Exciton-polariton based topological photonics and topological lasers

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- Introduction.
- Z topological insulator.
- Quantum Valley Hall effect.
- Quantum fluids: Z$_2$ Topological Insulator for vortices.
- Topological Lasers
2D lattices (photonic)

→ Planar Fabry Perot cavity:
  2D parabolic dispersion for radiative photon modes

\[ E_C(k) = E_C(0) + \frac{\hbar^2 k^2}{2m_C} \]

→ Lateral etching: 0D modes (photonic atoms).

→ Coupled cavities:
  Molecules, Lattices.
  Each atomic states gives a dispersive branch.

Good description with tight binding approach. but
Radiative modes TE and TM modes are close.

Exciton+photon → Exciton-Polariton
  - Interacting photons.
  - Zeeman splitting under magnetic field.
Berry curvature and Chern number

• Spinor Wave function in a lattice: $\psi_k = \begin{pmatrix} u_k^- \\ u_k^+ \end{pmatrix} e^{i\phi} = \begin{pmatrix} \cos \frac{\theta_k}{2} e^{i\phi} \\ \sin \frac{\theta_k}{2} \end{pmatrix} e^{i\phi}$

• Pseudo spin vector $S_k$ associated to the wave function.

• Berry curvature is related to the change of $S_k$ in reciprocal space:

$$B = \frac{1}{2} \sin \theta (\partial_x \theta \partial_y \phi - \partial_y \theta \partial_x \phi)$$

Chern Number: Integral of the Berry curvature over a band in the first Brillouin zone.
A gap should close to change topology. The vacuum is trivial. Gap Closure on the interface.

One way edge modes, which cannot be elastically scattered.
Intrinsic Chirality of Photons

2 spin projections coupled by TE-TM Splitting

Spin-orbit coupling for light
Optical Spin Hall effect


TE-TM + Zeeman splitting

Berry curvature for photons (PRL 102, 046407, 2009)

Photon/Polariton anomalous Hall effect in a planar cavity

Arxiv 2016, PRL 121, 020401 (2018). See O. Bleu Poster
Chiral photons combined with a good lattice $\rightarrow$ Topological gaps

$\rightarrow$ Initial proposal Haldane-Raghu PRL 100, 013904 (2008).
$\rightarrow$ Observed at GHz frequencies Soljacic Group Nature 2009.

Proposal for Exciton-polaritons at optical frequencies

For realistic parameters
Chern number $\pm 2$

Defect

T. Jacqmin & al,
Edge modes under magnetic field reported in S. Klembt et al. (Hofling group) Nature 562, 552, (2018).
**$\mathbb{Z}_2$ topological insulator**

Quantum *Spin* Hall Effect (2005)

Total Chern number of Bands is zero.
But Spin Chern number $1/2(C_+ - C_-)$ is 1.
Spin current on the sample edge.
Time Reversal Symmetry (No magnetic field).
No spin conversion for electrons.

**Does not work for any spinor!!**
(for instance polarised photons)

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C.L.Kane, E.J.Mele,
Quantum (pseudo)-spin Hall effect

Quantum Valley Hall Effect

How to make it robust (against disorder scattering for instance)?
Honeycomb lattice (scalar case)

Real space

Reciprocal space

Tight-Binding Hamiltonian

\[
H_{\text{graphene}} = -\begin{pmatrix}
0 & Jf_k \\
Jf_k^* & 0
\end{pmatrix}
\]

\[
f_k = \sum_{j=1}^{3} \exp(-ikd_{\varphi_j})
\]

Dispersion:

Close to K or K'

\[
H \sim \tau_z \sigma_x k_x + \sigma_y k_y
\]

Effective field representation

\[
H = \Omega_{\text{eff}} \hat{\sigma}
\]

Sub-lattice pseudo-spin

\[
\tau_z = \pm 1
\]

\[
\tilde{\Omega}_{\text{eff}} \approx \nu \begin{pmatrix}
\tau_z k_x, k_y, 0
\end{pmatrix}^T
\]

Opposite winding at K and K'
Let us make A and B different (staggered lattice).

\[
H_{\text{staggered}} = -\begin{pmatrix}
-\Delta & Jf_k^- \\
Jf_k^+ & \Delta \\
\end{pmatrix} \approx -\left( J \left( \tau_z k_x \sigma_x + k_y \sigma_y \right) + \Delta \sigma_z \right)
\]


- **Massive Dirac Hamiltonian.**
- Gap opening.
- Berry curvature of opposite sign at K and K'.
- Valley dependent angular momentum.

Valley = pseudo-spin

- Definition of a Valley Chern Number \( C_K = -C_{K'} \)

\[
C_{KK'} = C_K - C_{K'} = 1 \quad Z_2 \text{ topological invariant, like in the Quantum Spin Hall effect at the zigzag interface between lattices of opposite staggering.}
\]

Valley current of topological origin.

**Remark**

Topological states, but unprotected from inter-valley scattering.
Same as QSHE, un-protected from inter-spin scattering.
In photonic crystal slabs:

- ….

**Direct analog of Quantum Spin Hall Effect cannot be made for photons.**

because

Photon (pseudo)-spin is not protected by Time Reversal Symmetry.

TE-TM splitting couples counter propagating spin states.

One needs to cancel competly TE-TM, which is demanding.

Photonic Quantum Valley Hall effect

Zig-zag interface between 2 opposite staggered lattices

Domain wall topological invariant*

$$ N_{k,k'} = C_{k,k'}(l) - C_{k,k'}(r) = \pm 1 $$

→ 1 interface state in each valley
→ One valley, one group velocity.

However, no protection against inter-valley scattering!!

Valley pseudo-spin is protected by a spatial symmetry which is not fulfilled by random disorder.

Condition: Presence of a Bose Einstein Condensate of exciton-polaritons at the Gamma point.

**BEC excitation:** Quantized vortices in 2D. **Vortex Core**

**Staggered honeycomb lattice.**
- Vortex core composed by states near $K$ and $K'$ possessing an angular momentum.
- The quantum vortex winding is linked with the Valley.
- The Valley imposes a well defined propagation direction.

**Winding - Valley coupling**

**Valley – Propagation direction** coupling.

Quantized vortex

$$\psi = \sqrt{n(r)}e^{ip\theta}$$

Vortex density

$$\rho = \frac{\hbar}{\sqrt{c\alpha n}m}$$
Robust Quantum Valley Hall effect

Vortex core inherits linear states chirality

\[
\frac{i\hbar}{\partial t} = -\frac{\hbar^2}{2m}\Delta \psi + \alpha|\psi|^2 \psi + U\psi - \mu \psi
\]

→ Robust chiral propagation thanks to combination of real and momentum space topologies

Non-linear analog of QSHE: vortex winding replacing electron spin

Topological protection of vortex winding replaces the TRS protection of electron spin.

[Image: Simulation of chiral vortex propagation]
Topological lasers

Get Lasing in a topological mode.
Not evident in the microwave range where a lot of experiments are carried out…

Initial proposal for a 1D topological laser

Pump on the edge.
Condensation in the edge state.
1D: Dimer chain and edge states

\( t' > t \): tightly bound pairs = “molecules” \( \text{AB} \), no «extra» atoms
Two bands: \( \text{AB} \) in phase/out of phase (like s and p states of a single site)

\( t' < t \): tightly bound “molecules” \( \text{BA} \); two «extra» atoms on the edges

Topological quantity characterizing edge states – the Zak phase

\[
\gamma_n = \int_{-\pi/a}^{-\pi/a} \frac{2\pi}{a} \int_{0}^{a} u_{nk}^* (x) i \frac{\partial u_{nk} (x)}{\partial k} \, dx \, dk
\]
Dimerization of a zigzag chain for polaritons

- Polarization-dependent coefficients $t$ and $t'$
- Tight-binding calculation of the eigenstates
- 0 edge states in D-polar, 2 edge states in A-polar

Same result with Rashba field

Same can be done with $p$-orbitals
Edge states in the condensation

• Edge states favored by higher overlap

• Localized pumping

\[ i\hbar \frac{\partial \psi_{\pm}}{\partial t} = - (1 - i\Lambda) \frac{\hbar^2}{2m} \Delta \psi_{\pm} + \beta \left( \frac{\partial}{\partial x} \mp i \frac{\partial}{\partial y} \right)^2 \psi_{\pm} \]  
\[ + U \psi_{\pm} - \frac{i\hbar}{2\sigma} \psi_{\pm} + ((U_R + i\gamma(n)) \psi_{\pm} + \xi) \exp \left( - \frac{(r-r_0)^2}{\sigma^2} \right) \]  

D. Solnyshkov et al., PRA 89, 033626 (2014).

Pump on the edge.
Condensation in the edge state.

Observation of a 1D topological laser
2D Topological Lasers

Based on Quantum Anomalous Hall Effect

Based on Quantum Spin Hall Effect
Conclusion

- Intrinsic chirality of Photon modes $\rightarrow$ Z topological insulator.
- Quantum fluids makes Quantum Valley Hall effect robust.
- Topological lasers in 1D-chains.