

# Exciton-polariton based topological photonics and topological lasers

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Collaboration: J.Bloch's group, C2N, CNRS Paris, A. Amo, Phlam, Lille

- Introduction.
- Z topological insulator.
- Quantum Valley Hall effect.
- Quantum fluids:  $Z_2$  Topological Insulator for vortices.
- Topological Lasers

# 2D lattices (photonic)

→ Planar Fabry Perot cavity:  
2D parabolic dispersion for radiative photon modes

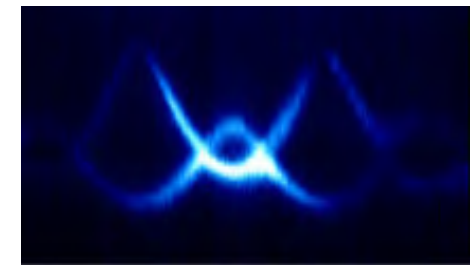
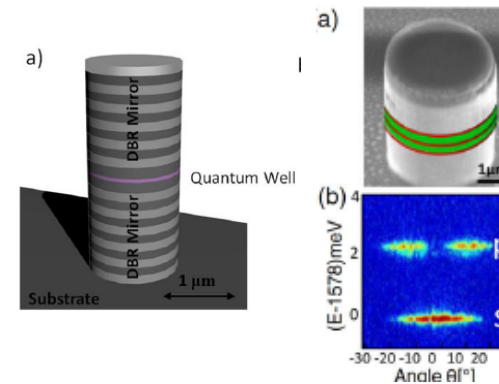
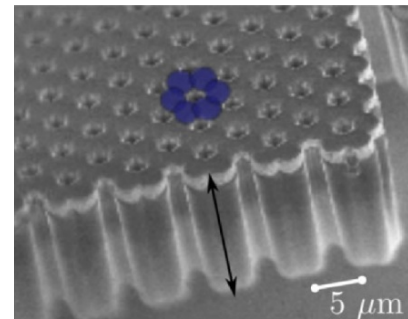
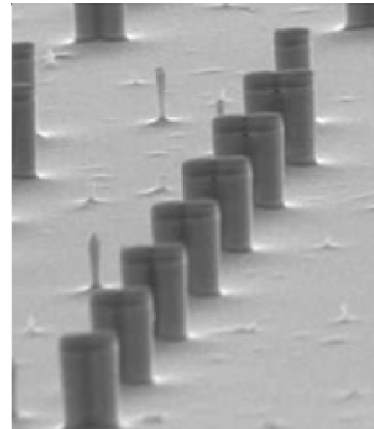
$$E_c(k) = E_c(0) + \frac{\hbar^2 k^2}{2m_c}$$

→ Lateral etching: 0D modes (photonic atoms).

→ Coupled cavities:  
Molecules, Lattices.  
Each atomic states  
gives a dispersive  
branch.

Good description with  
tight binding approach.  
but

Radiative modes  
TE and TM modes are  
close.



T. Jacqmin & al, Phys. Rev. Lett. 112, 116402 (2014) - C2N

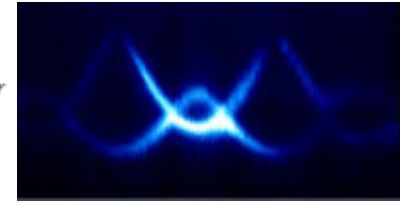
Exciton+photon → Exciton-Polariton

- Interacting photons.

- Zeeman splitting under magnetic field.

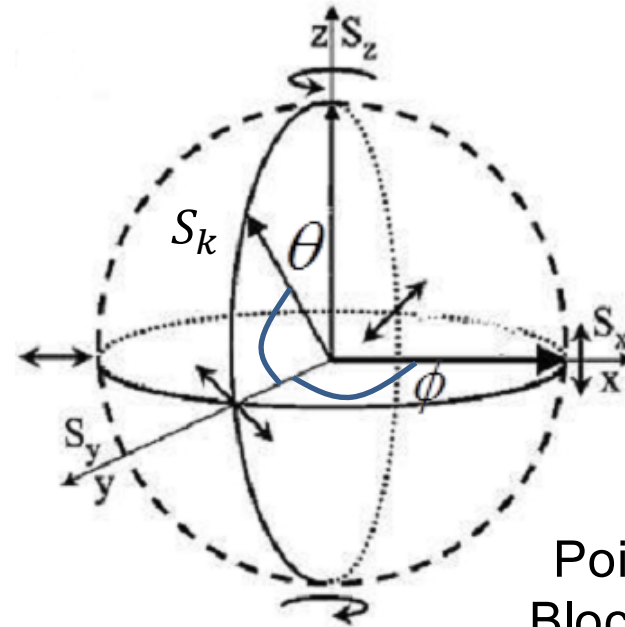
# Berry curvature and Chern number

- Spinor Wave function in a lattice:  $\psi_k = \begin{pmatrix} u_k^+ \\ u_k^- \end{pmatrix} e^{ikr} = \begin{pmatrix} \cos \frac{\theta_k}{2} e^{i\phi_k} \\ \sin \frac{\theta_k}{2} \end{pmatrix} e^{ikr}$

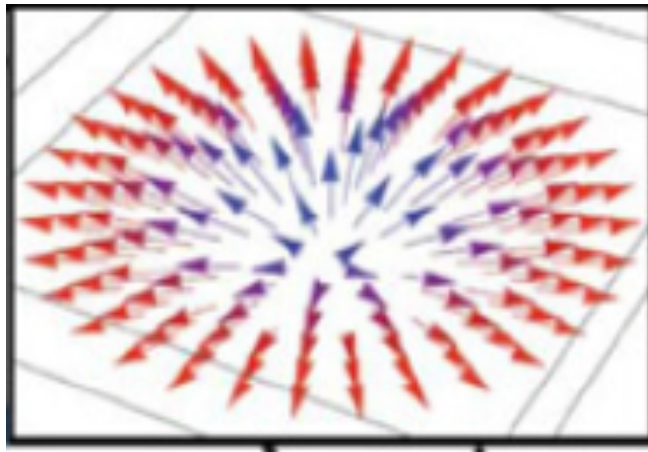


- Pseudo spin vector  $S_k$  associated to the wave function.
- Berry curvature is related to the change of  $S_k$  in reciprocal space:

$$B = \frac{1}{2} \sin \theta (\partial_x \theta \partial_y \phi - \partial_y \theta \partial_x \phi)$$



Poincaré or Bloch Sphere



**Pseudo spin texture in k-space**  
**Non-zero Berry curvature.**

**Chern Number:** Integral of the Berry curvature over a band in the first Brillouin zone.

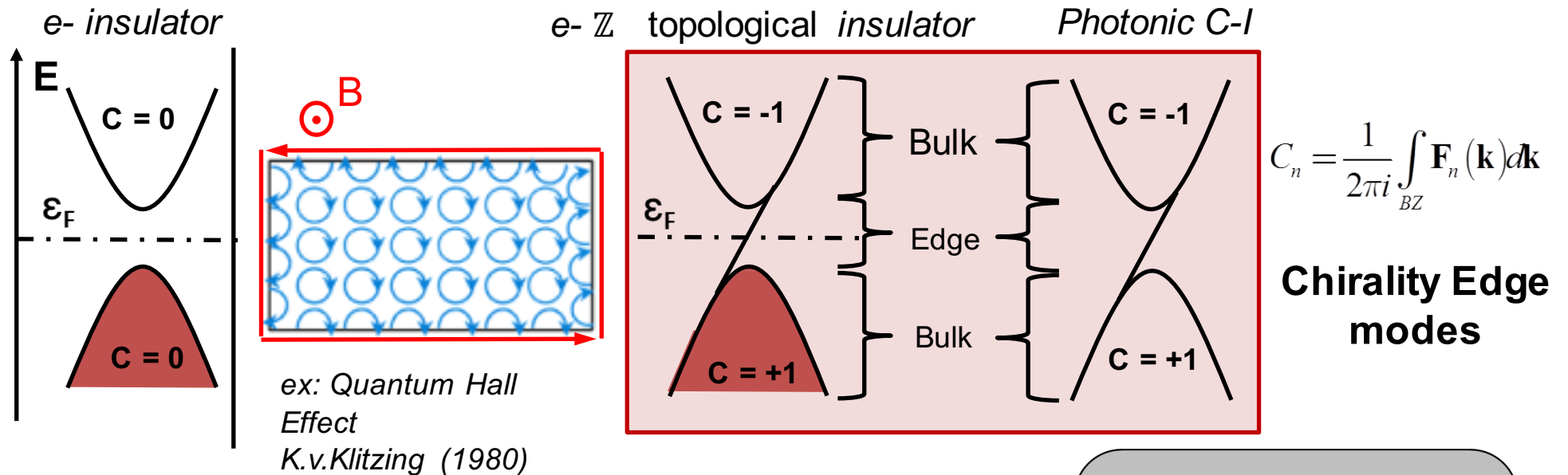
# $\mathbb{Z}$ topological insulator

Chern number of bands is a non-zero integer.

Quantum Hall Effect (1980).

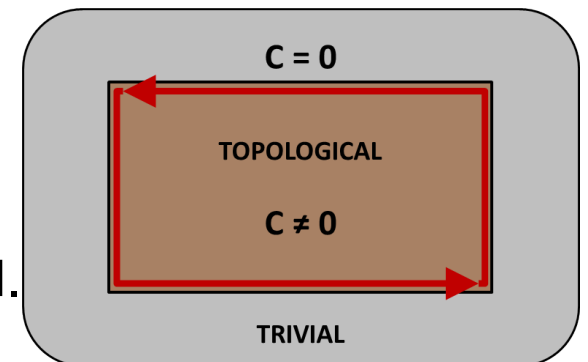
Quantum Anomalous Hall Effect (1988).

Broken Time Reversal symmetry (magnetic field).



A gap should close to change topology.  
The vacuum is trivial. Gap Closure on the interface.

One way edge modes, which cannot be elastically scattered.



# Intrinsic Chirality of Photons

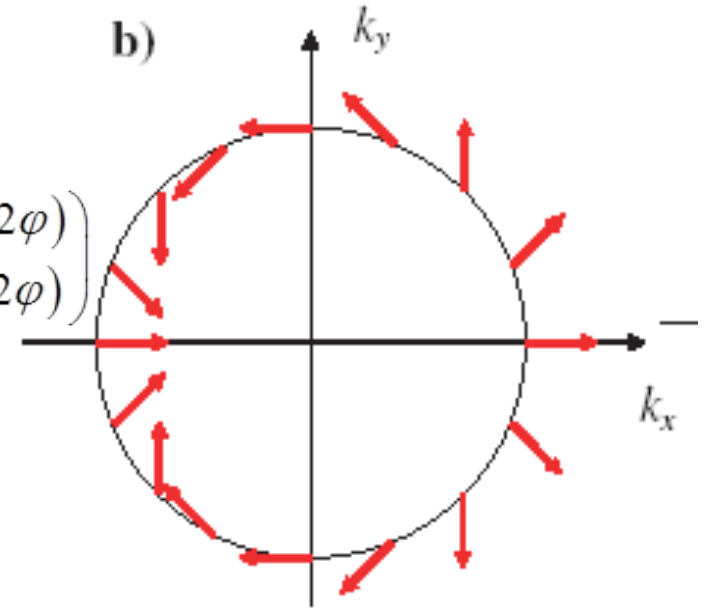
## 2 spin projections coupled by TE-TM Splitting

$$H = \begin{pmatrix} H_0(k) & \Omega_{LT}(k)e^{-2i\varphi} \\ \Omega_{LT}(k)e^{2i\varphi} & H_0(k) \end{pmatrix} = H_0\mathbf{I} + \vec{\Omega}_{LT} \cdot \vec{\sigma}$$

Spin-orbit coupling for light  
Optical Spin Hall effect

A. Kavokin, G. Malpuech, M. Glazov, *PRL*, 95, 136601 (2005).  
*Nature Phys.* 3, 628, (2007) .

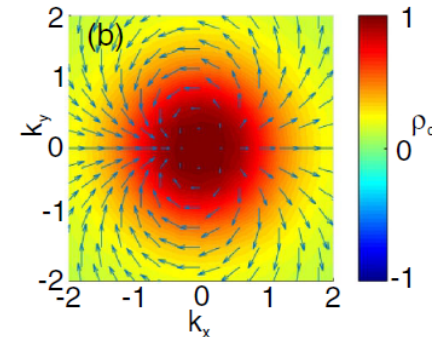
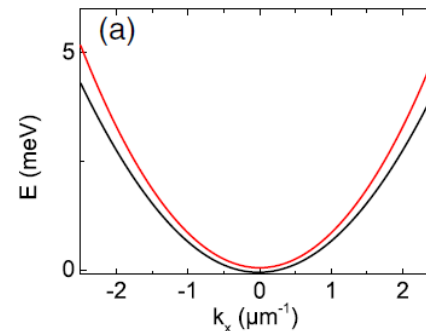
$$\Omega_{LT} \sim \beta k^2 \begin{pmatrix} \cos(2\varphi) \\ \sin(2\varphi) \end{pmatrix}$$



## TE-TM + Zeeman splitting

Berry curvature for photons (*PRL* 102, 046407, 2009)

$$H_0 = \begin{pmatrix} \frac{\hbar^2 k^2}{2m^*} + \Delta & \beta k^2 e^{2i\varphi} \\ \beta k^2 e^{-2i\varphi} & \frac{\hbar^2 k^2}{2m^*} - \Delta \end{pmatrix},$$



Photon/Polariton anomalous Hall effect in a planar cavity

*Arxiv* 2016, *PRL* 121, 020401 (2018). See O. Bleu Poster



# Chiral photons combined with a good lattice → Topological gaps

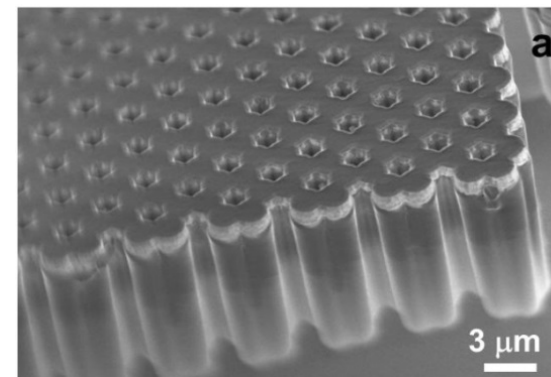
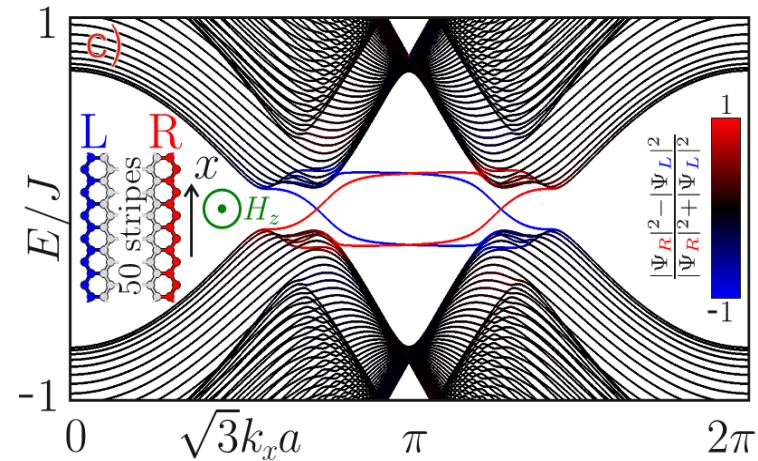
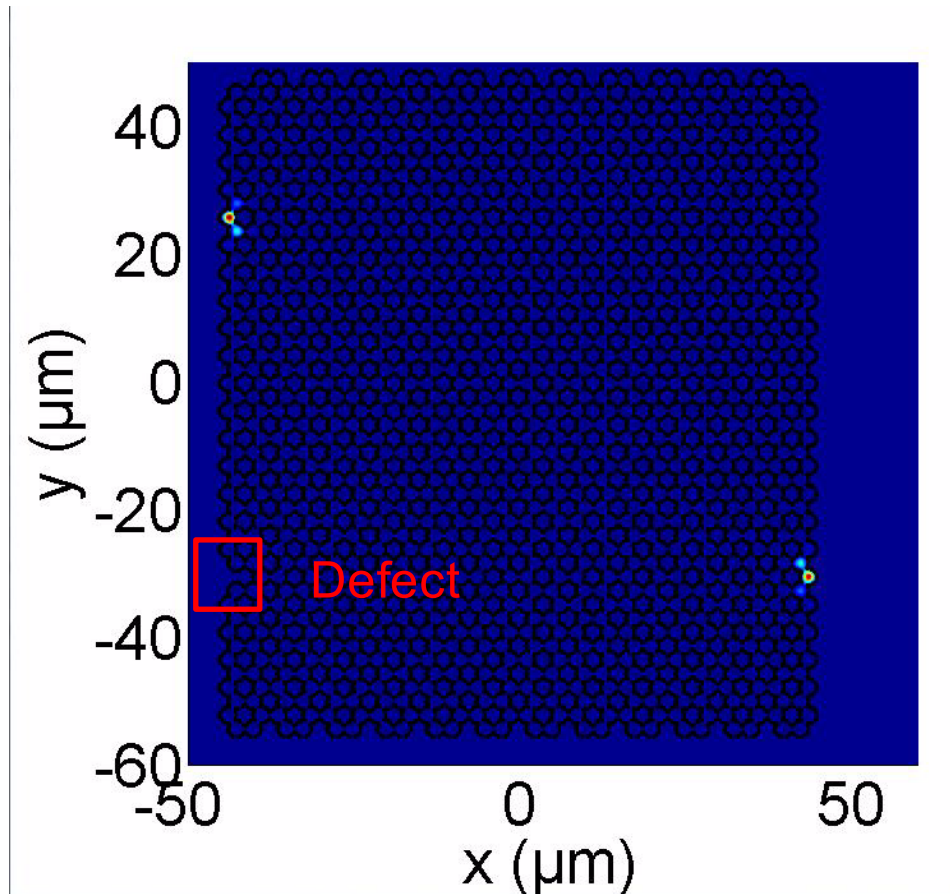
- Initial proposal Haldane-Raghu PRL 100, 013904 (2008).
- Observed at GHz frequencies Soljacic Group Nature 2009.

## Proposal for Exciton-polaritons at optical frequencies

A. Nalitov, D. Solnyshkov, and G. Malpuech, PRL 114, 116401, (2015).

O. Bleu, D. Solnyshkov, G. Malpuech, PRB, 95, 115415 (2017).

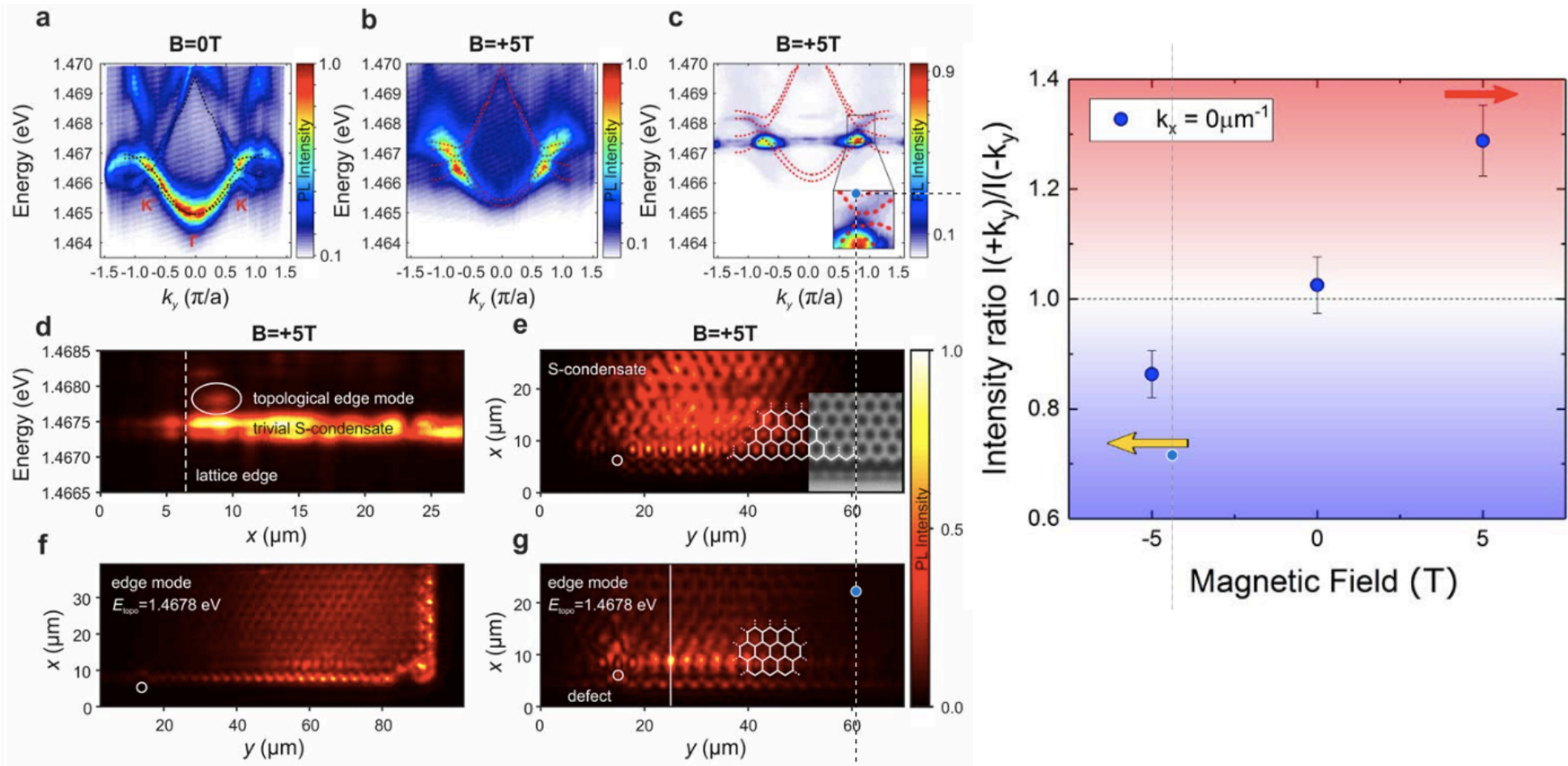
For realistic parameters  
Chern number  $\pm 2$



T. Jacqmin & al,  
Phys. Rev. Lett. 112, 116402 (2014)

# Edge modes under magnetic field reported in

S. Klembt et al. (Hofling group) Nature 562, 552, (2018).



# $\mathbb{Z}_2$ topological insulator

Quantum Spin Hall Effect (2005)

Total Chern number of Bands is zero.

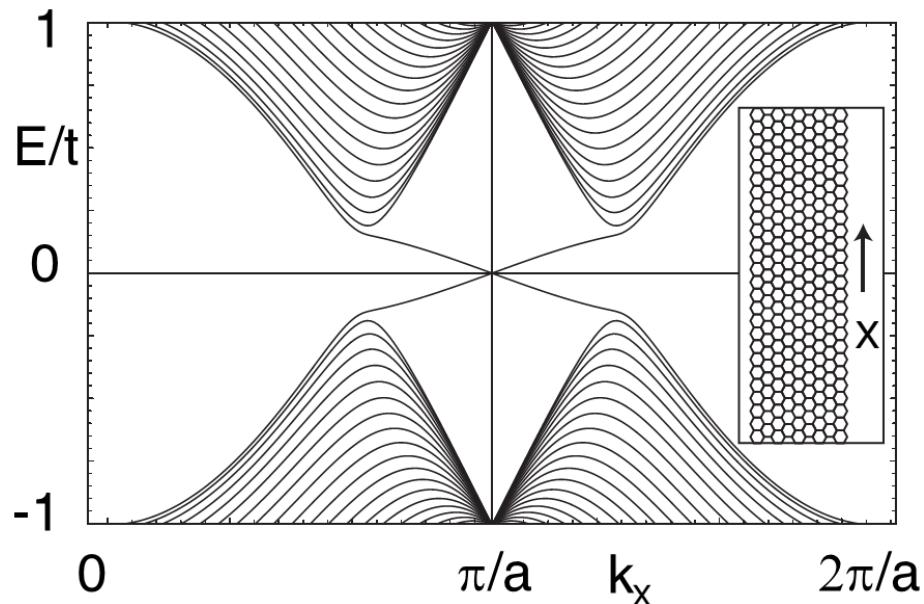
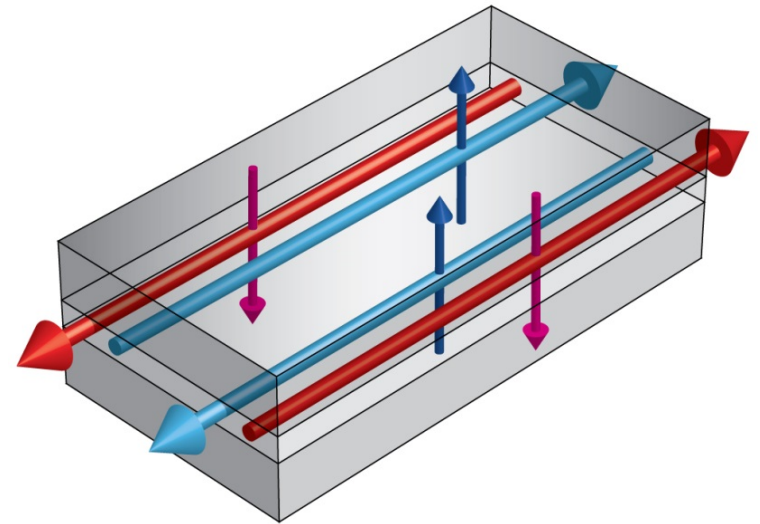
But Spin Chern number  $1/2(C_+ - C_-)$  is 1.

Spin current on the sample edge.

Time Reversal Symmetry (No magnetic field).

No spin conversion for electrons.

**Does not work for any spinor !!  
(for instance polarised photons)**



C.L.Kane, E.J.Mele,

Phys. Rev. Lett. **95**, 226801 (2005)





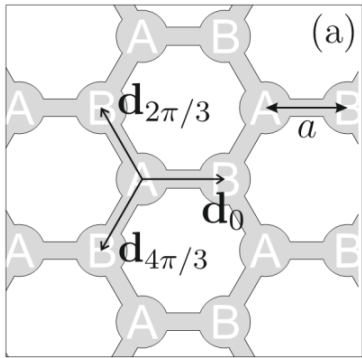
**Quantum (pseudo)-spin Hall effect**

**Quantum Valley Hall Effect**

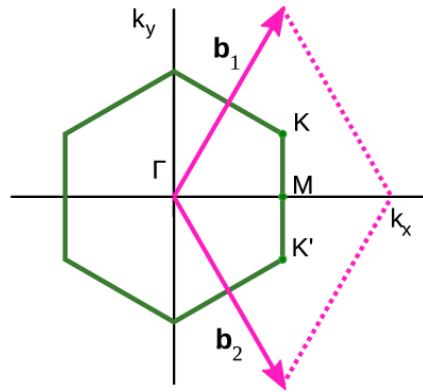
**How to make it robust (against disorder scattering  
for instance) ?**

# Honeycomb lattice (scalar case)

real space



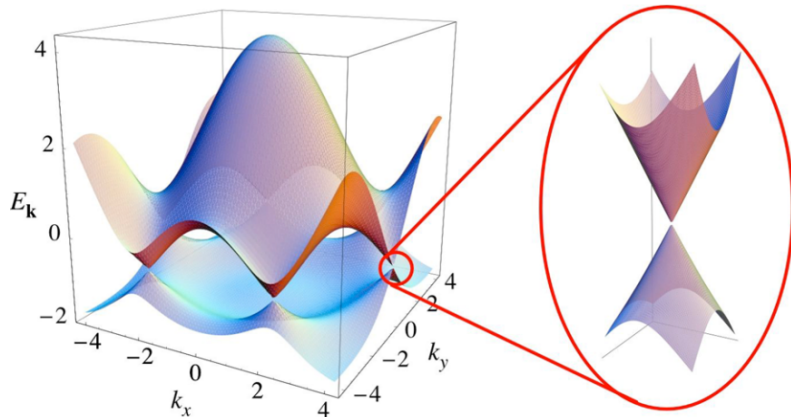
reciprocal space



Tight-Binding Hamiltonian

$$H_{\text{graphene}} = - \begin{pmatrix} 0 & Jf_k \\ Jf_k^* & 0 \end{pmatrix} \begin{matrix} A \\ B \end{matrix} \quad f_k = \sum_{j=1}^3 \exp(-ikd_{\varphi_j})$$

dispersion:



Close to K or K'

$$H \sim \tau_z \sigma_x k_x + \sigma_y k_y$$

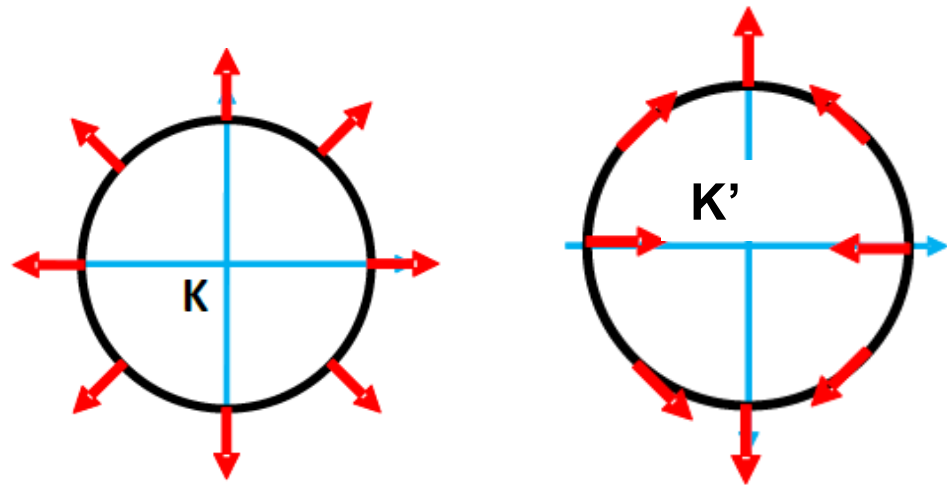
Effective field representation

$$H = \Omega_{\text{eff}} \hat{\sigma}$$

Sub-lattice pseudo-spin

$$\tau_z = \pm 1$$

$$\vec{\Omega}_{\text{eff}} \approx v(\tau_z k_x, k_y, 0)^T$$



Monopolar

Dresselhaus

Opposite winding at K and K'

# Quantum Valley Hall Effect

Let us make A and B different (staggered lattice).

$$H_{staggered} = - \begin{pmatrix} -\Delta & Jf_k \\ Jf_k^+ & \Delta \end{pmatrix} \approx - \left( J(\tau_z k_x \sigma_x + k_y \sigma_y) + \Delta \sigma_z \right)$$

Di Xiao, Wang Yao, and Qian Niu, *Phys. Rev. Lett.* 99, 236809, (2007).

## - Massive Dirac Hamiltonian.

- Gap opening.
- Berry curvature of opposite sign at K and K'.
- Valley dependent angular momentum.

**Valley = pseudo-spin**

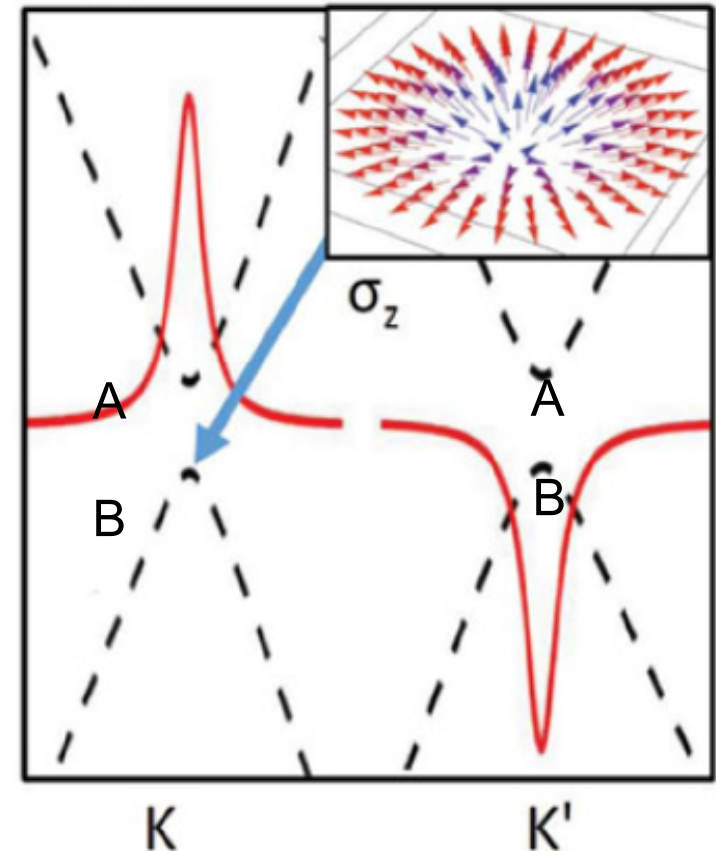
## -Definition of a Valley Chern Number $C_K = -C_{K'}$

$C_{KK'} = C_K - C_{K'} = 1$   $Z_2$  topological invariant, like in the Quantum Spin Hall effect at the zigzag interface between lattices of opposite staggering.

**Valley current of topological origin.**

## Remark

Topological states, but unprotected from inter-valley scattering.  
Same as QSHE, un-protected from inter-spin scattering.



# Photonic Quantum Valley Hall Effect

## In photonic crystal slabs:

- L.-H. Wu and X. Hu, Phys. Rev. Lett. 114 , 223901 (2015).
- T. Ma, A. B. Khanikaev, S. H. Mousavi, and G. Shvets, Phys. Rev. Lett. 114 , 127401 (2015).
- T. Ma and G. Shvets, New Journal of Physics, 18, 025012 (2016).
- L. Xu, H. Wang, Y. D. Xu, H. Y. Chen, and J.-H. Jiang, Optics Express 24, 18059 (2016).
- X.-D. Chen and J.-W. Dong, arXiv:1602.03352.
- ... ,Hafezi, New J. Phys. 18, 113013 (2016).
- .....
- Fan Zhang, « *Topological Valleytronics brought to light* » Nature Physics **14**, 111 (2018).

**Direct analog of Quantum Spin Hall Effect cannot be made for photons.**  
because

Photon (pseudo)-spin is not protected by Time Reversal Symmetry.

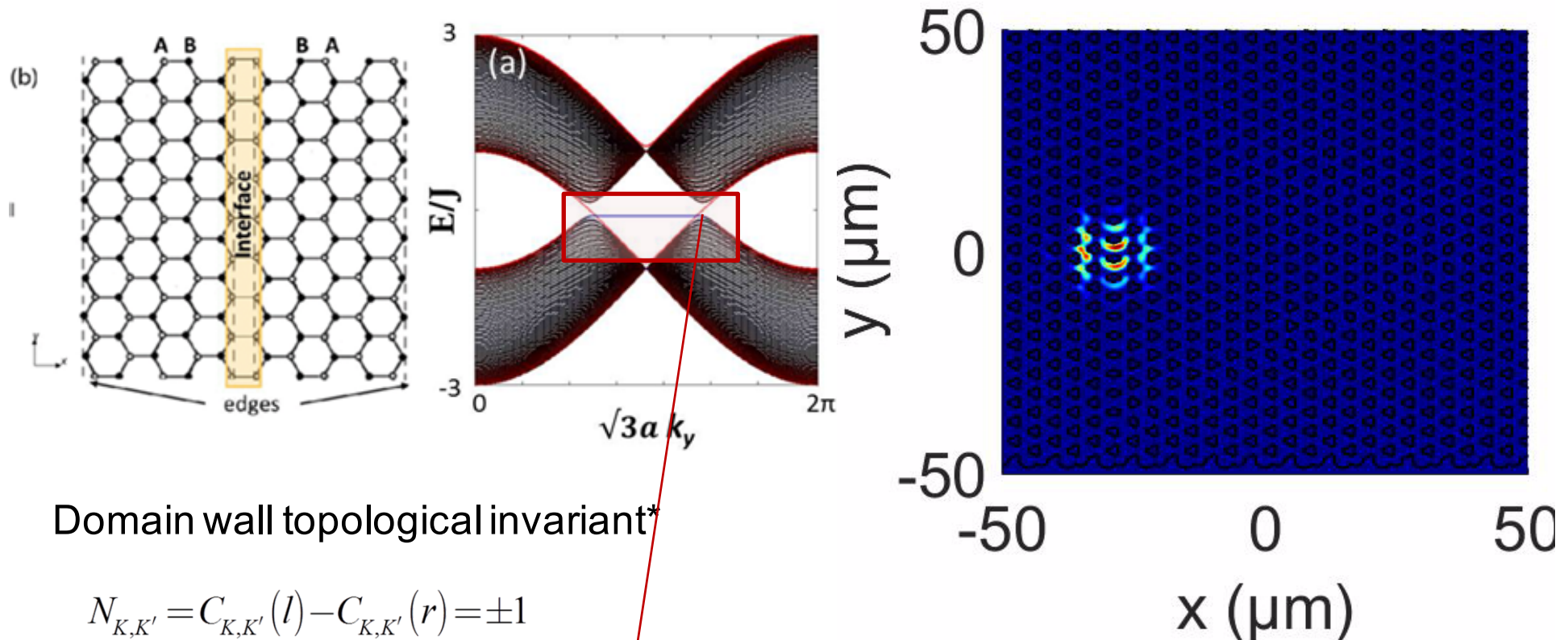
TE-TM splitting couples counter propagating spin states.

One needs to cancel competly TE-TM, which is demanding.

*A.B Khanikaev et al., Nature Materials 12, 233 (2013). Nat. Phot. 2017.*

# Photonic Quantum Valley Hall effect

Zig-zag interface between 2 opposite staggered lattices



Domain wall topological invariant\*

$$N_{K,K'} = C_{K,K'}(l) - C_{K,K'}(r) = \pm 1$$

→ 1 interface state in each valley

→ One valley, one group velocity.

**However, no protection against inter-valley scattering !!**

Valley pseudo-spin is protected by a spatial symmetry which is not fulfilled by random disorder.



# Interacting Quantum fluids in topological lattices

Condition: Presence of a **Bose Einstein Condensate** of exciton-polaritons at the Gamma point.

**BEC excitation: Quantized vortices in 2D. Vortex Core**

## Staggered honeycomb lattice.

- Vortex core composed by states near K and K' possessing an angular momentum.
- The quantum vortex winding is linked with the Valley.
- The Valley imposes a well defined propagation direction.

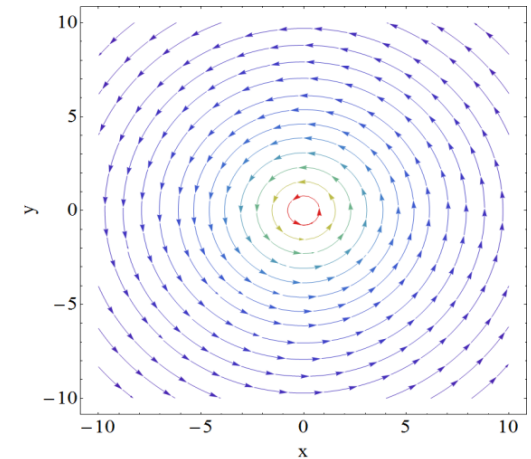
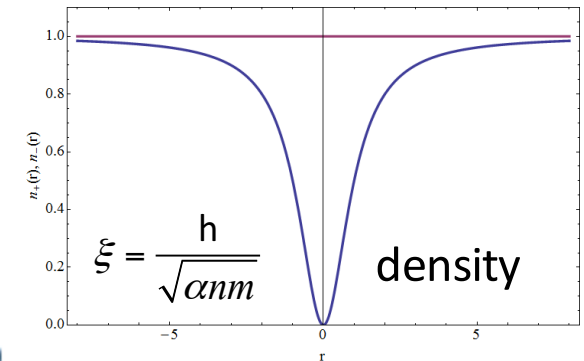
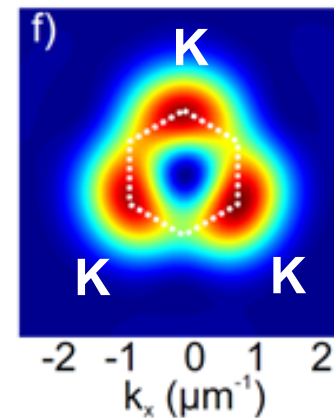
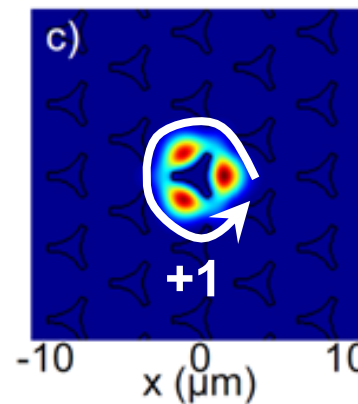
**Winding - Valley** coupling



**Valley - Propagation direction** coupling.

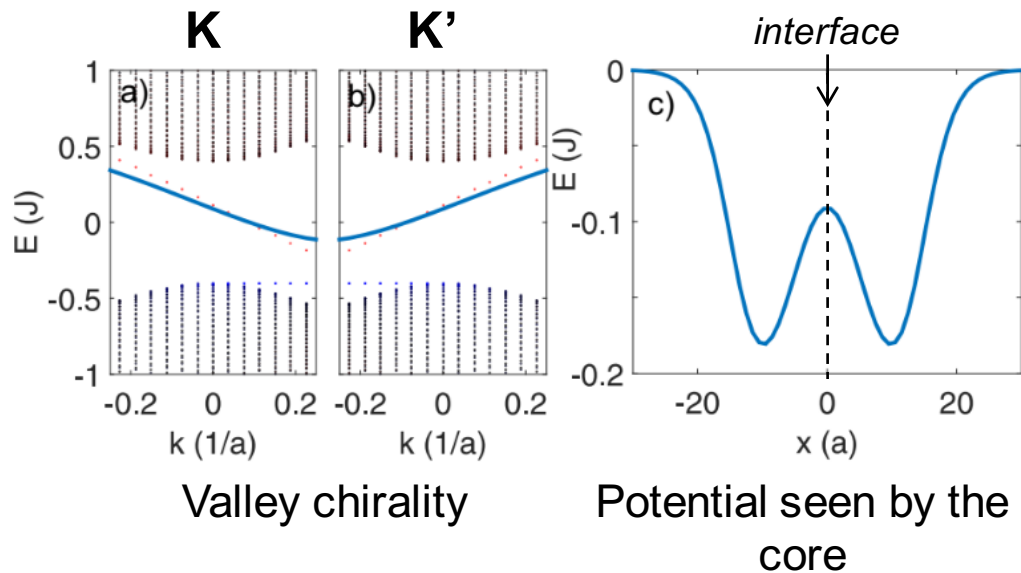
## Quantized vortex

$$\psi = \sqrt{n(r)} e^{ip\theta}$$



# Robust Quantum Valley Hall effect

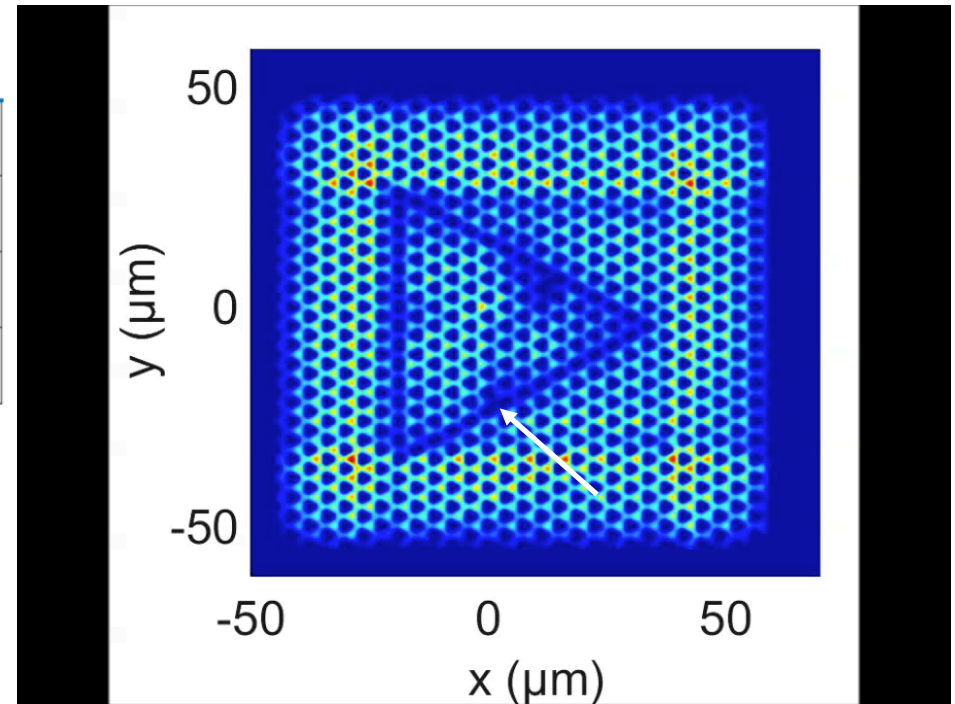
Vortex core inherits linear states chirality



GPE equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + \alpha |\psi|^2 \psi + U\psi - \mu\psi$$

Simulation of chiral vortex propagation



*Nat. Com. 9, 3991, (2018).*

→ **Robust chiral propagation** thanks to combination of real and momentum space topologies

**Non-linear analog of QSHE** : **vortex winding** replacing **electron spin**

**Topological protection of vortex winding** replaces the **TRS** protection of **electron spin**.

# Topological lasers

**Get Lasing in a topological mode.**

Not evident in the microwave range where a lot of experiments are carried out...

## Initial proposal for a 1D topological laser

PRL 116, 046402 (2016)

PHYSICAL REVIEW LETTERS

week ending  
29 JANUARY 2016

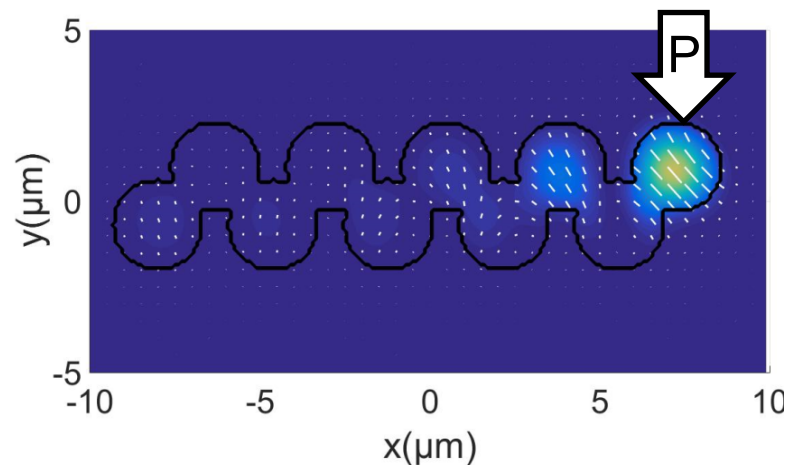
### Kibble-Zurek Mechanism in Topologically Nontrivial Zigzag Chains of Polariton Micropillars

D. D. Solnyshkov,<sup>1</sup> A. V. Nalitov,<sup>1,2</sup> and G. Malpuech<sup>1</sup>

<sup>1</sup>*Institut Pascal, PHOTON-N2, Université Clermont Auvergne, CNRS, 4 Avenue Blaise Pascal, 63178 Aubière Cedex, France*

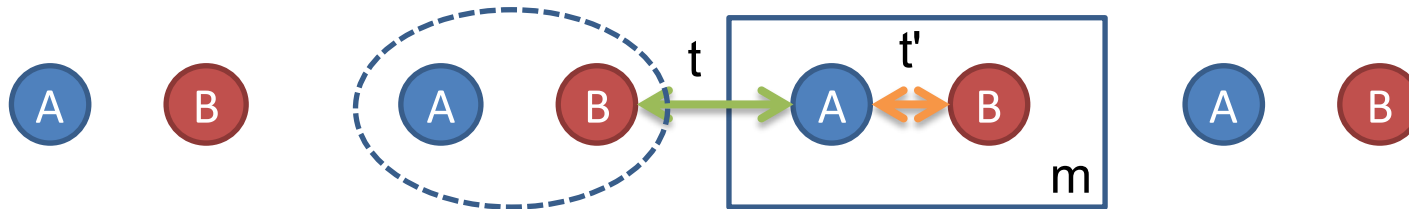
<sup>2</sup>*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*

(Received 15 June 2015; revised manuscript received 7 October 2015; published 29 January 2016)

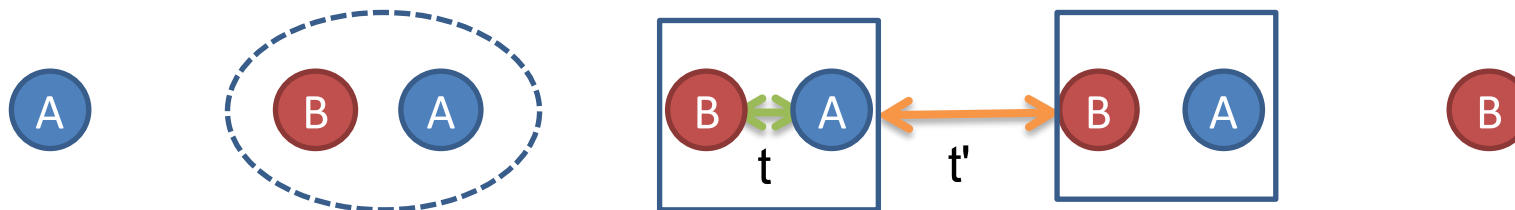


Pump on the edge.  
Condensation in the edge state.

# 1D: Dimer chain and edge states



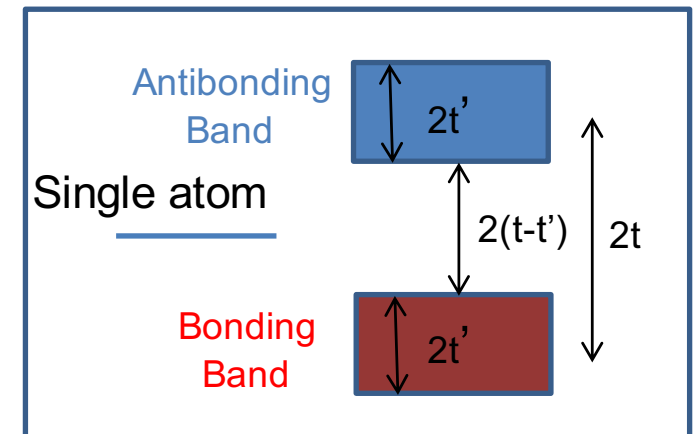
$t' > t$ : tightly bound pairs = “molecules” AB, no «extra» atoms  
 Two bands: AB in phase/out of phase (like s and p states of a single site)



$t' < t$ : tightly bound “molecules” BA; two «extra» atoms on the edges

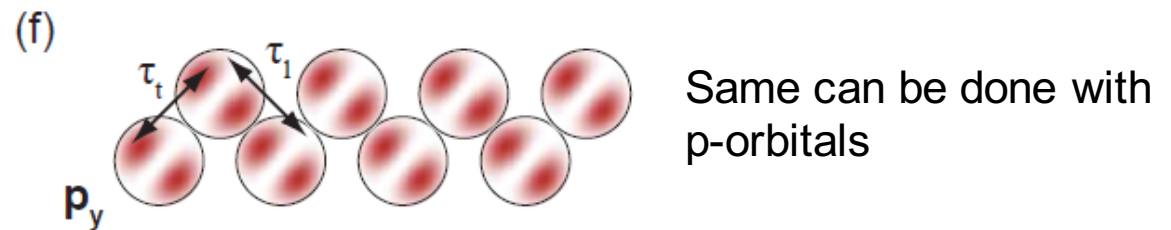
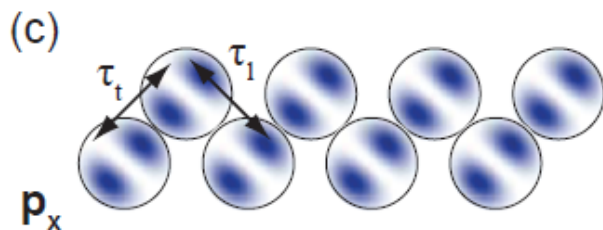
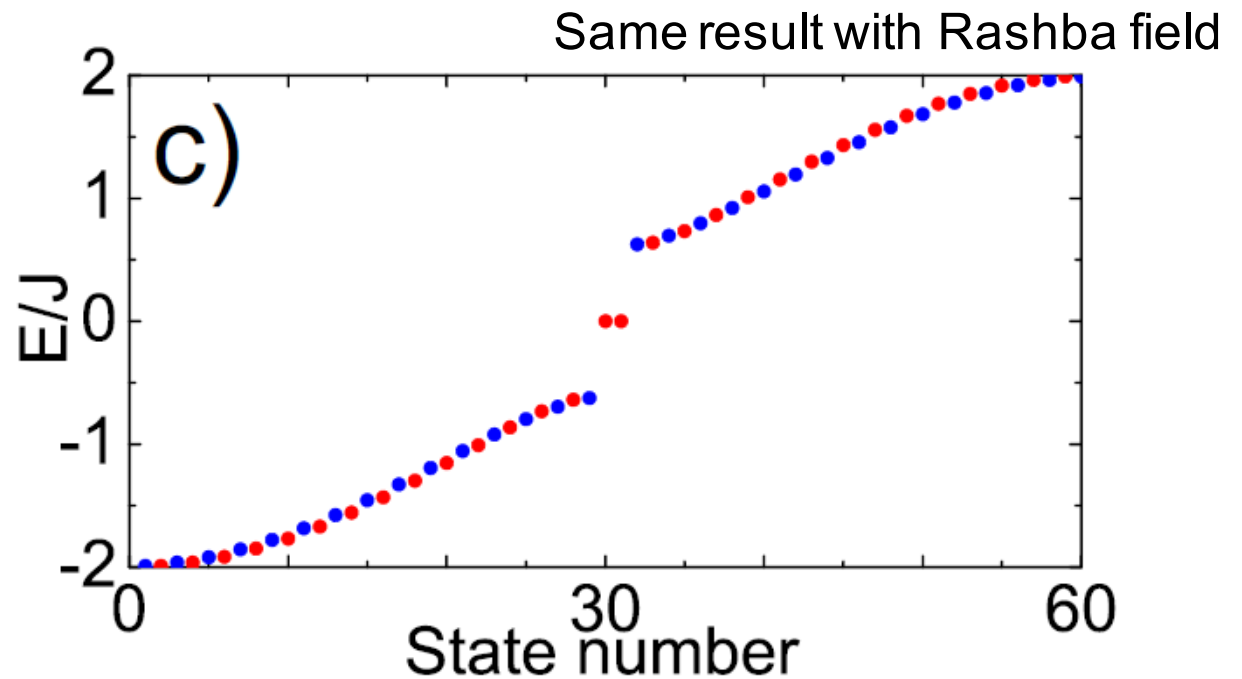
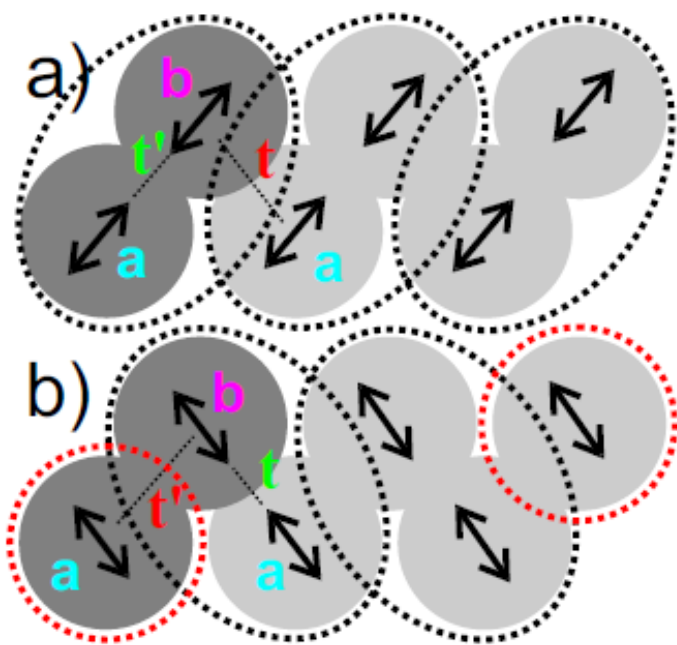
Topological quantity characterizing edge states –  
 the **Zak phase**

$$\gamma_n = \int_{-\pi/a}^{\pi/a} \frac{2\pi}{a} \int_0^a u_{nk}^*(x) i \frac{\partial u_{nk}(x)}{\partial k} dx dk$$



# Dimerization of a zigzag chain for polaritons

- Polarization-dependent coefficients  $t$  and  $t'$
- Tight-binding calculation of the eigenstates
- 0 edge states in D-polar, 2 edge states in A-polar





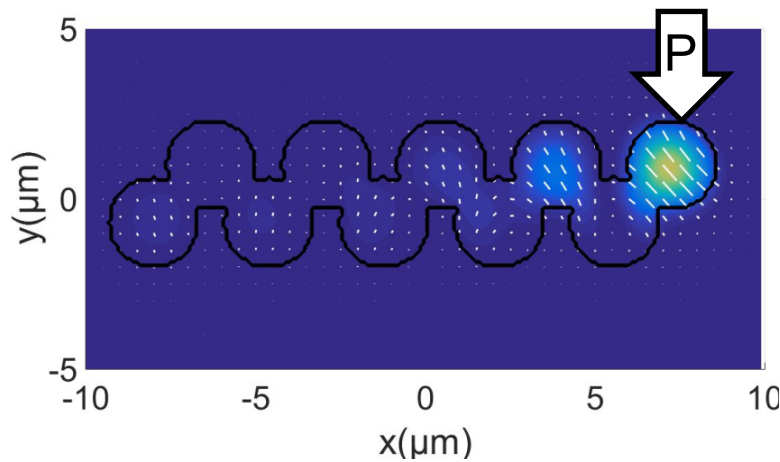
# Edge states in the condensation

- Edge states favored by higher overlap
- **Localized pumping**

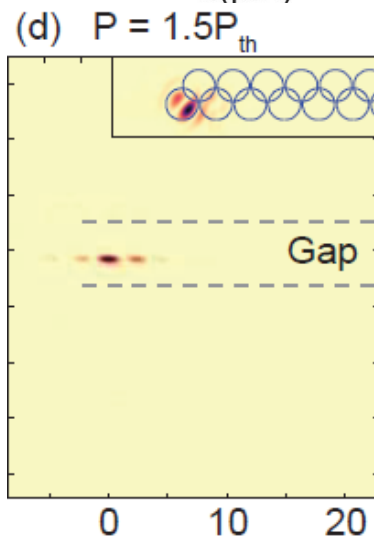
$$i\hbar \frac{\partial \psi_{\pm}}{\partial t} = - (1 - i\Lambda) \frac{\hbar^2}{2m} \Delta \psi_{\pm} + \beta \left( \frac{\partial}{\partial x} \mp i \frac{\partial}{\partial y} \right)^2 \psi_{\mp} \quad (4)$$

$$+ U \psi_{\pm} - \frac{i\hbar}{2\tau} \psi_{\pm} + ((U_R + i\gamma(n)) \psi_{\pm} + \xi) \exp \left( -\frac{(\mathbf{r}-\mathbf{r}_0)^2}{\sigma^2} \right)$$

*D. Solnyshkov et al., PRA 89, 033626 (2014).*



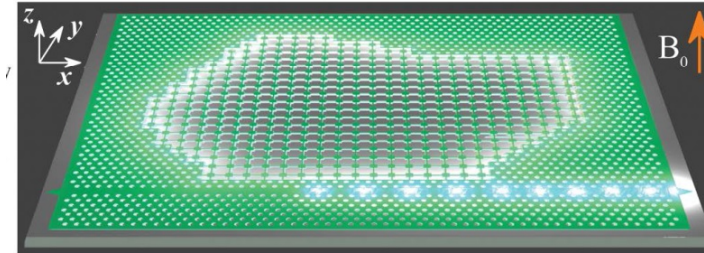
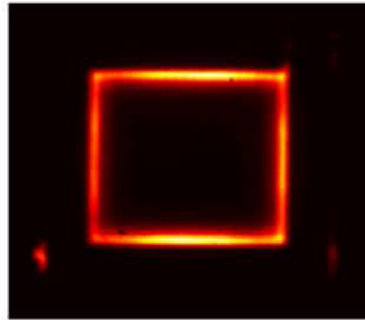
Pump on the edge.  
Condensation in the edge state.



## Observation of a 1D topological laser

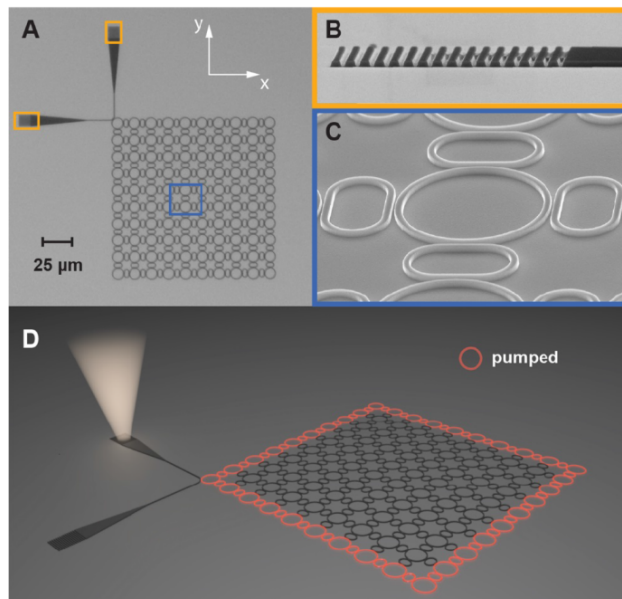
*P. Saint Jean et al. Nature Photonics, 11,651, (2017).*

# 2D Topological Lasers



**Based on Quantum Anomalous Hall Effect**

*Bahari et al, Science, 10.1126/scienceaa04551 (2017).*



**Based on Quantum Spin Hall Effect**

*Brandes et al, Science,  
10,1126/Scienceaar4005 (2018).*

# Conclusion

- **Intrinsic chirality of Photon modes → Z topological insulator.**
- **Quantum fluids makes Quantum Valley Hall effect robust.**
- **Topological lasers in 1D-chains.**