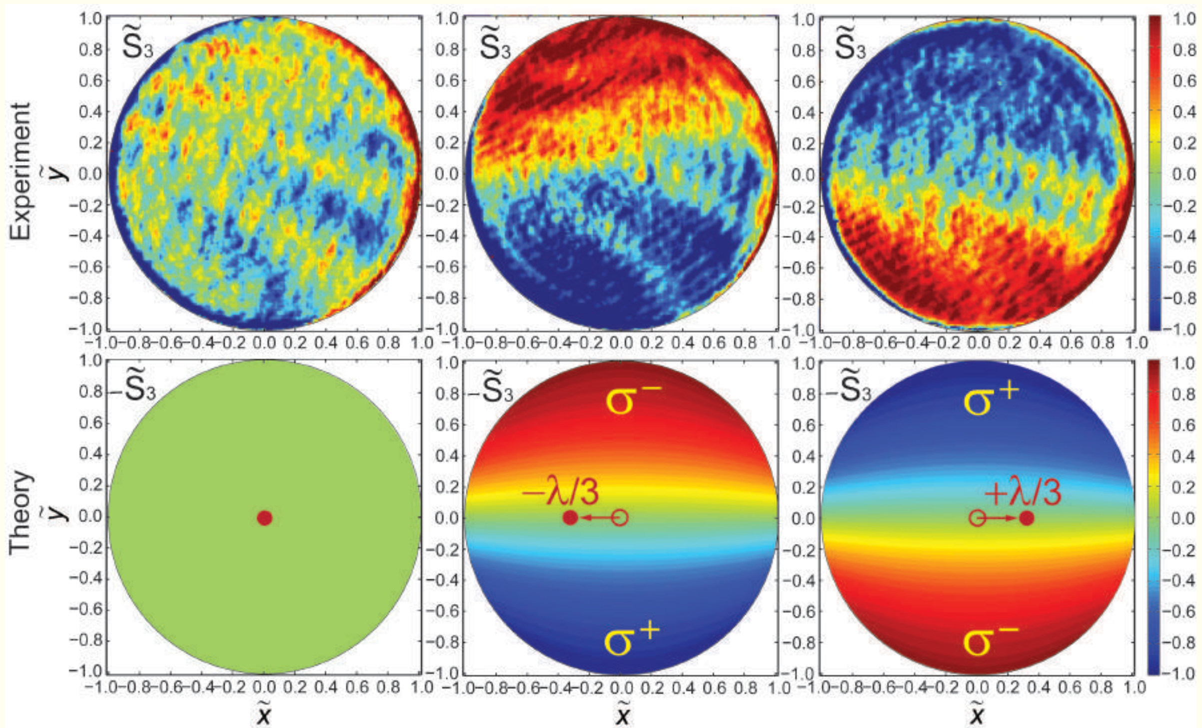
Focusing, scattering, and imaging

If the scatterer is placed a bit **off-axis**, the output polarization demonstrates **giant spin Hall effect**:

$$\hat{J} \propto \hat{J}^{(0)} + \alpha \begin{pmatrix} \rho_s e^{-i\phi} & \rho_s^* e^{-i\phi} \\ \rho_s e^{i\phi} & \rho_s^* e^{i\phi} \end{pmatrix}$$

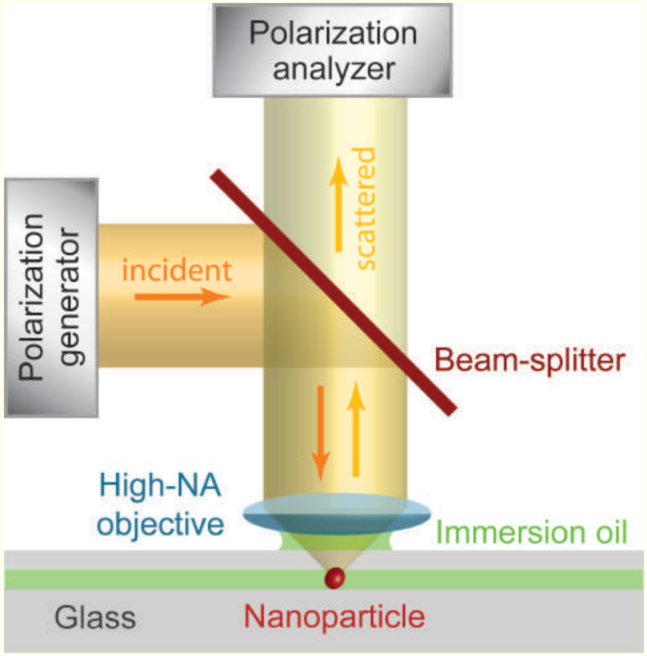
$$\rho_s = k\left(x_s + i y_s\right)$$

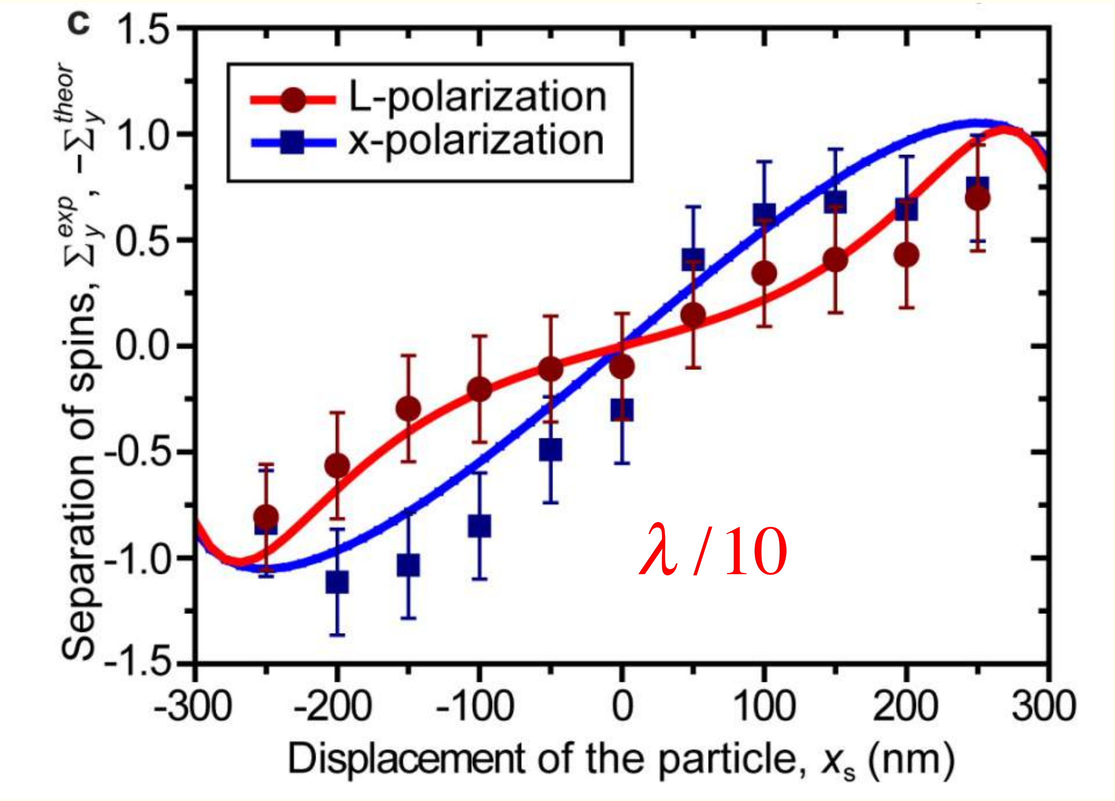
Macro-separation of spins induced by subwavelength shift of the particle !



Focusing, scattering, and imaging

High sensitivity of the SOI effects can be employed as a new method for subwavelength probing. We determined position of the scatterer using spin-Hall effect:





Rodrigues-Herrera et al., PRL 2010

Summary of the SOI of light

- SOI is everywhere: propagation, refraction, reflection, focusing, scattering, diffraction, anisotropy, nonlinearity, etc.
- We can consider SOI as undesirable aberrations or employ it for fine manipulations with light using internal degrees of freedom.
- In any case, SOI effects cannot be ignored anymore as we deal with nano-optics and subwavelength scales.
- AM theory, energy flows, and geometric phases provide efficient description of SOI.

- Spin-orbit interaction in paraxial beams
  - Propagation in gradient-index media
  - Reflection/transmission at an interface
- Spin-orbit coupling in nonparaxial fields
  - Spin and orbital AM in free space
  - Focusing, scattering, imaging
- **Spin-orbit coupling in inhomogeneous anisotropic structures (metasurfaces)**
- Transverse spin-momentum locking

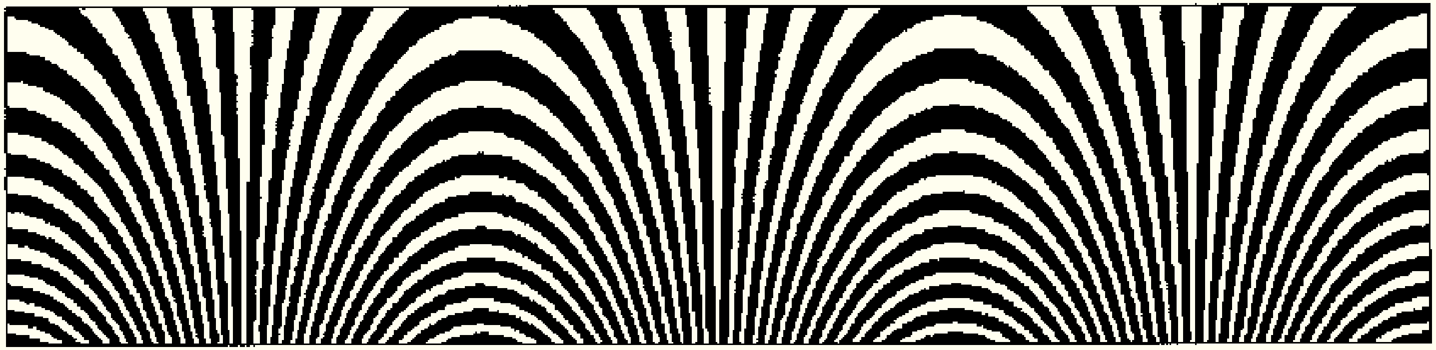
SOI in anisotropic nanostructures

An efficient way of manipulation of SOI using **artificial geometric phase** was developed by E. Hasman *et al.*

Space-variant Pancharatnam–Berry phase optical elements with computer-generated subwavelength gratings

Ze'ev Bomzon, Gabriel Biener, Vladimir Kleiner, and Erez Hasman

Anisotropic structure with space-varying anisotropy axis:

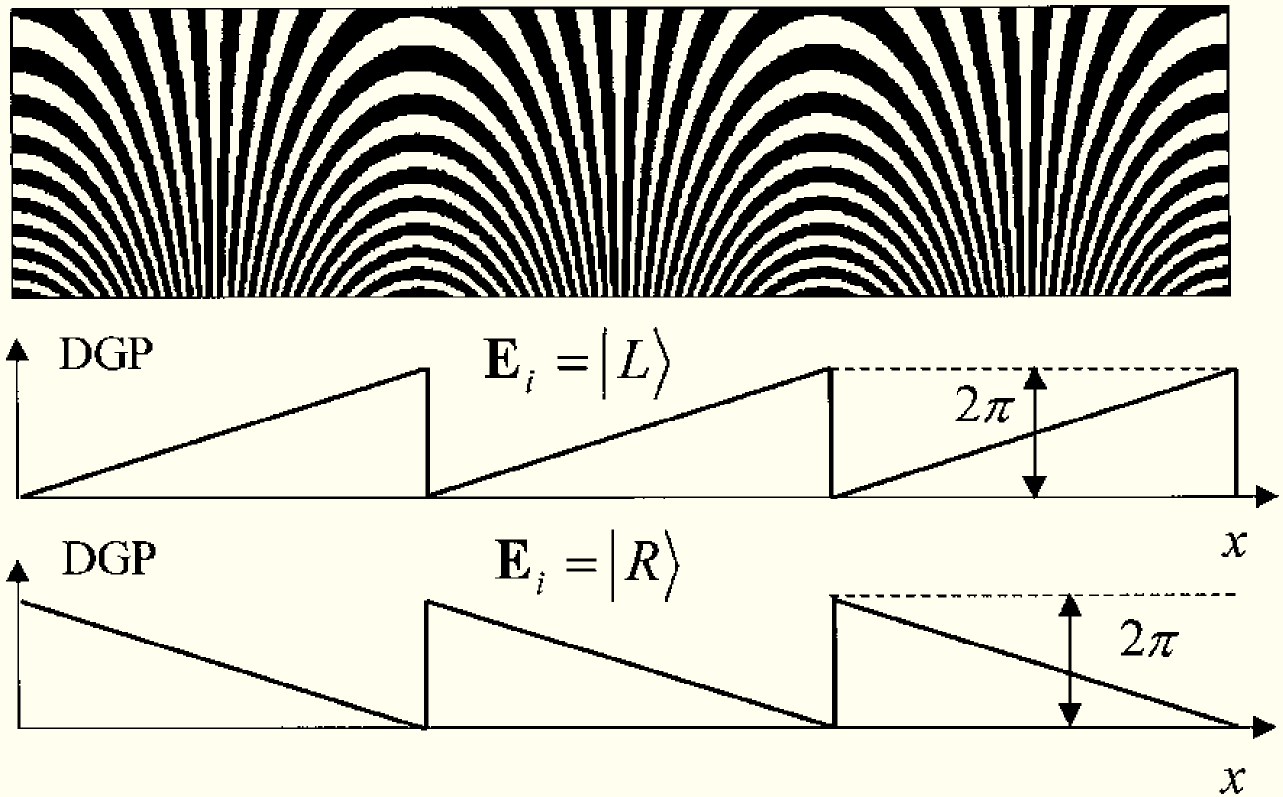


Bomzon et al., OL 2002

SOI in anisotropic nanostructures

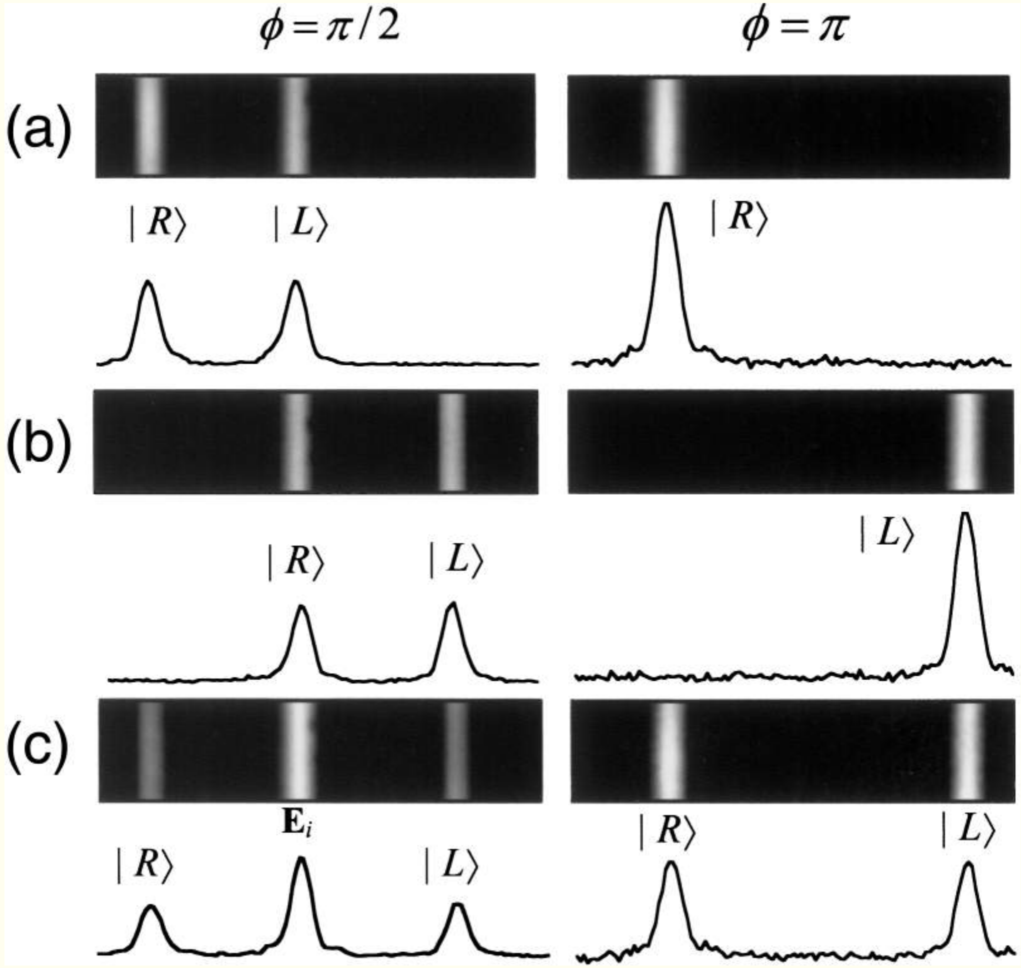
This results in space-variant geometric phase:

$$\hat{J} \propto \begin{pmatrix} a & -ibe^{2i\varphi(x,y)} \\ -ibe^{-2i\varphi(x,y)} & a \end{pmatrix} \qquad \begin{aligned} a &= \cos(\delta/2) \\ b &= \sin(\delta/2) \end{aligned}$$



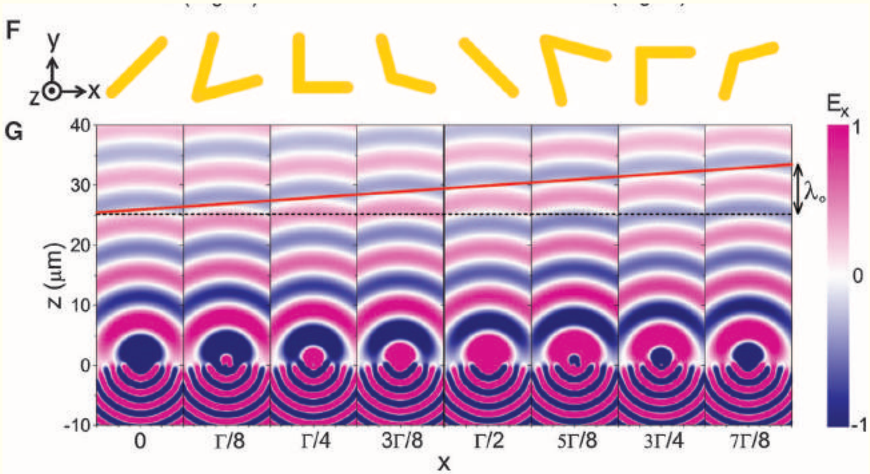
SOI in anisotropic nanostructures

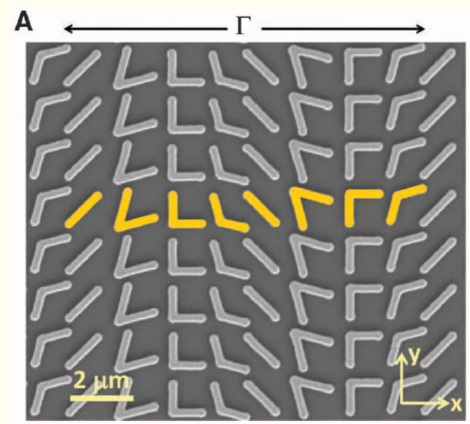
Polarization-dependent diffraction (spin-Hall effect):

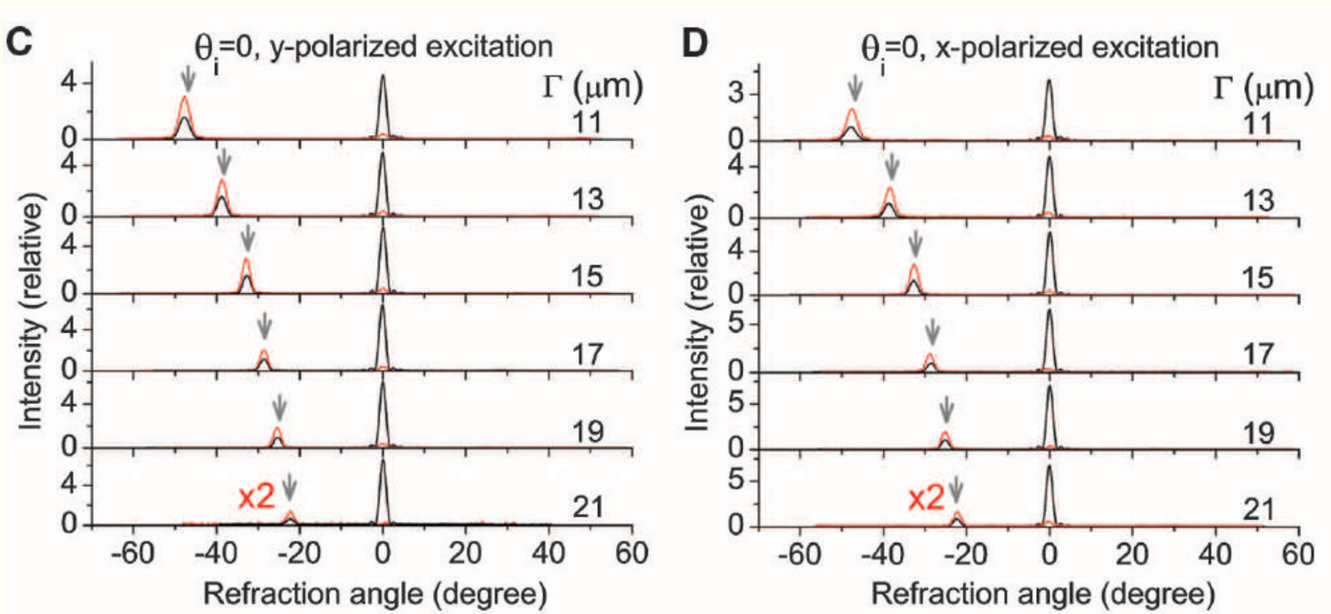


SOI in anisotropic nanostructures

Cf. work by F. Capasso et al. in Science 2011:





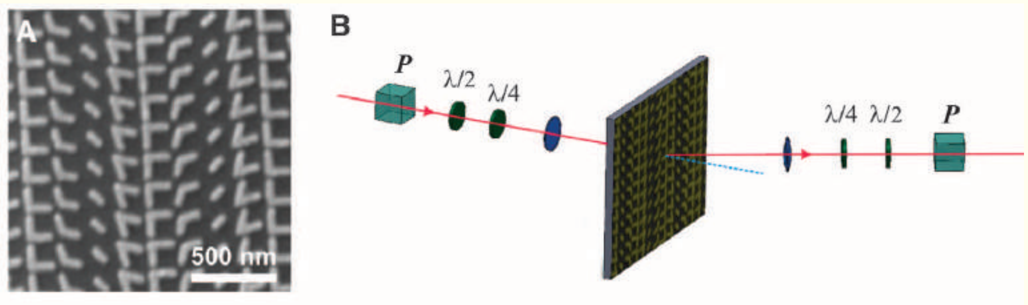


SOI in anisotropic nanostructures

Cf. work by X. Zhang et al. in Science 2013:

Photonic Spin Hall Effect at Metasurfaces

Xiaobo Yin,[1,2] Ziliang Ye,[1] Junsuk Rho,[1] Yuan Wang,[1] Xiang Zhang[1,2]*



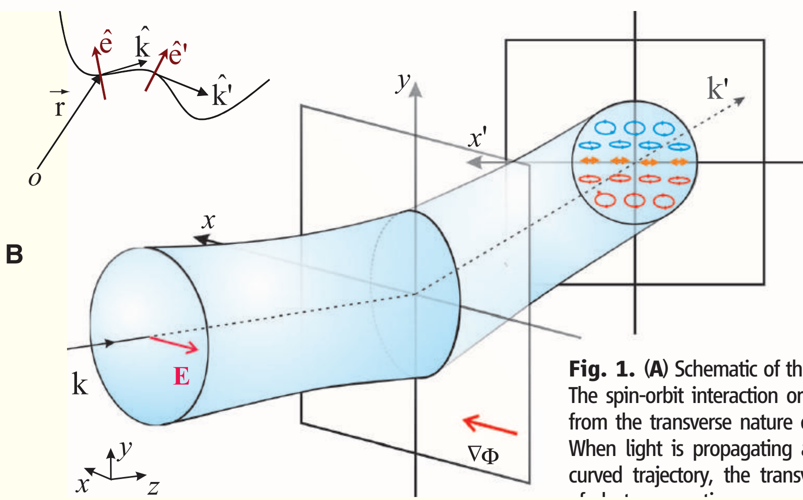
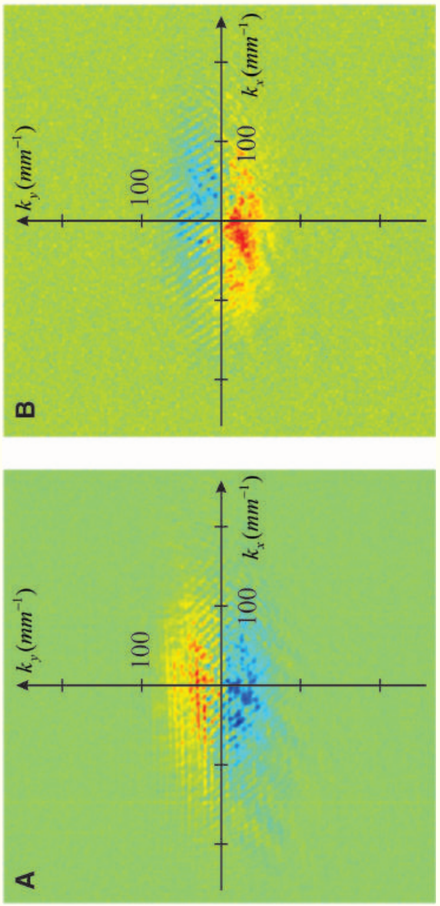


Fig. 1. (A) Schematic of the P
The spin-orbit interaction origin
from the transverse nature of li
When light is propagating alor
curved trajectory, the transvers

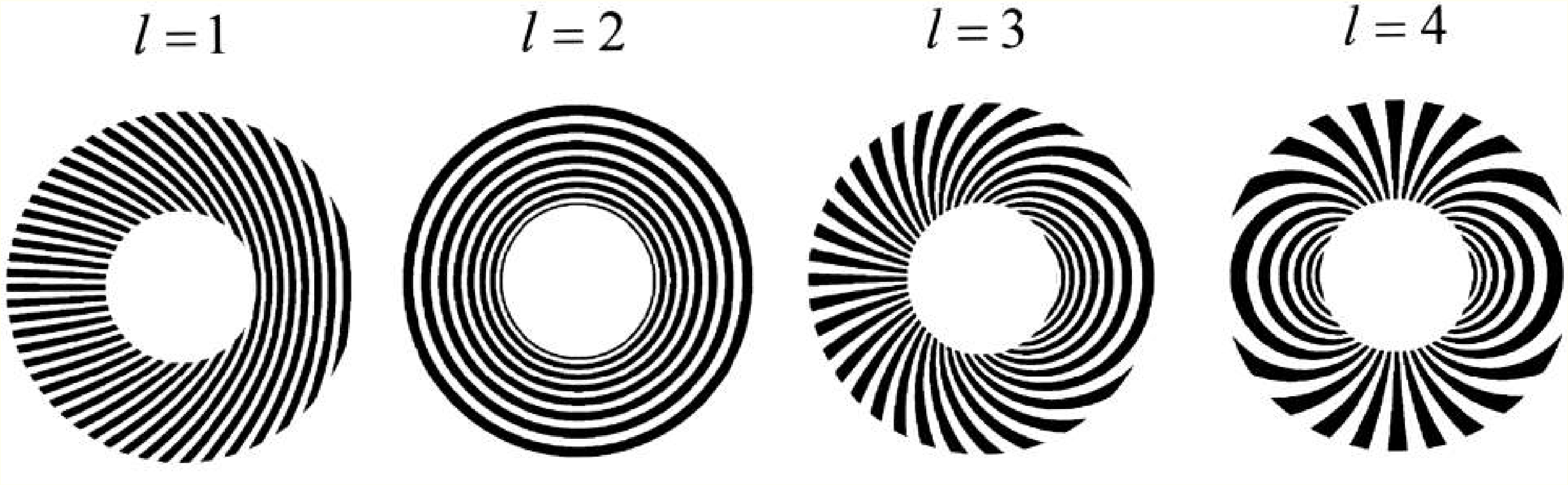


SOI in anisotropic nanostructures

In cylindrical geometry: vortex (spin-to-orbit conversion)

Formation of helical beams by use of Pancharatnam–Berry phase optical elements

Gabriel Biener, Avi Niv, Vladimir Kleiner, and Erez Hasman



Bomzon et al., OL 2002

SOI in anisotropic nanostructures

$\delta = \pi$

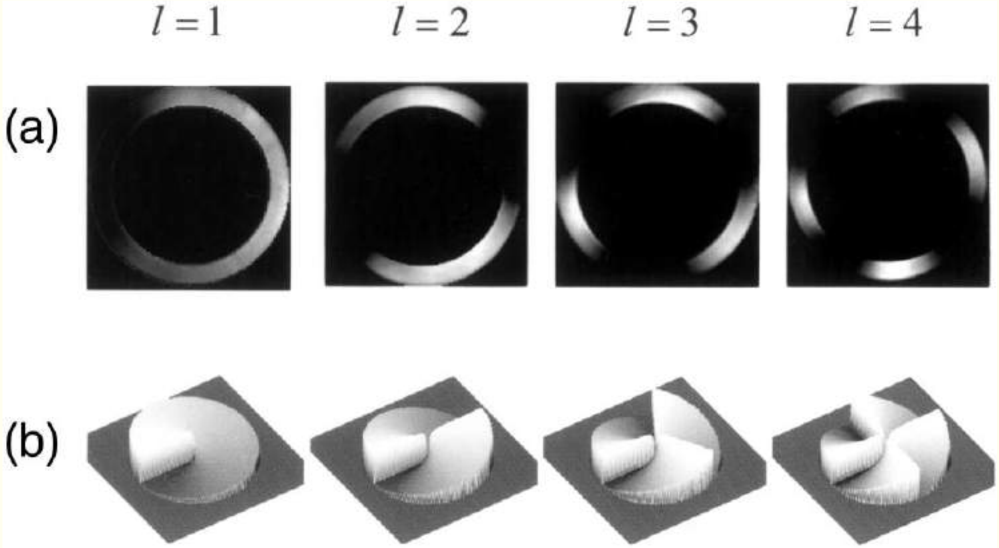


Fig. 3. (a) Interferogram measurements of the spiral PBOEs. (b) The corresponding spiral phases for different topological charges.

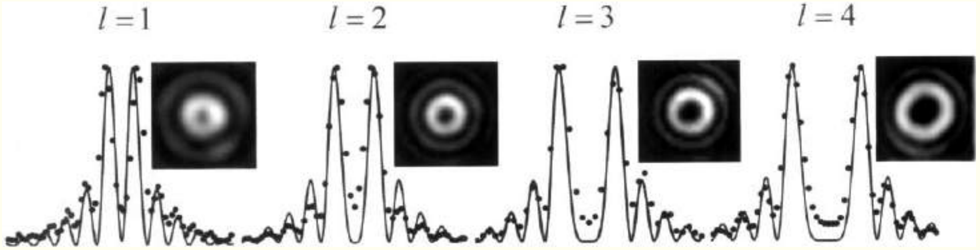
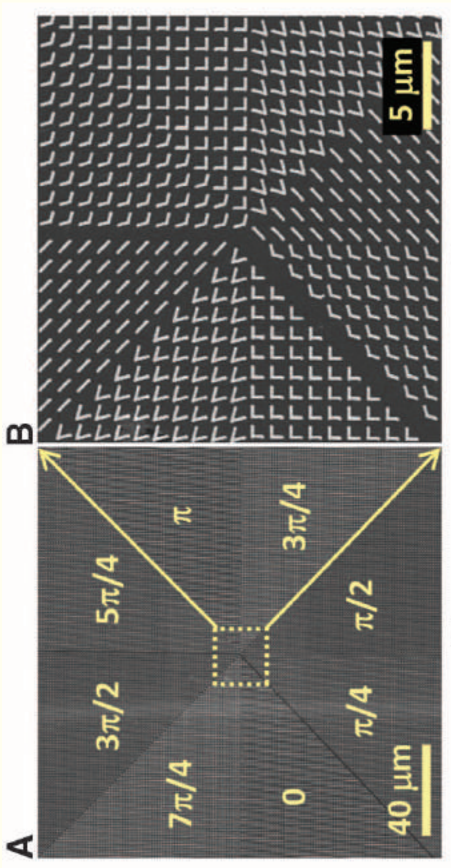
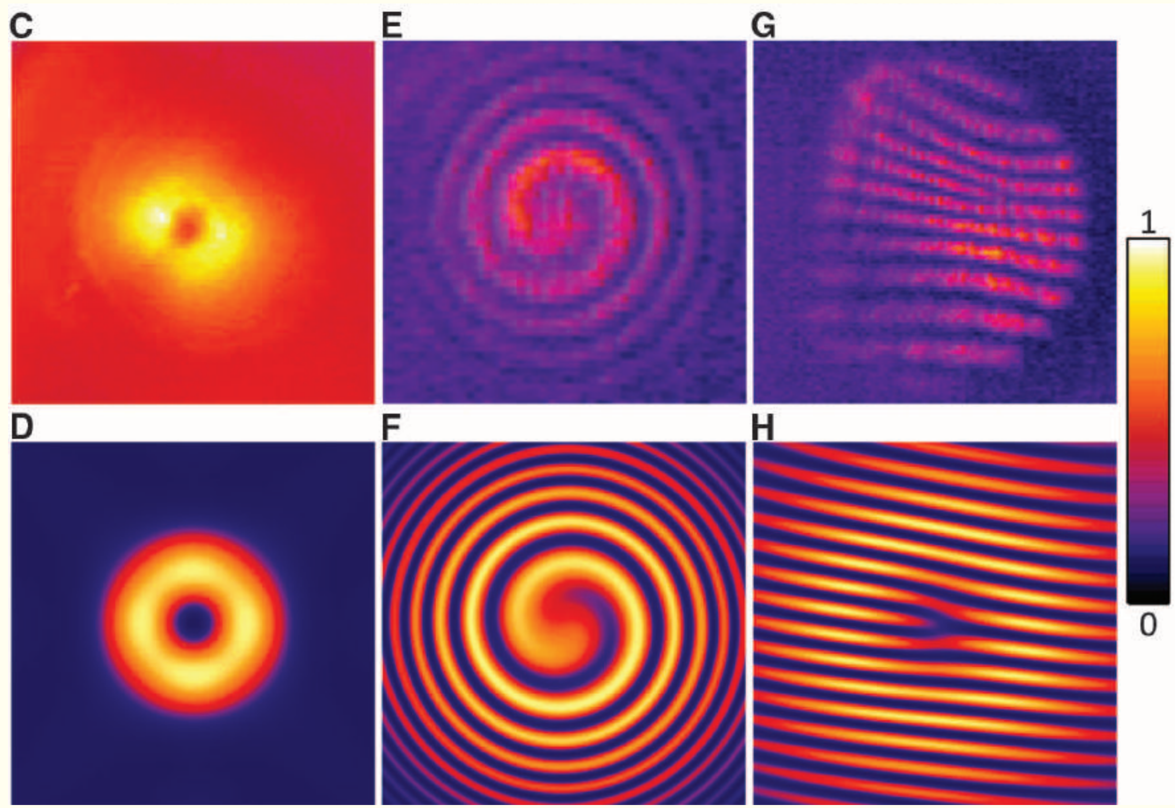


Fig. 4. Experimental far-field images and their calculated and measured cross sections for the helical beams with $l = 1-4$.

SOI in anisotropic nanostructures

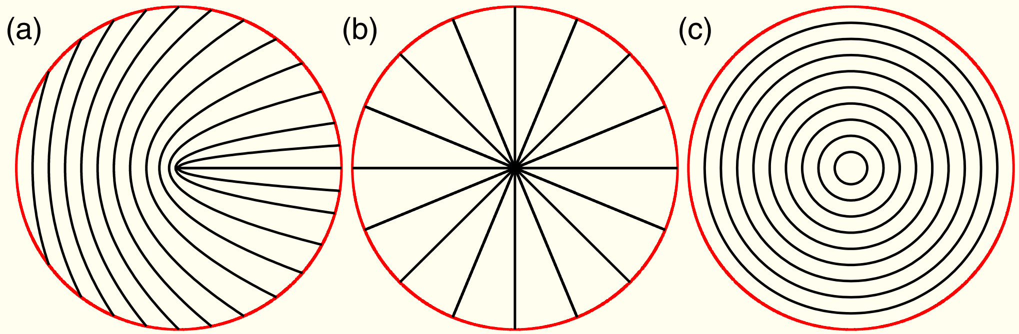
Cf. work by F. Capasso et al. in Science 2011:



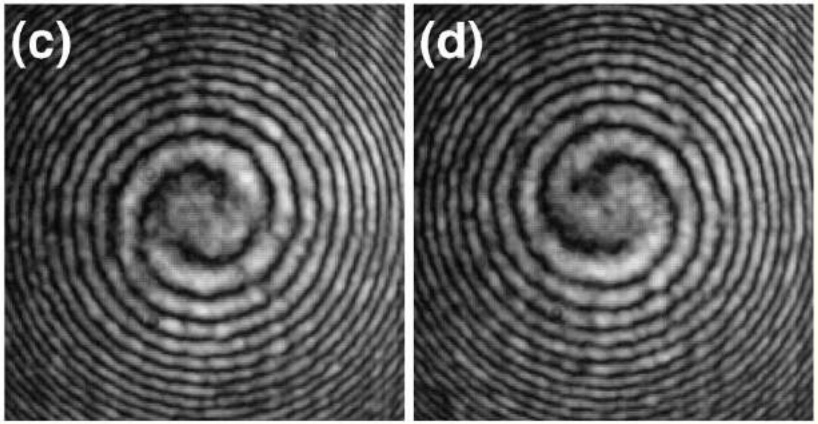


SOI in anisotropic nanostructures

Cf. work by L. Marrucci et al. in PRL 2011 (q-plates):



$$\alpha(r,\varphi) = q\varphi + \alpha_0,$$



$$\mathbf{M} = \mathbf{R}(-\alpha)\cdot\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\cdot\mathbf{R}(\alpha) = \begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{pmatrix},$$

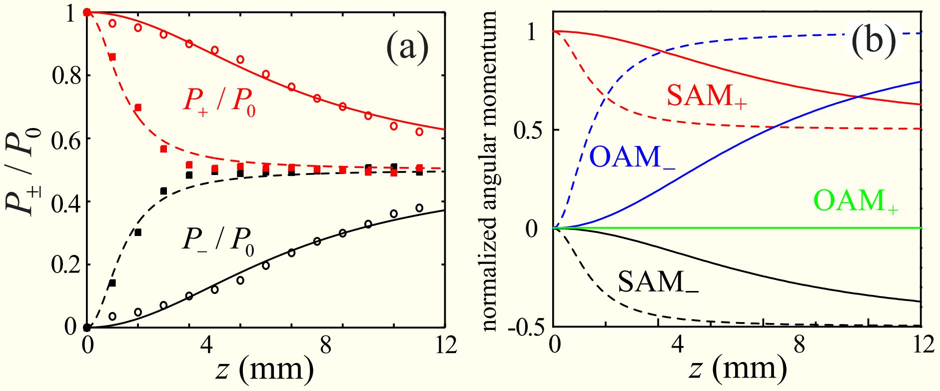
Marrucci et al., PRL 2006

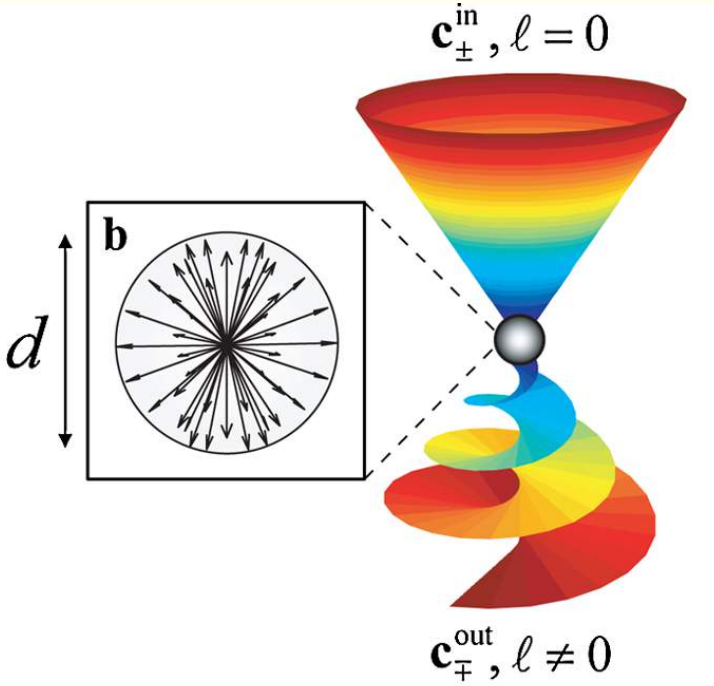
SOI in anisotropic nanostructures

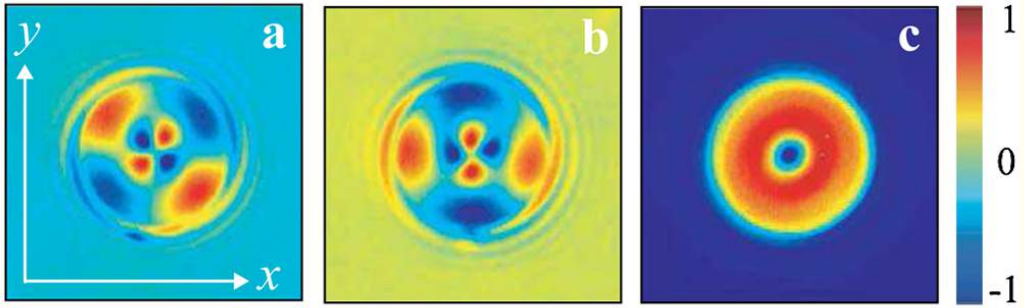
Particular case: azimuthal or radial anisotropy (q=2)

$$\hat{\mathbf{M}} = \begin{pmatrix} C & Se^{-2i\varphi} \\ Se^{2i\varphi} & C \end{pmatrix} = C\hat{\boldsymbol{\sigma}}_0 + S\hat{\mathbf{T}},$$

$$\hat{\mathbf{T}} = \hat{\boldsymbol{\sigma}}_x \cos(2\varphi) + \hat{\boldsymbol{\sigma}}_y \sin(2\varphi),$$





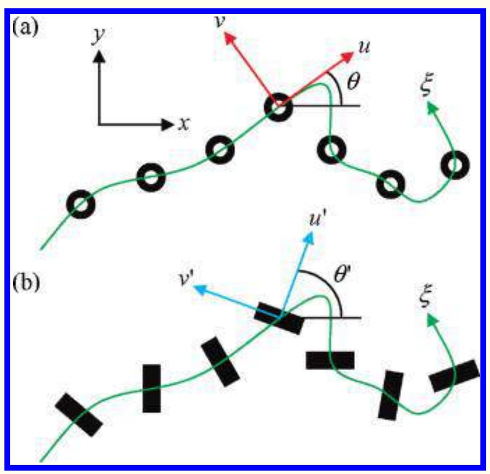


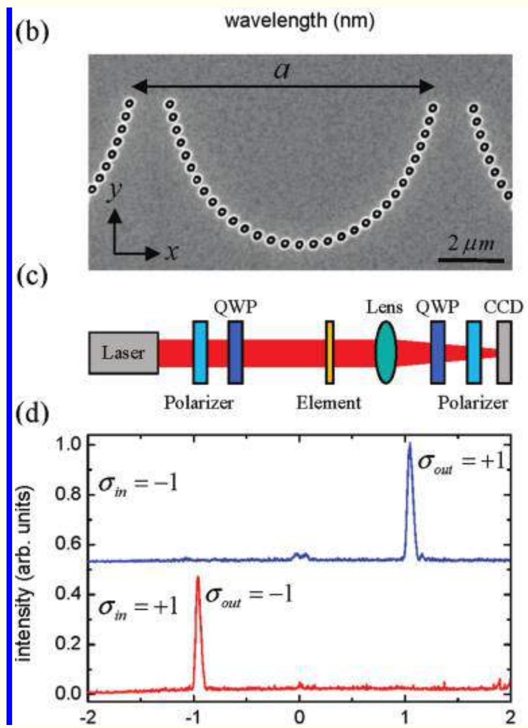
Brasselet et al., OL 2009, PRL 2009

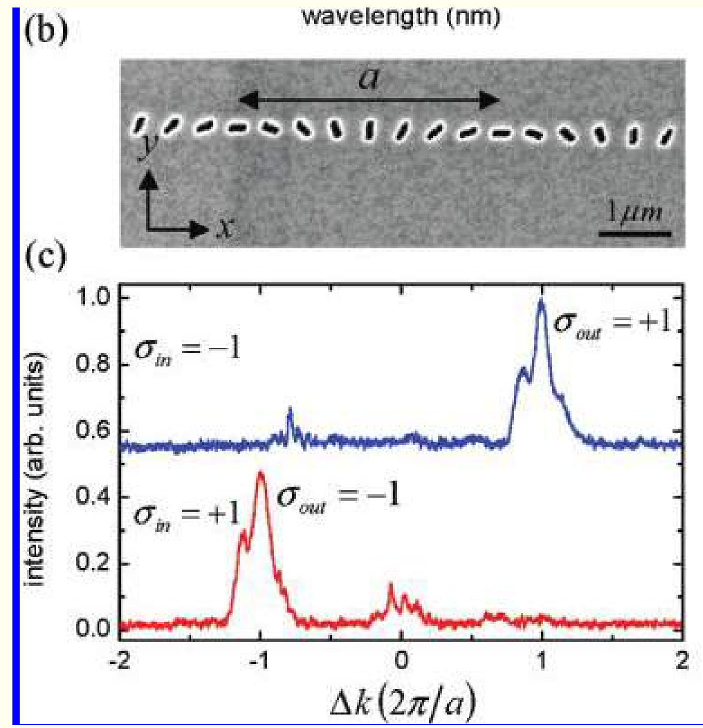
SOI in anisotropic nanostructures

Optical Spin Hall Effects in Plasmonic Chains

Nir Shitrit, Itay Bretner, Yuri Gorodetski, Vladimir Kleiner, and Erez Hasman*





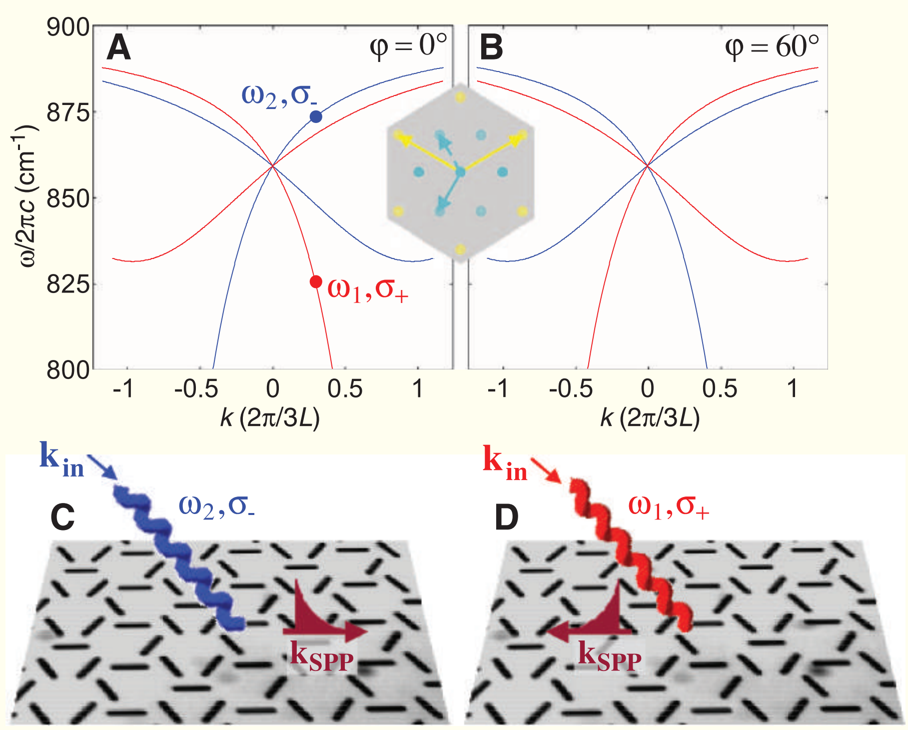


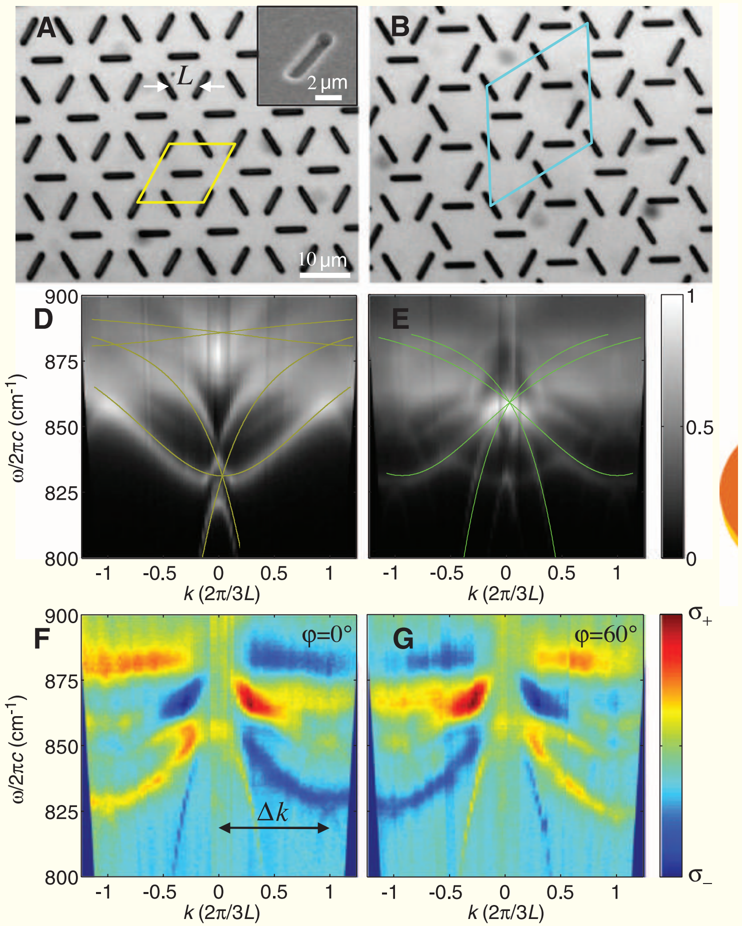
Shitrit et al., Nano Lett. 2011

SOI in anisotropic nanostructures

Spin-Optical Metamaterial Route to Spin-Controlled Photonics

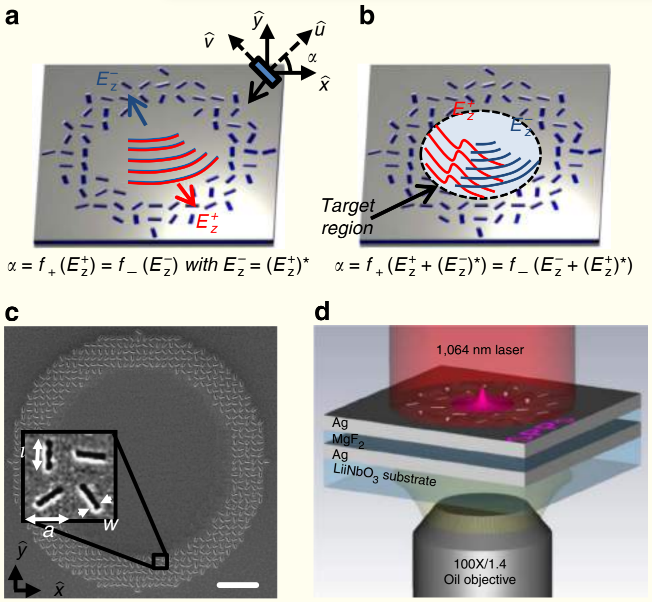
Nir Shitrit, Igor Yulevich, Elhanan Maguid, Dror Ozeri, Dekel Veksler, Vladimir Kleiner, Erez Hasman*

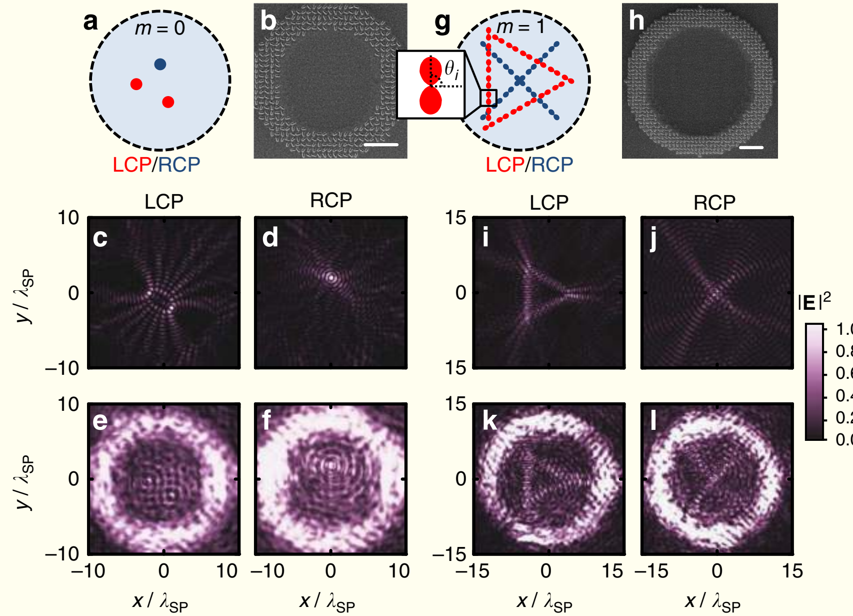


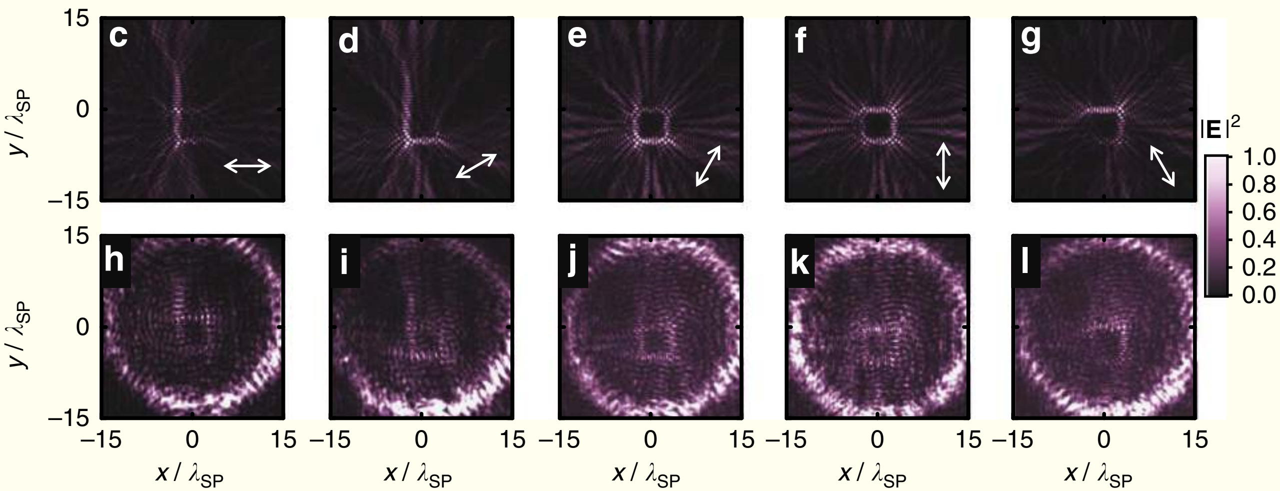


Shitrit et al., Science 2013

SOI in anisotropic nanostructures

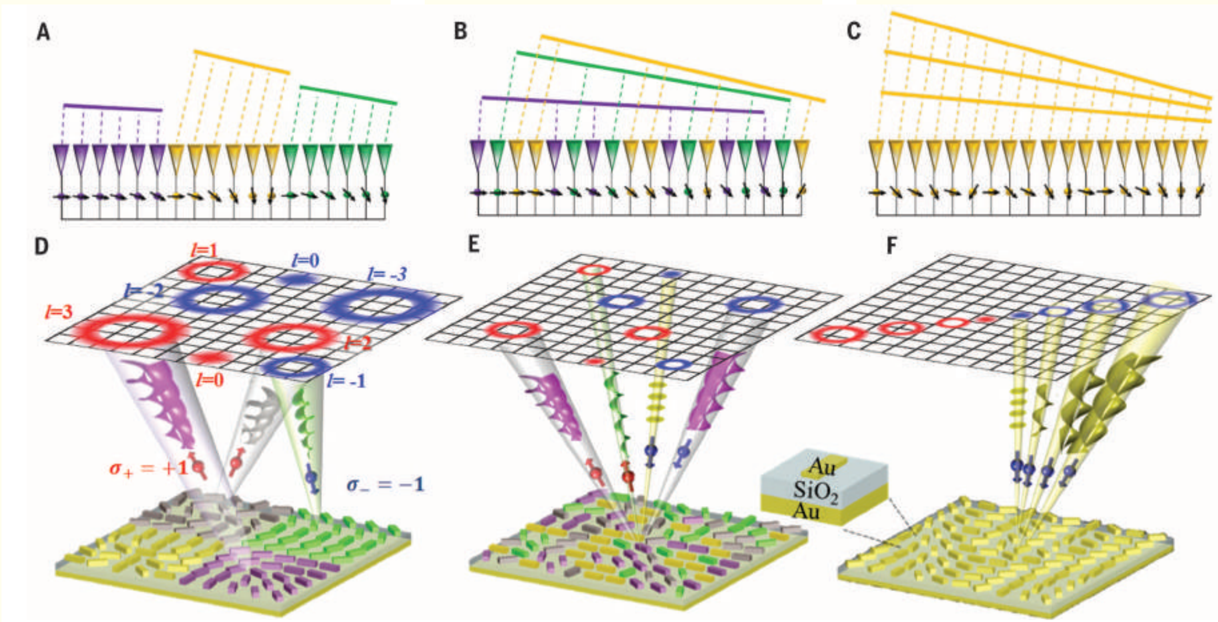


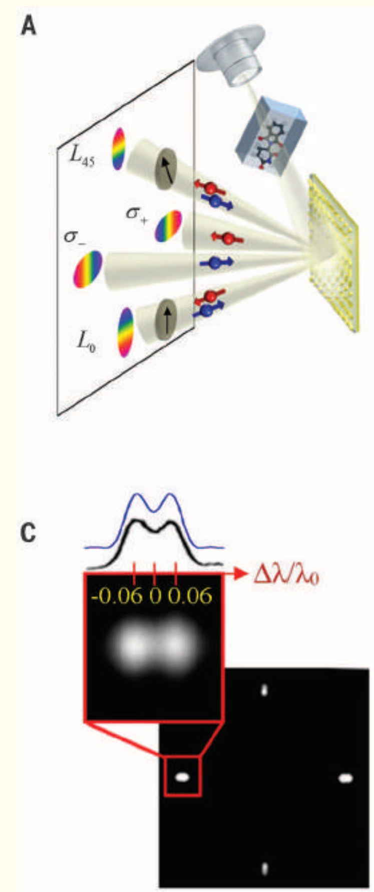


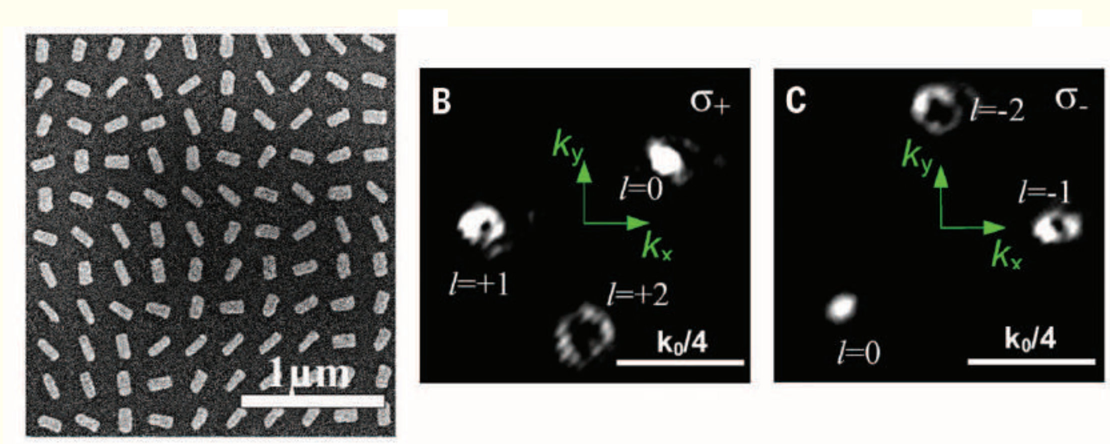


Xiao et al., Nature Commun. 2015

SOI in anisotropic nanostructures



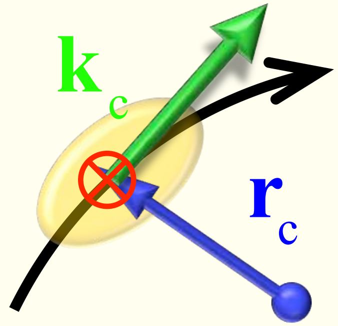




Maguid et al., Science 2016

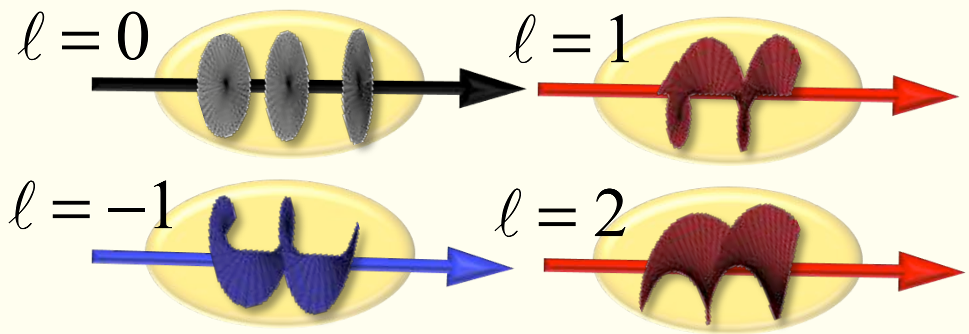
- **Spin-orbit interaction in paraxial beams**
  - Propagation in gradient-index media
  - Reflection/transmission at an interface
- **Spin-orbit coupling in nonparaxial fields**
  - Spin and orbital AM in free space
  - Focusing, scattering, imaging
- **Spin-orbit coupling in inhomogeneous anisotropic structures (metasurfaces)**
- **Transverse spin-momentum locking**

1. Extrinsic orbital AM (trajectory)



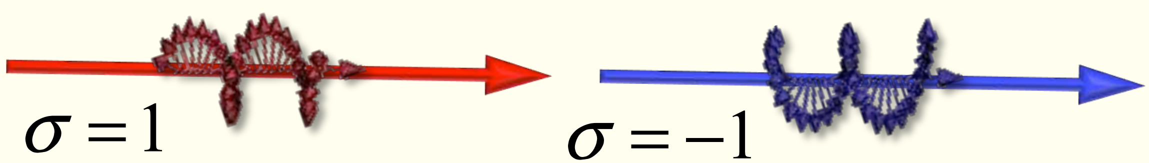
$$\mathbf{L}_{ext} = \mathbf{r}_c \times \mathbf{k}_c$$

2. Intrinsic orbital AM (vortex)



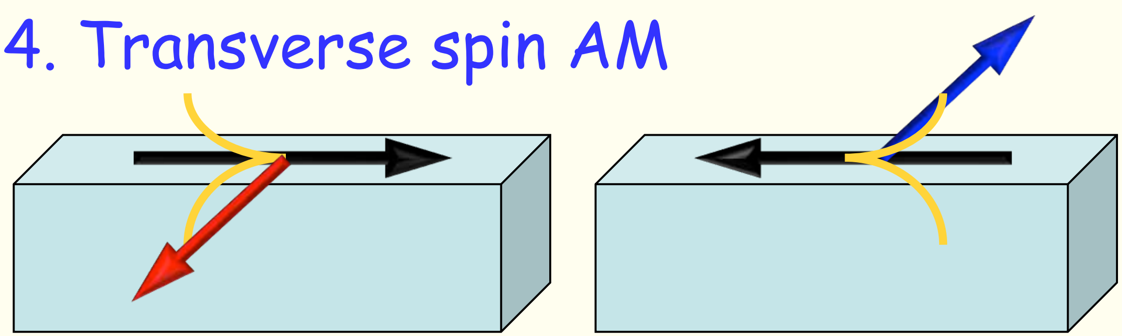
$$L_{int} = \ell \boldsymbol{\kappa}_c$$

3. Longitudinal spin AM



$$\mathbf{S} = \sigma \boldsymbol{\kappa}_c$$

4. Transverse spin AM

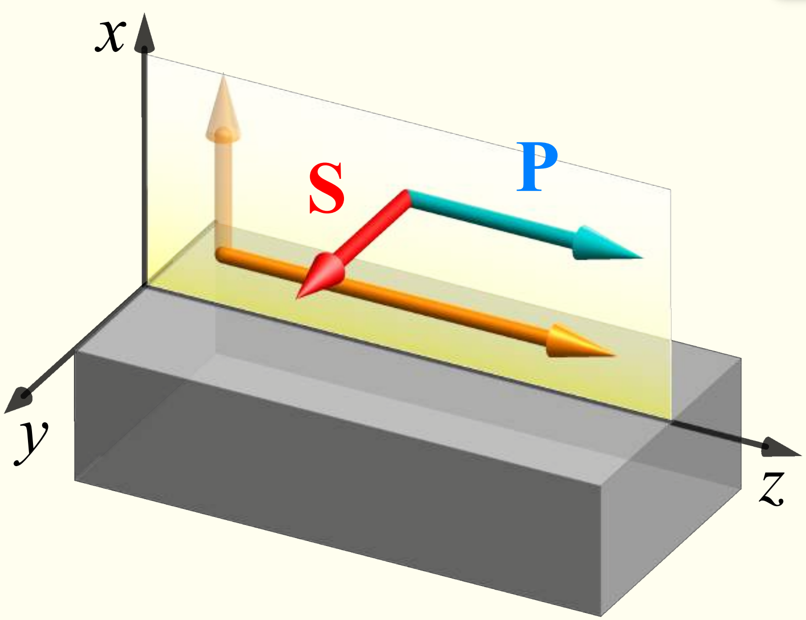


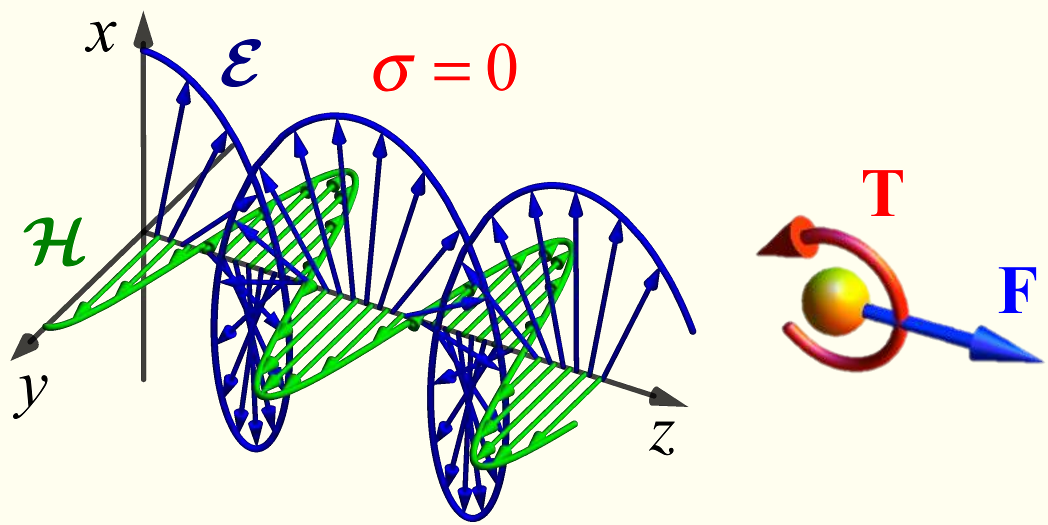
$$\mathbf{S}_{\perp} \propto \mathrm{Re}\, \mathbf{k} \times \bar{\mathbf{n}}$$

Transverse spin angular momentum

Evanescent waves carry longitudinal canonical momentum and **transverse helicity-independent spin AM**:

$$\bar{\mathbf{P}} = \mathrm{Re}\,\mathbf{k}, \qquad \boxed{\bar{\mathbf{S}}_{\perp} = \frac{\mathrm{Re}\,\mathbf{k} \times \mathrm{Im}\,\mathbf{k}}{(\mathrm{Re}\,\mathbf{k})^2}}$$

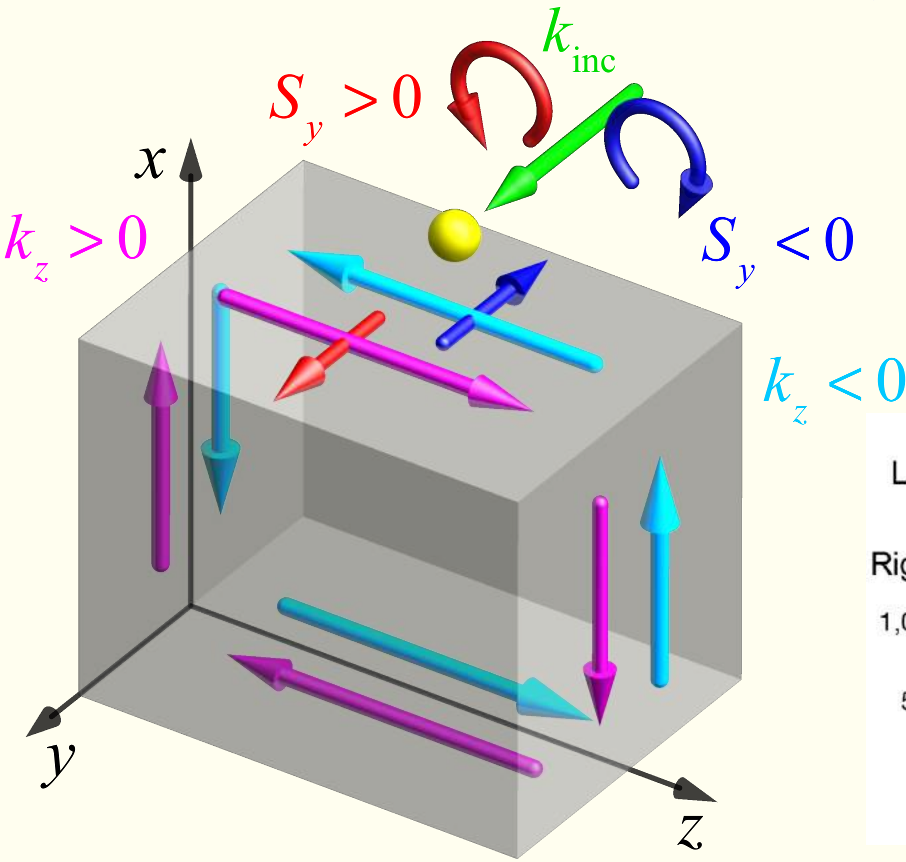


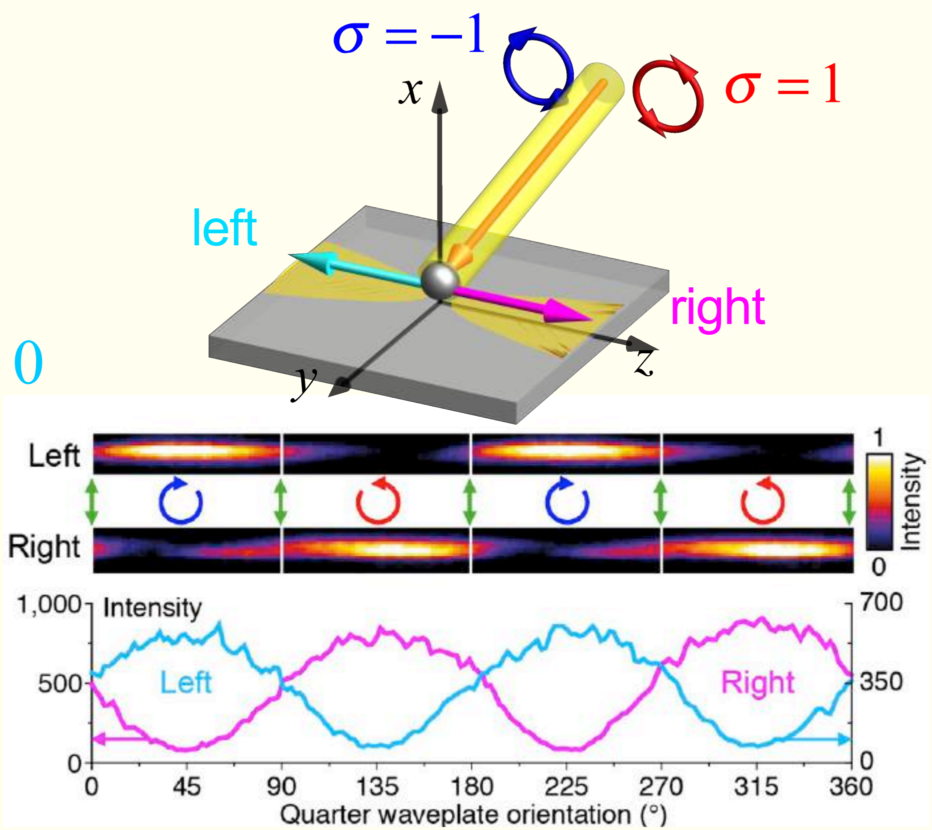


Bliokh & Nori, PRA 2012

Transverse spin-momentum locking

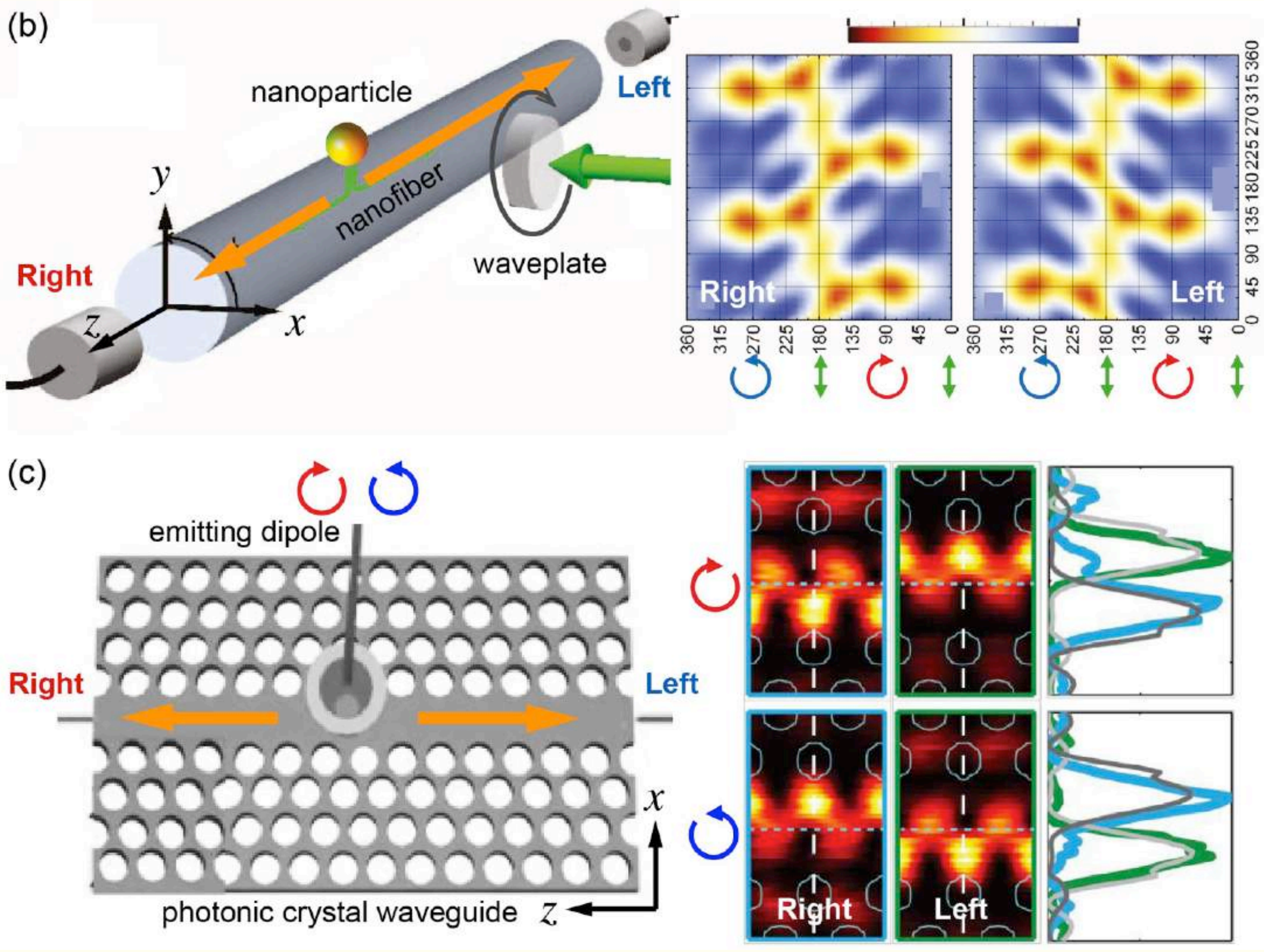
This extraordinary property provides transverse **spin-momentum locking**, analogues to that in topological insulators, or the "**quantum spin Hall effect of light**":





Science: Rodriguez–Fortuno et al. 2013; Petersen et al. 2014; Bliokh et al. 2015

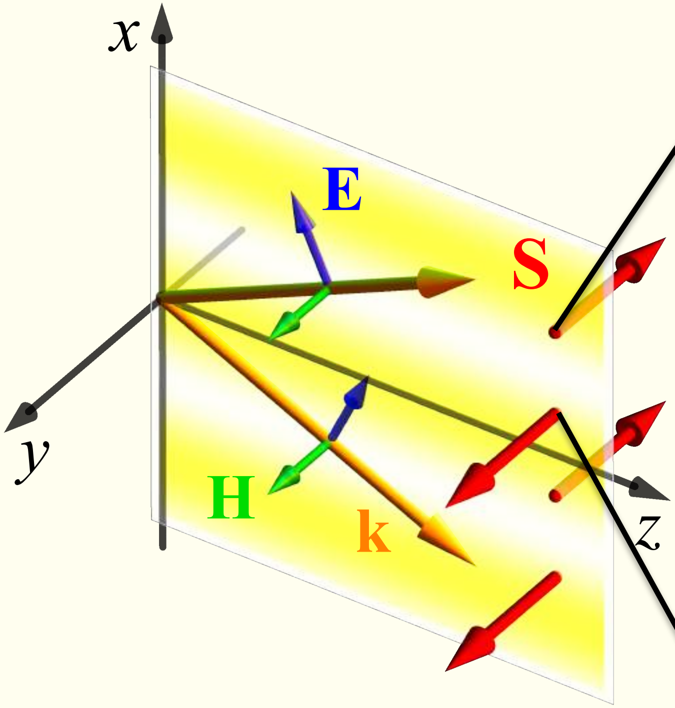
Transverse spin-momentum locking



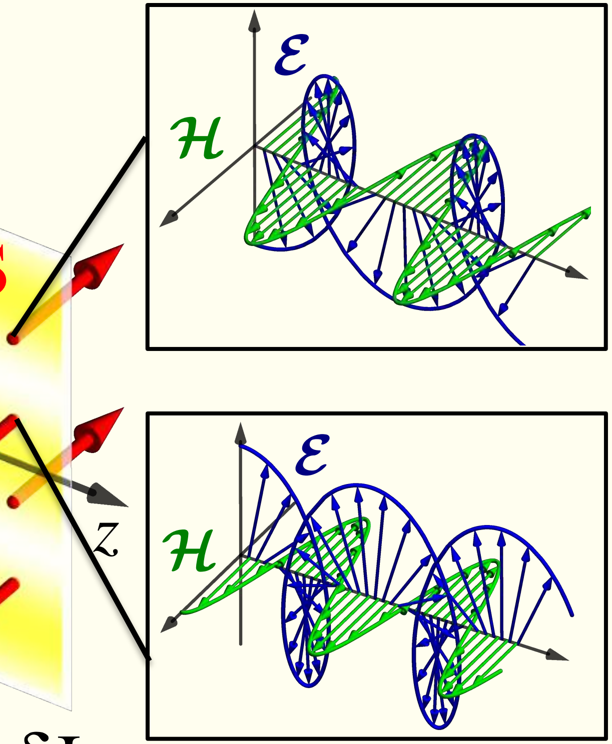
Petersen et al., Science 2014; Mitsch et al., Nat. Com. 2014; le Feber et al. Nat. Com. 2015

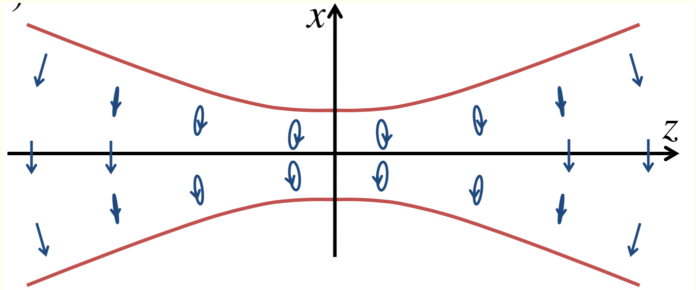
Transverse spin angular momentum

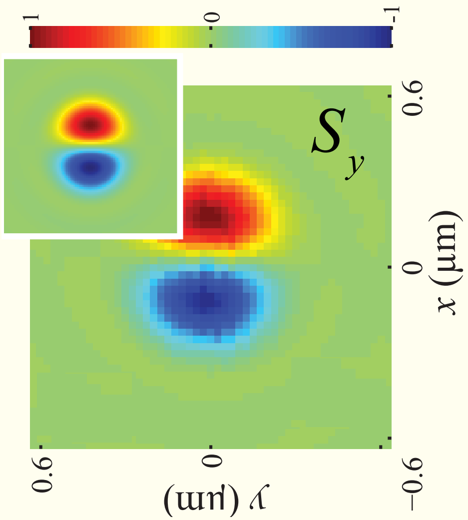
The transverse polarization-independent spin AM density also appears in **propagating interference fields**, but its integral value vanishes in such fields:



$$\mathbf{S}_{\perp} \propto \langle \mathbf{k} \rangle \times \delta\mathbf{k} \sin \delta\Phi$$







Bekshaev et al. PRX 2015; Mathevet & Rikken, OE 2014; Neugebauer et al. PRL 2015

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Transverse and longitudinal angular momenta of light

Konstantin Y. Bliokh[a,b,*], Franco Nori[a,c]

[a] Center for Emergent Matter Science, RIKEN, Wako-shi, Saitama 351-0198, Japan

[b] Nonlinear Physics Centre, RSPhysE, The Australian National University, Canberra, ACT 0200, Australia

REVIEW ARTICLE | FOCUS

PUBLISHED ONLINE: 27 NOVEMBER 2015 | DOI: 10.1038/NPHOTON.2015.201

Spin–orbit interactions of light

K. Y. Bliokh[1,2*], F. J. Rodríguez-Fortuño[3], F. Nori[1,4] and A. V. Zayats[3]

nature photonics

PUBLISHED ONLINE: 27 NOVEMBER 2015 | DOI: 10.1038

From transverse angular momentum to photonic wheels

Andrea Aiello[1,2], Peter Banzer[1,2,3*], Martin Neugebauer[1,2] and Gerd Leuchs[1,2,3]

REVIEW

doi:10.1038/nature21037

Chiral quantum optics

Peter Lodahl[1], Sahand Mahmoodian[1], Søren Stobbe[1], Arno Rauschenbeutel[2], Philipp Schneeweiss[2], Jürgen Volz[2], Hannes Pichler[3,4] & Peter Zoller[3,4]

SOI with the transverse spin AM

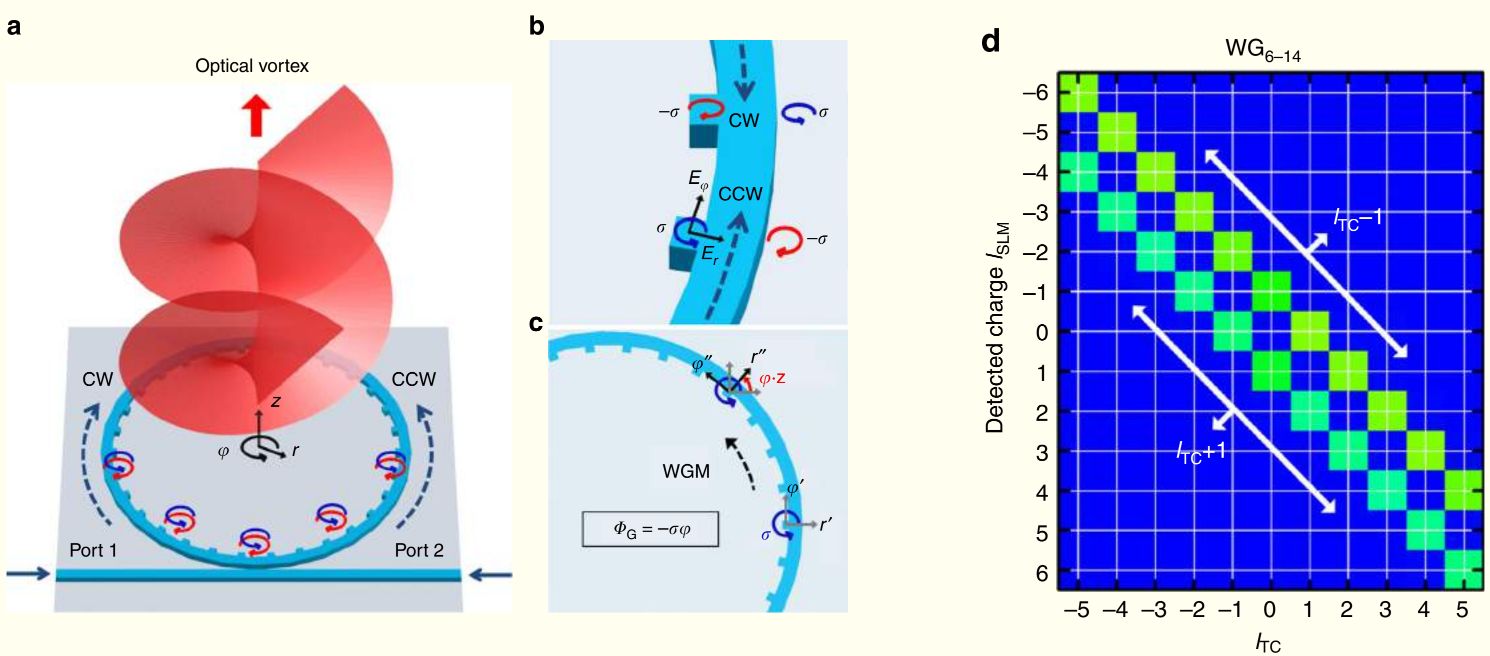
One can also employ the transverse spin for the usual geometric-phase-related SOI phenomena:

ARTICLE

DOI: 10.1038/s41467-018-03237-5 OPEN

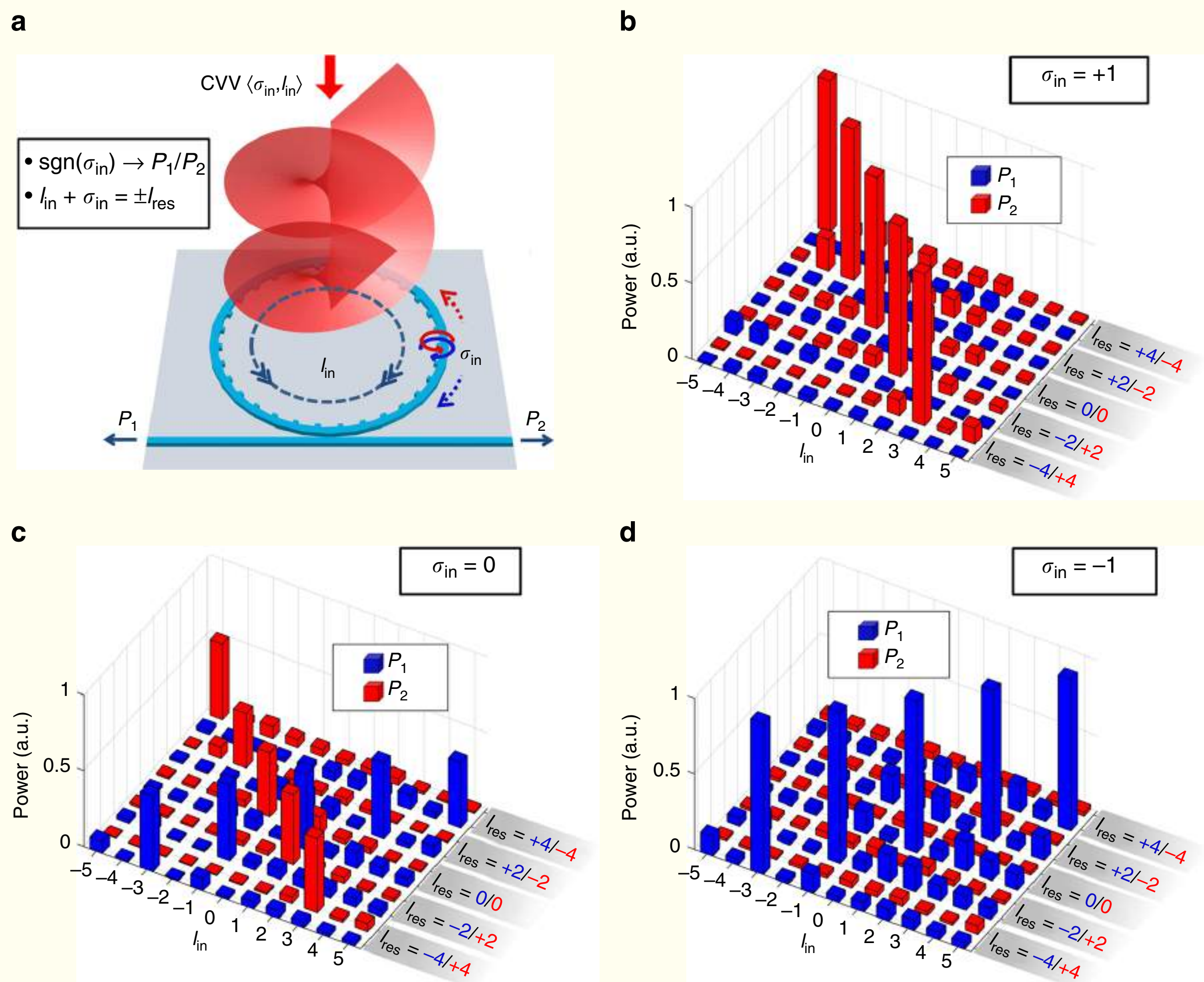
Spin-orbit interaction of light induced by transverse spin angular momentum engineering

Zengkai Shao[1], Jiangbo Zhu[2], Yujie Chen[1], Yanfeng Zhang[1] & Siyuan Yu[1,2]



SOI with the transverse spin AM

One can also employ the transverse spin for the usual geometric-phase-related SOI phenomena:



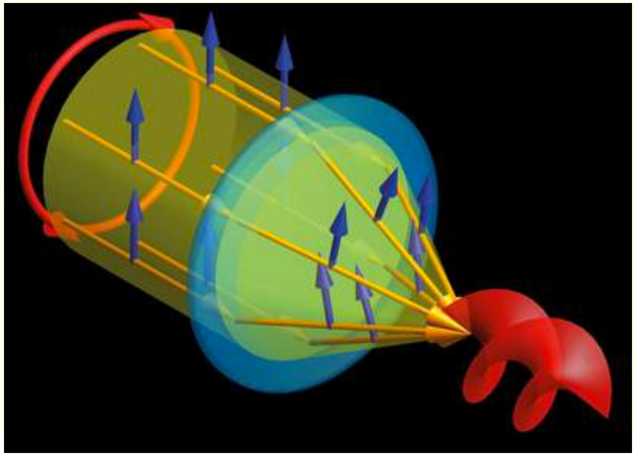
REVIEW ARTICLE | FOCUS
PUBLISHED ONLINE: 27 NOVEMBER 2015 | DOI: 10.1038/NPHOTON.2010.201

nature photonics

Spin–orbit interactions of light

K. Y. Bliokh[1,2]*, F. J. Rodríguez-Fortuño[3], F. Nori[1,4] and A. V. Zayats[3]

Light carries both spin and orbital angular momentum. These dynamical properties are determined by the polarization and spatial degrees of freedom of light. Nano-optics, photonics and plasmonics tend to explore subwavelength scales and additional degrees of freedom of structured — that is, spatially inhomogeneous — optical fields. In such fields, spin and orbital properties become strongly coupled with each other. In this Review we cover the fundamental origins and important applications of the main spin–orbit interaction phenomena in optics. These include: spin-Hall effects in inhomogeneous media and at optical interfaces, spin-dependent effects in nonparaxial (focused or scattered) fields, spin-controlled shaping of light using anisotropic structured interfaces (metasurfaces) and robust spin-directional coupling via evanescent near fields. We show that spin–orbit interactions are inherent in all basic optical processes, and that they play a crucial role in modern optics.



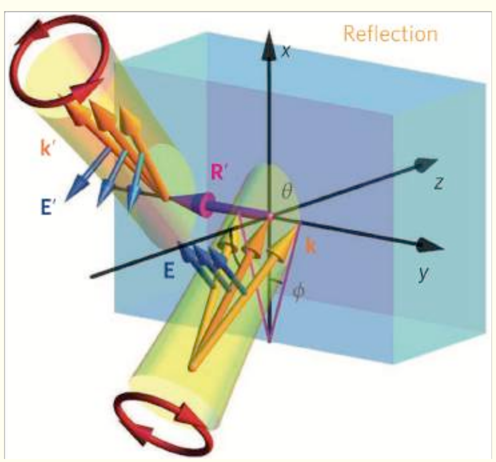
❑ Recent extensions from our group

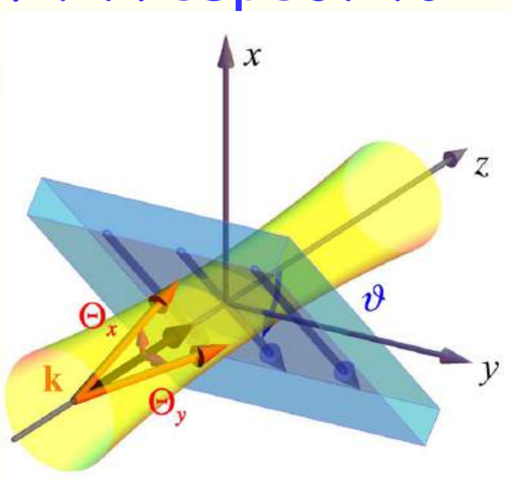
- **Spin-Hall effect for anisotropic wave plates**
- **Weak measurements and shifts in time domain**

Beam shifts at an anisortopic plate

The Goos-Hänchen and spin-Hall beam shifts are caused by the varying wavevectors orientation with respect to the normal to the interface. Changing the propagation direction of the beam is crucial for these phenomena.

Remarkably, entirely similar phenomena appear without changing the beam propagation direction, in the **transmission through an anisotropic wave plate**. There, the wavevector orientation varies with respect to the **anisotropy axis**.





Bliokh et al., Optica 2016

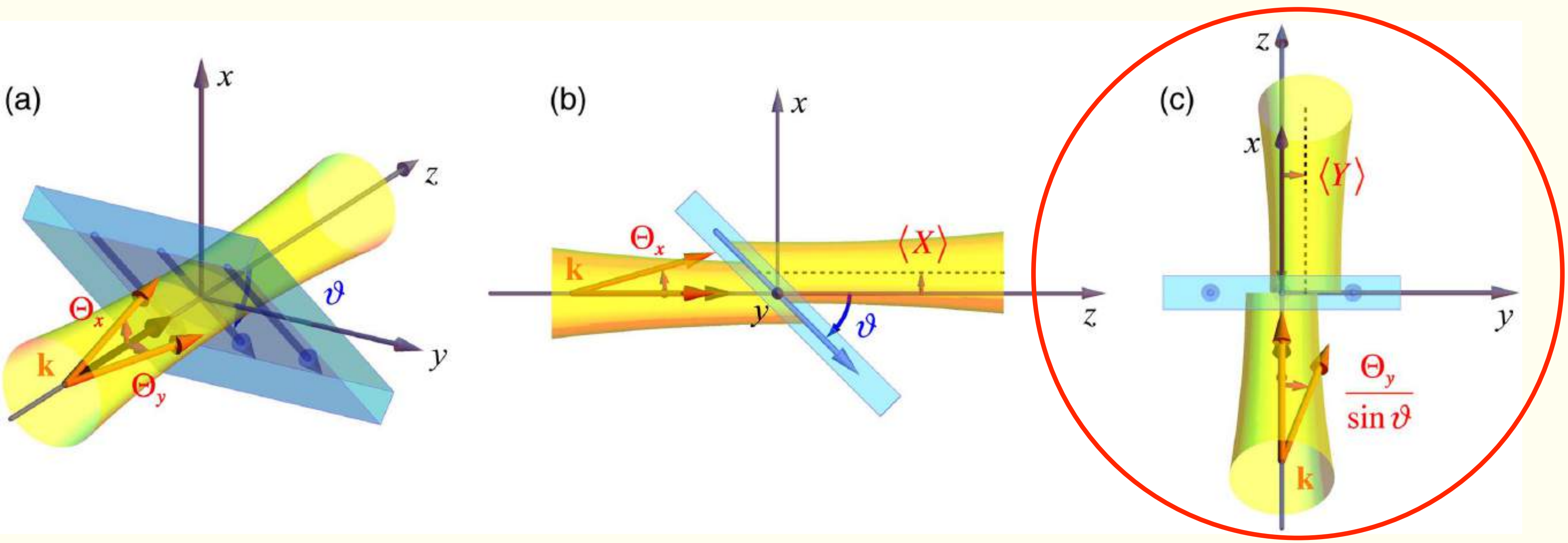
Spin-Hall effect at uniaxial wave plates

Research Article Vol. 3, No. 10 / October 2016 / Optica 1039

optica

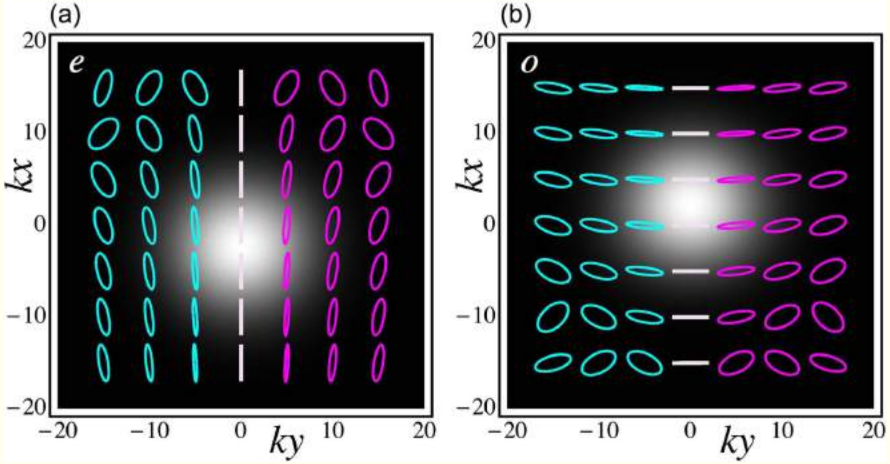
Spin-Hall effect and circular birefringence of a uniaxial crystal plate

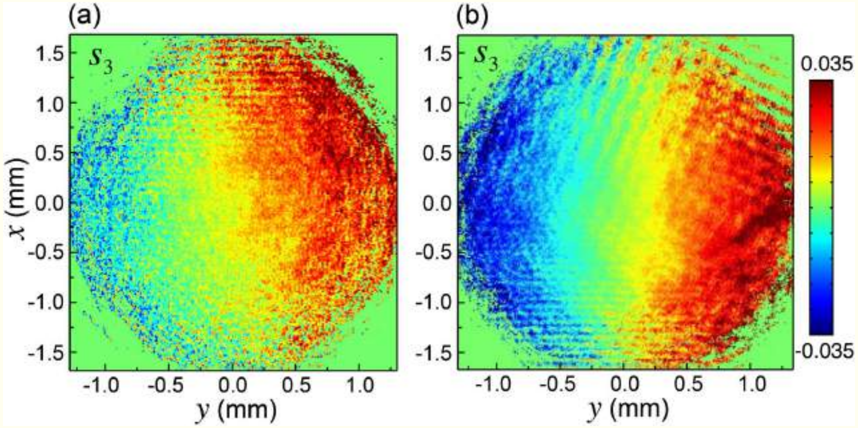
KONSTANTIN Y. BLIOKH,[1,2,*] C. T. SAMLAN,[3] CHANDRAVATI PRAJAPATI,[3] GRACIANA PUENTES,[4] NIRMAL K. VISWANATHAN,[3] AND FRANCO NORI[1,5]

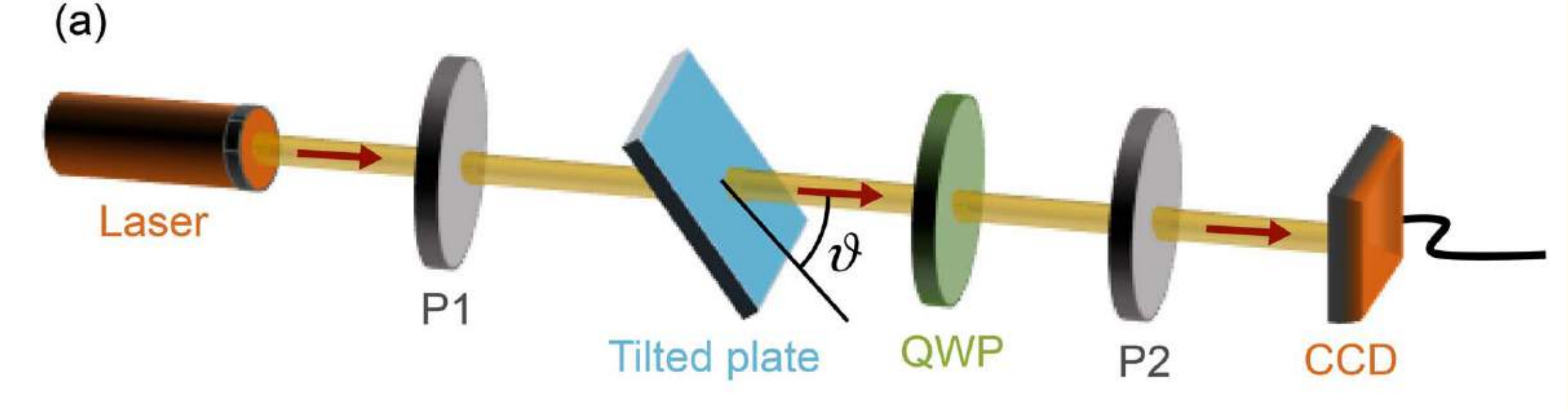


Spin-Hall effect at uniaxial wave plates

Output polarization distribution in the ordinary or extraordinary linearly-polarized Gaussian beam:

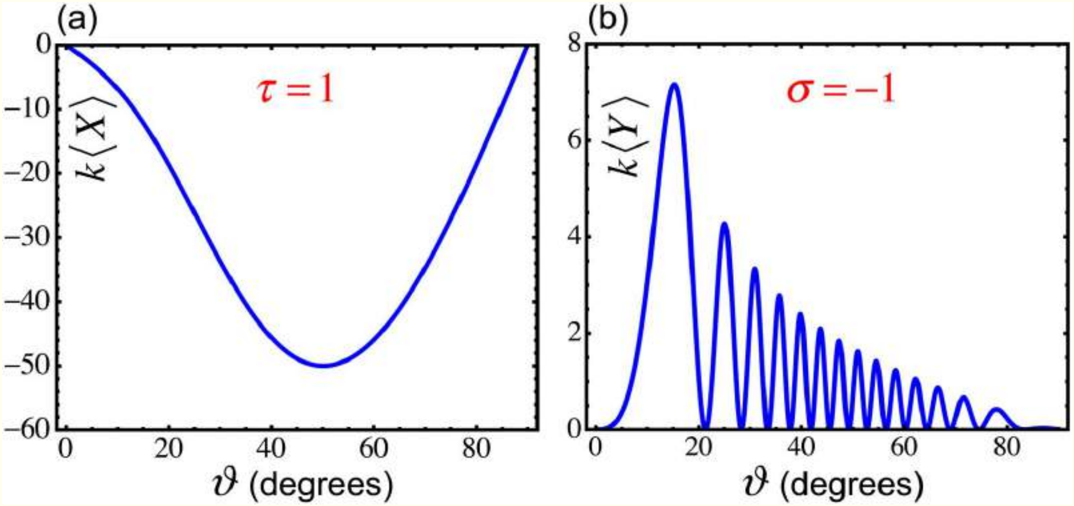


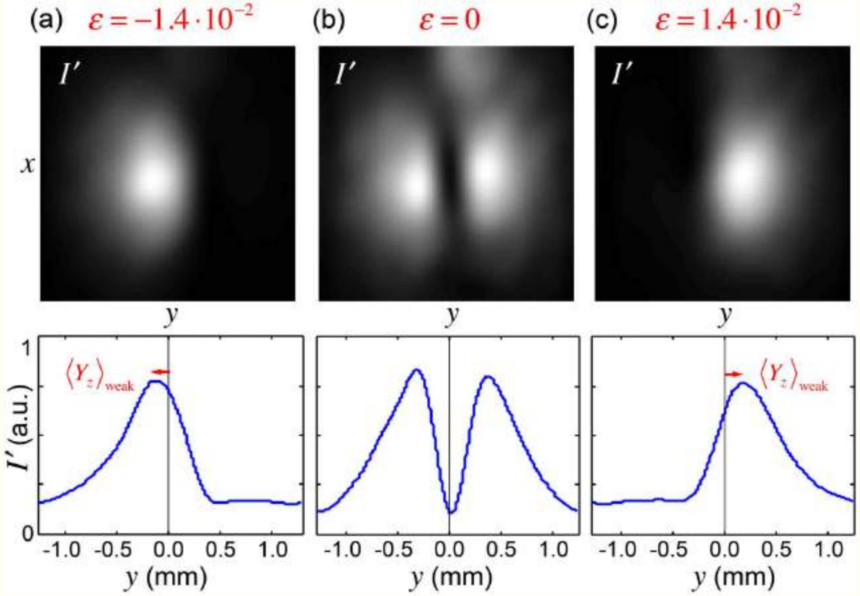


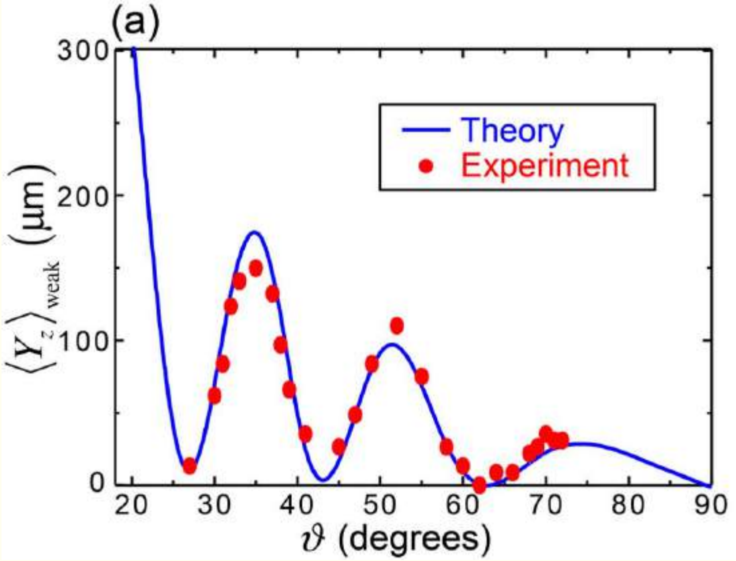


Spin-Hall effect at uniaxial wave plates

Beam shifts and weak measurements:







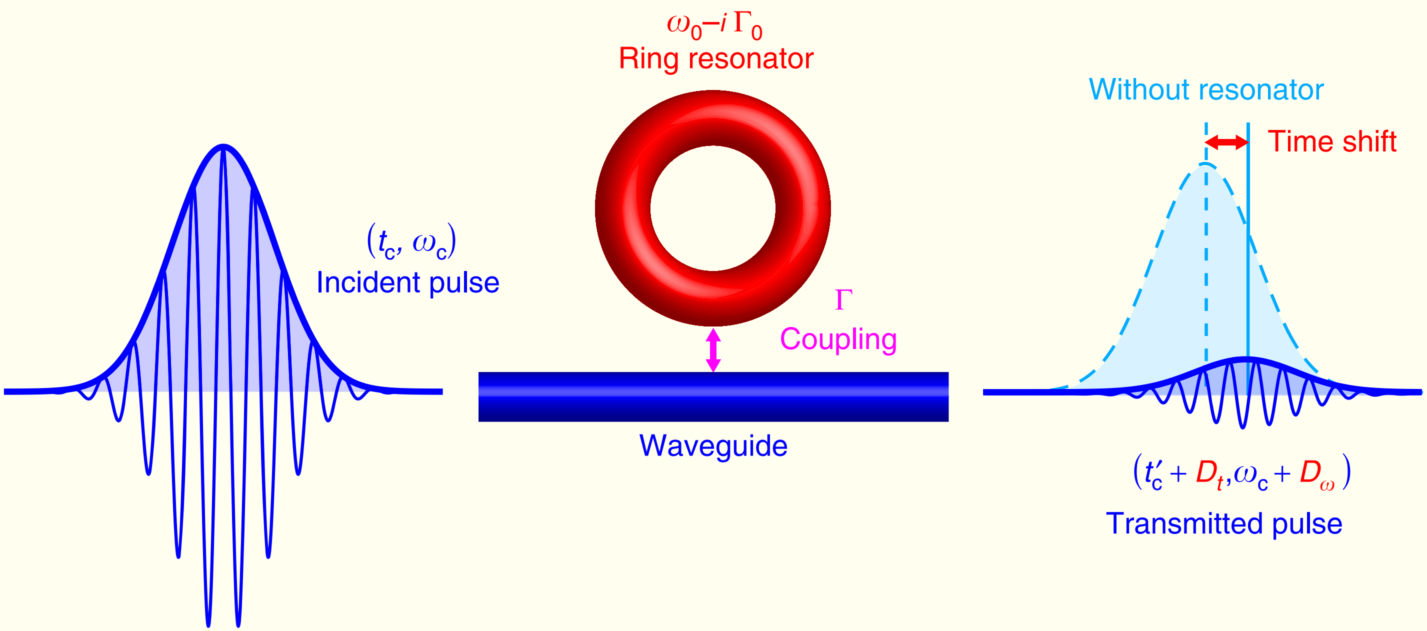
Beam shifts and weak measurements time domain

ARTICLE

Received 28 Jun 2016 | Accepted 7 Oct 2016 | Published 14 Nov 2016 DOI: 10.1038/ncomms13488 OPEN

Anomalous time delays and quantum weak measurements in optical micro-resonators

M. Asano[1,*], K.Y. Bliokh[2,3,*], Y.P. Bliokh[4,*], A.G. Kofman[2,5], R. Ikuta[1], T. Yamamoto[1], Y.S. Kivshar[3], L. Yang[6], N. Imoto[1], Ş.K. Özdemir[6] & F. Nori[2,7]



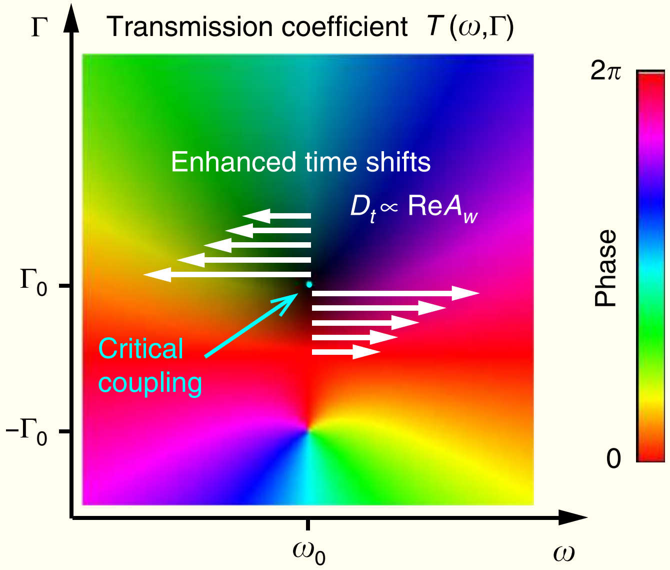
Wigner time delay and frequency shift

Beam shifts and weak measurements time domain

$$A_w = -i\frac{\partial \ln T(\omega_c)}{\partial \omega_c} = D$$

– complex Wigner time delay

It diverges near the **zero** of the transmission coefficient (**critical coupling** in non-Hermitian resonators):

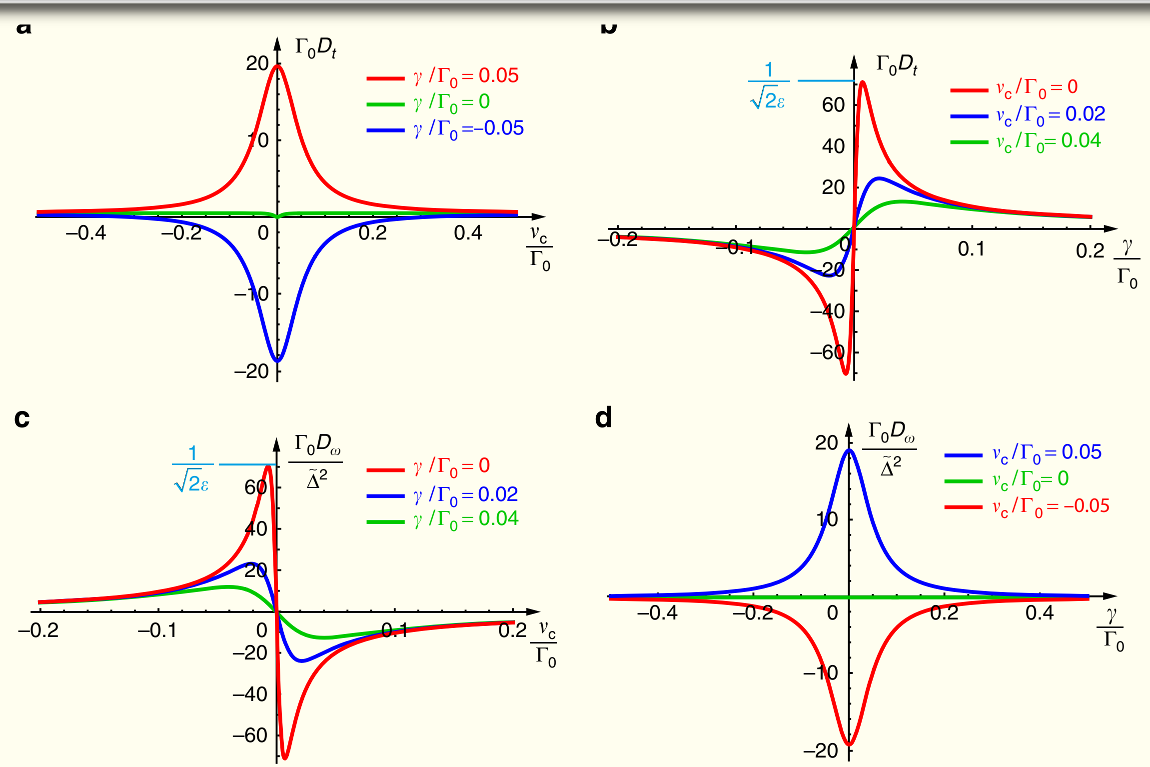


$$T(\omega,\Gamma) = \frac{(\omega-\omega_0) - i(\Gamma-\Gamma_0)}{(\omega-\omega_0) + i(\Gamma+\Gamma_0)}$$

Beam shifts and weak measurements time domain

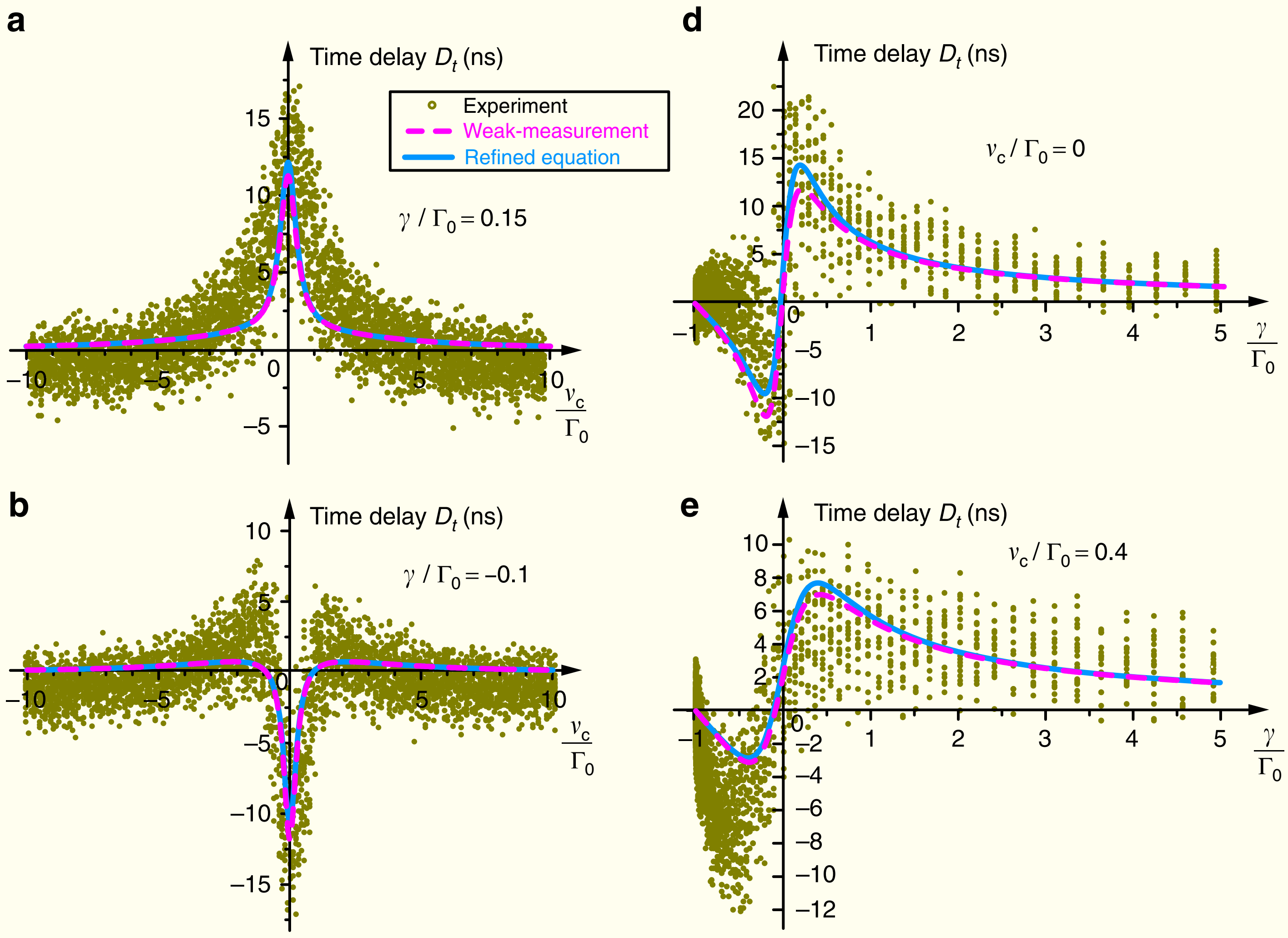
In this regime, the anomalous time/frequency shifts are given by the universal weak-measurement equations:

$$D_t = \frac{\mathrm{Re}\,A_w}{1+\tilde{\Delta}^2 |A_w|^2/2}, \qquad D_\omega = -\frac{\tilde{\Delta}^2\,\mathrm{Im}\,A_w}{1+\tilde{\Delta}^2 |A_w|^2/2}$$



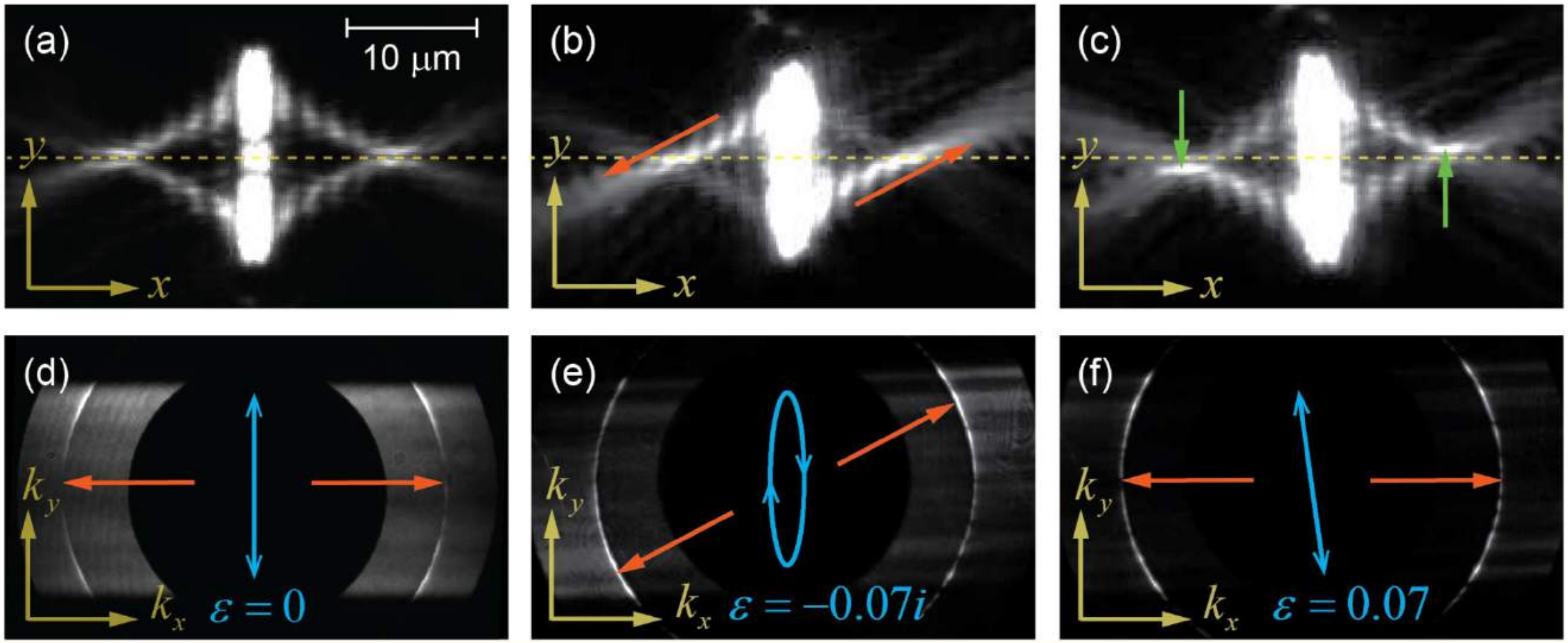
Beam shifts and weak measurements time domain

In this regime, the anomalous time/frequency shifts are given by the universal weak-measurement equations:



Reflection/refraction at an interface

The results SPP beam fields in the real and momentum (Fourier) spaces at different input polarizations:

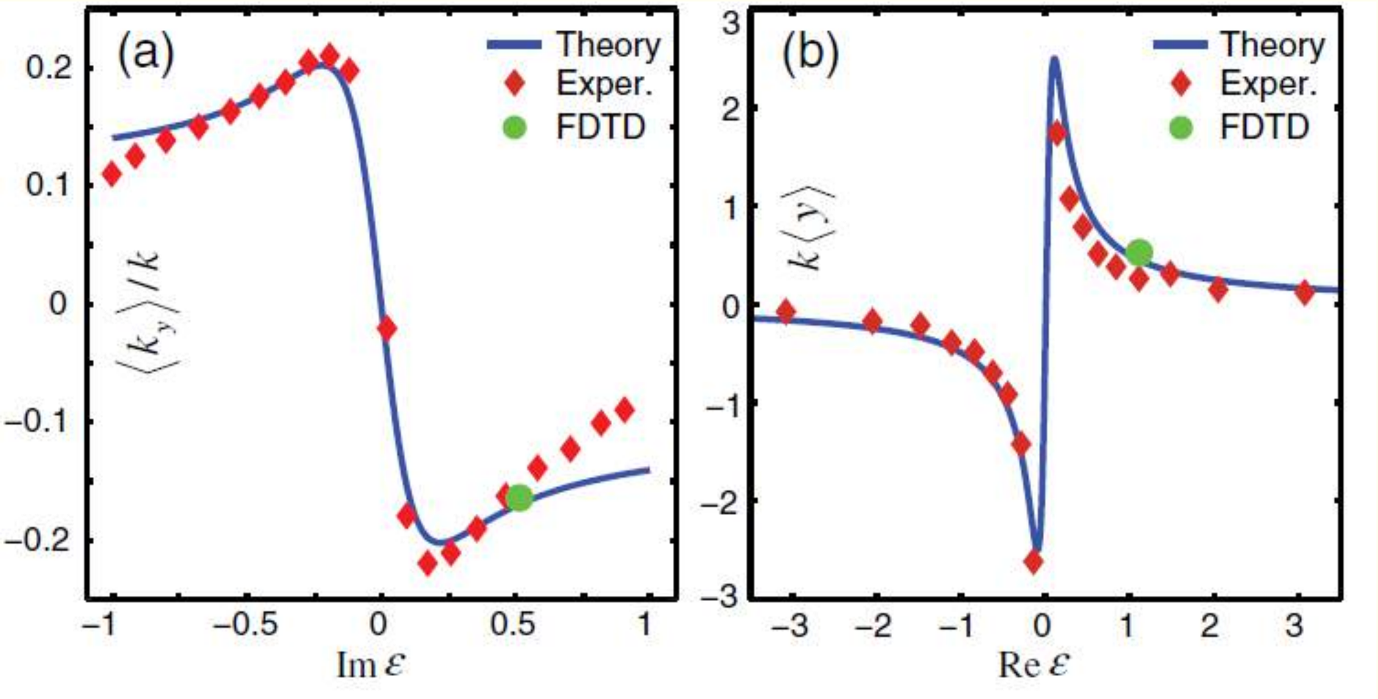


Small real and imaginary ε induces strong deformations and coordinate and momentum spin-Hall shifts in the SPP beams.

Gorodetski et al., PRL 2012

Reflection/refraction at an interface

The results are in perfect agreement with the FDTD and analytical calculations:



$$\langle y \rangle = \frac{1}{k\theta_c} \frac{2\chi\theta_c \operatorname{Re}\varepsilon}{2\chi^2 |\varepsilon|^2 + \theta_c^2/2} \qquad \langle k_y \rangle = -k\theta_c \frac{\chi\theta_c \operatorname{Im}\varepsilon}{2\chi^2 |\varepsilon|^2 + \theta_c^2/2}$$

THANK YOU!