Orbital edge states in a photonic honeycomb lattice

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Laboratoire de Physique des Solides
Université Paris-Sud, Orsay, France

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BEC Trento → RIKEN

Aussois, November 27th 2018

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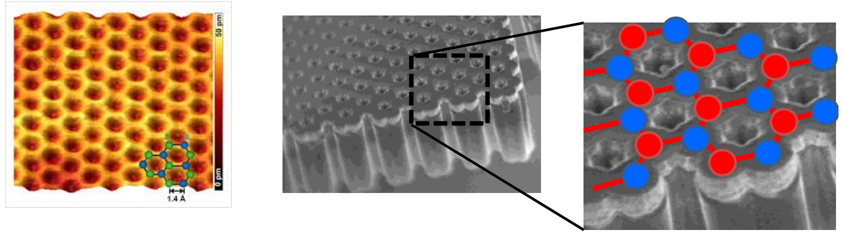
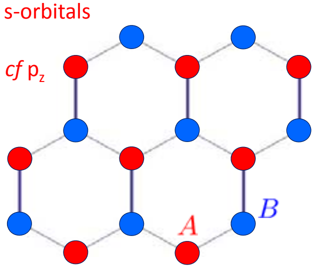
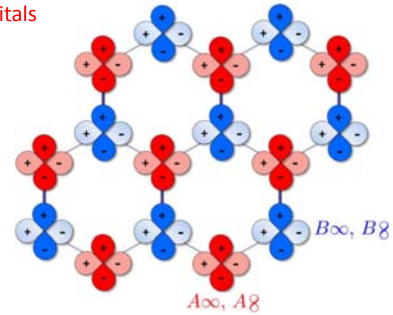
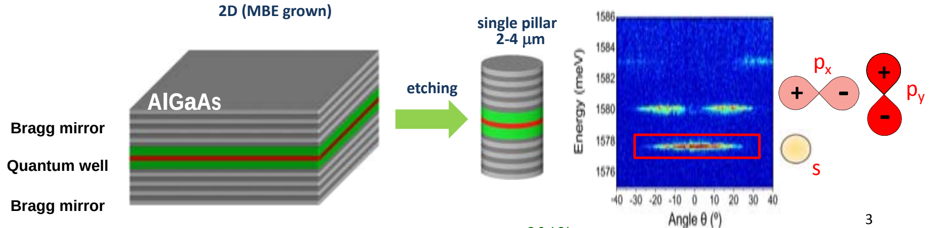


C atoms ←→ semiconducting cavities
electrons ←→ polaritons

s and p orbitals







I. Carusotto & C. Ciuti. Quantum fluids of light. Rev. Mod. Phys. **85,** 299 (2013)

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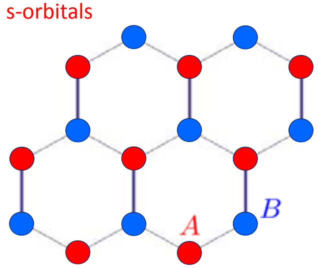
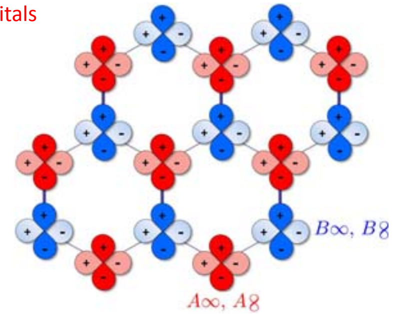
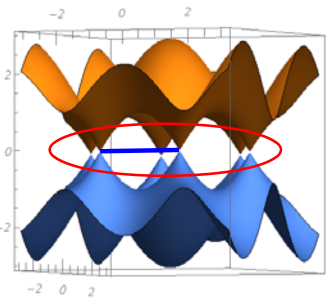
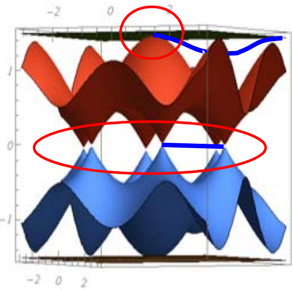
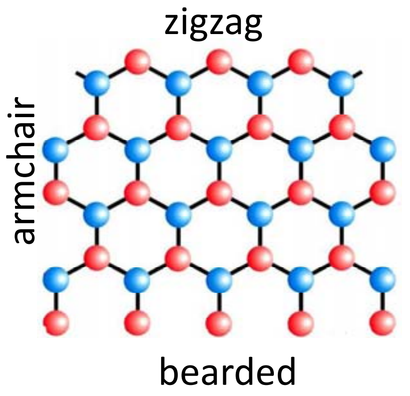
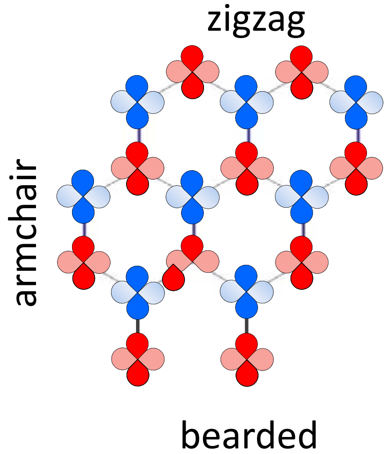


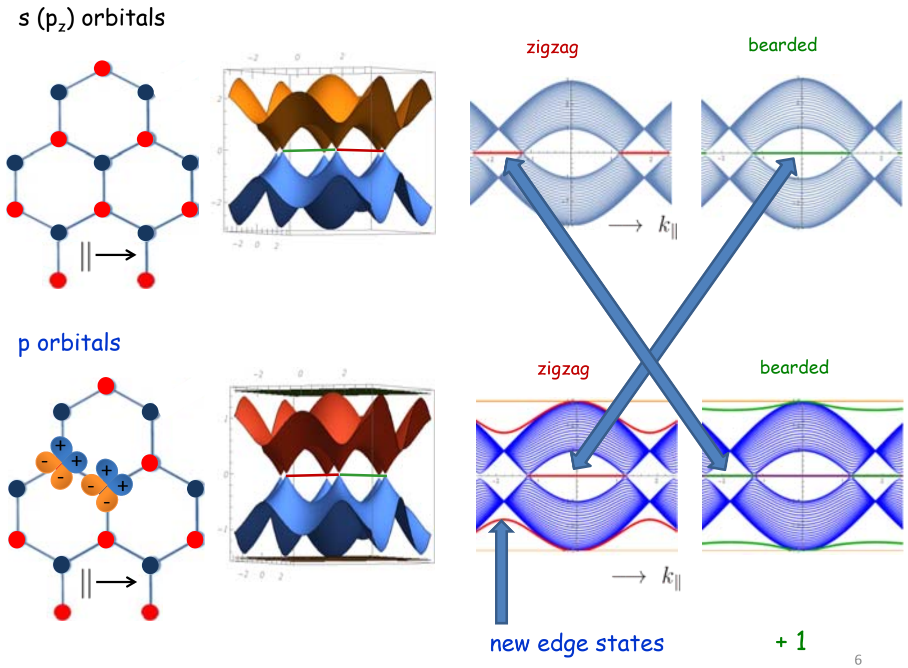




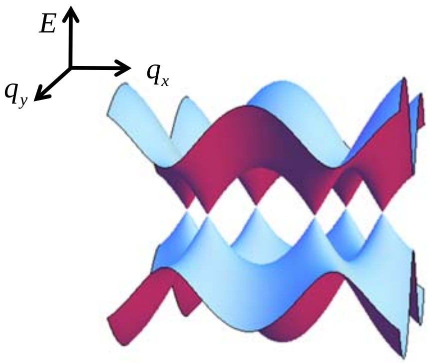


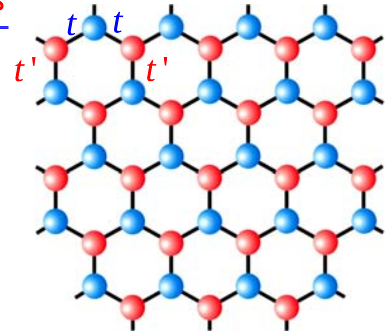






Motion and merging of Dirac points





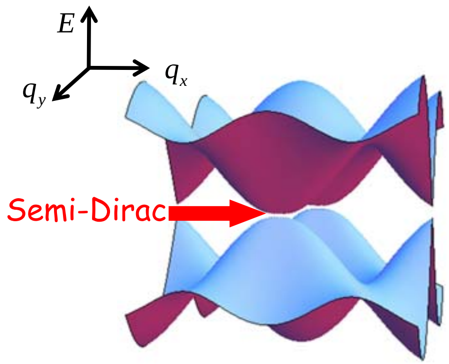
$$\beta = \frac{t'}{t}: \quad 1 \rightarrow 2$$

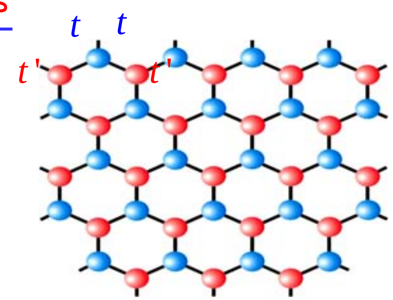
*Y. Hasegawa, R. Konno, H. Nakano, and M. Kohmoto, Phys. Rev. B **74**, 033413 2006.*

*G. M., F. Piéchon, J.N. Fuchs, M.O. Goerbig, Phys. Rev. B **80**, 153412 (2009)*

https://users.lps.u-psud.fr/montambaux/publis-dirac.htm

Motion and merging of Dirac points





$$\beta = \frac{t'}{t}: \quad 1 \rightarrow 2$$

Universal scenario
for merging of Dirac points

Photonic crystals
Microwaves
Ultracold atoms in optical lattices
α-(BEDT-TTF)$_2$I$_3$
Phosphorene

*Y. Hasegawa, R. Konno, H. Nakano, and M. Kohmoto, Phys. Rev. B **74**, 033413 2006.*

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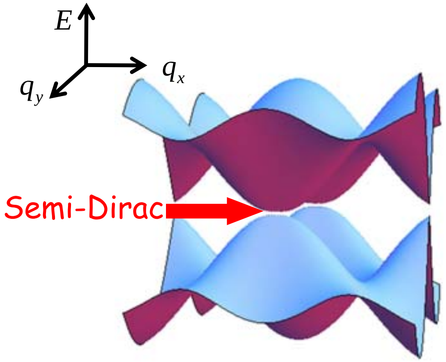
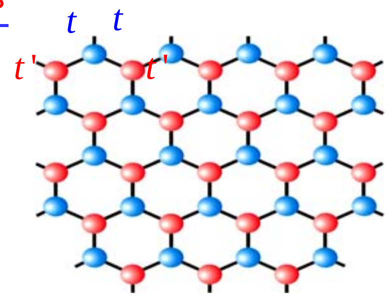
Motion and merging of Dirac points





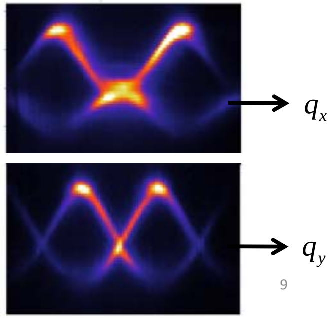
$\beta = \frac{t'}{t}$: $1 \to 2$

*Y. Hasegawa, R. Konno, H. Nakano, and M. Kohmoto, Phys. Rev. B **74**, 033413 2006.*

*G. M., F. Piéchon, J.N. Fuchs, M.O. Goerbig, Phys. Rev. B **80**, 153412 (2009)*

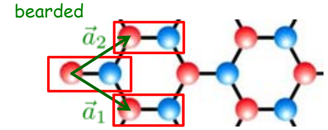
https://users.lps.u-psud.fr/montambaux/publis-dirac.htm

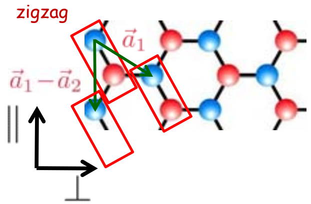
M. Milicevic, et al. , C2N



Reminder: « s »-edge states, topological argument

Write the Hamiltonian compatible with the boundary conditions





$$\hat{\mathcal{H}}_s = \begin{pmatrix} 0 & f_s \\ f_s^* & 0 \end{pmatrix} = \varepsilon \begin{pmatrix} 0 & e^{i\phi(\vec{k})} \\ e^{-i\phi(\vec{k})} & 0 \end{pmatrix}$$

$$f_s(\vec{k}) = 1 + e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}$$

bearded $\vec{u}_1 = \vec{a}_1$, $\vec{u}_2 = \vec{a}_2$

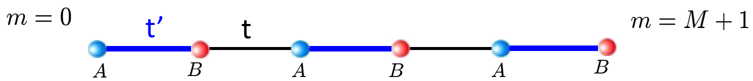
zigzag $\vec{u}_1 = \vec{a}_1$, $\vec{u}_2 = \vec{a}_1 - \vec{a}_2$

The number of edge states is related to the winding number

$$\mathcal{W}(k_\parallel) = \frac{1}{2\pi}\int \frac{\partial \phi(\vec{k})}{\partial k_\perp}\, dk_\perp \qquad \mathcal{W}(k_\parallel) = \frac{1}{2i\pi}\int \frac{\partial \ln f_s(k_\parallel, k_\perp)}{\partial k_\perp} dk_\perp$$

Ryu, Hatsugai (2002), Delplace, Ullmo, G.M (2011)

Reminder: « s »-edge states, topological argument (SSH chain)



Bulk solutions, infinite system $\qquad t' = 2t\cos K_\parallel$

$$|\psi_k\rangle = \frac{1}{\sqrt{2N}} \sum_m (|m, A\rangle, |m, B\rangle)\, e^{ikma_0} \begin{pmatrix} e^{i\phi_k} \\ 1 \end{pmatrix}$$

Bulk solutions, with appropriate boundary conditions :

$$|\psi_k\rangle = \frac{1}{\sqrt{2N}} \sum_m (|m, A\rangle, |m, B\rangle) \begin{pmatrix} \sin(kma_0 - \phi_k) \\ \sin(kma_0) \end{pmatrix} \qquad \langle M+1, A|\psi_k\rangle = 0, \quad \langle 0, B|\psi_k\rangle = 0$$

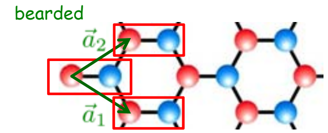
$k(M+1)a_0 - \phi_k = \kappa\pi\,, \quad \kappa = 1, \cdots, M$ has $M - \mathcal{W}$ bulk solutions

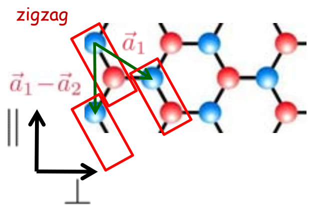
$$\mathcal{W} = \frac{1}{2\pi}\int \frac{\partial \phi_k}{\partial k}\, dk \qquad \Longrightarrow \mathcal{W} \text{ edge states}$$

Ryu, Hatsugai (2002), Delplace, Ullmo, G.M (2011)

Reminder: « s »-edge states, topological argument

Write the Hamiltonian compatible with the boundary conditions





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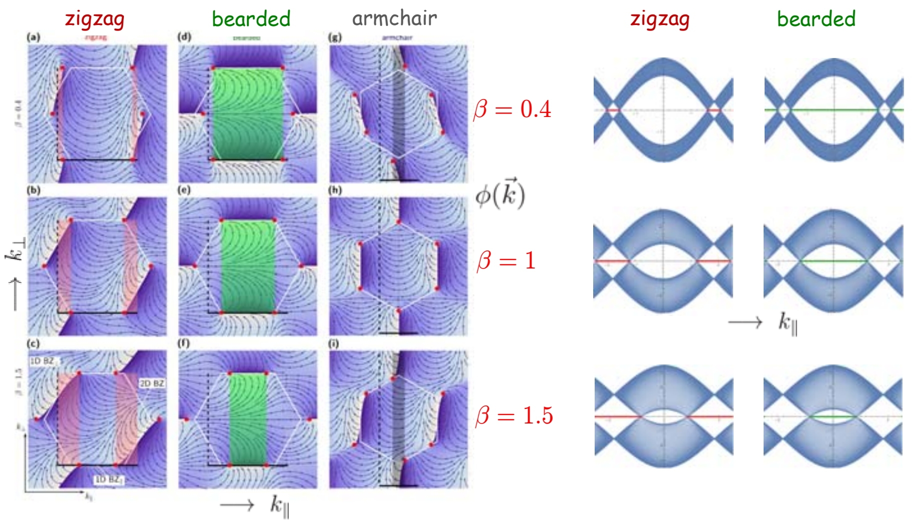
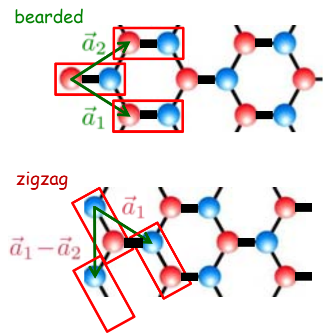


$\beta < 1$ bearded edge states at the expense of zigzag states
$\beta > 1$ zigzag edge states at the expense of bearded states
$\beta \neq 1$ appearance of edge states at the armchair edges

$$\mathcal{W}(k_\parallel) = \frac{1}{2\pi} \int \frac{\partial \phi(\vec{k})}{\partial k_\perp} dk_\perp$$

zigzag and bearded states are complementary



$$\hat{\mathcal{H}}_s = \begin{pmatrix} 0 & f_s \\ f_s^* & 0 \end{pmatrix} = \varepsilon \begin{pmatrix} 0 & e^{i\phi(\vec{k})} \\ e^{-i\phi(\vec{k})} & 0 \end{pmatrix}$$

$$f_s(\text{bearded}) = \beta + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}$$

$$f_s(\text{zigzag}) = 1 + \beta e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot(\vec{a}_1 - \vec{a}_2)}$$

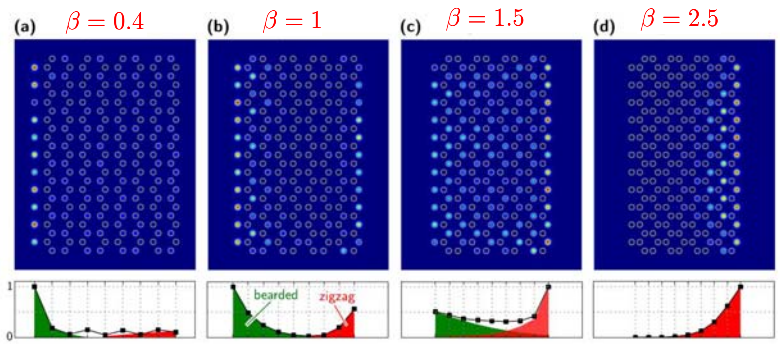
$$\mathcal{W}(k_\parallel) = \frac{1}{2i\pi} \int \frac{\partial \ln f_s(k_\parallel, k_\perp)}{\partial k_\perp} dk_\perp$$

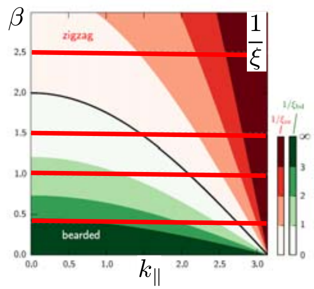
$$= e^{i\vec{k}\cdot\vec{a}_1}(\beta + e^{-i\vec{k}\cdot\vec{a}_1} + e^{-i\vec{k}\cdot\vec{a}_2})$$

$$f_s(\text{zigzag}) = e^{i\vec{k}\cdot\vec{a}_1} f_s^*(\text{bearded})$$

$$\mathcal{W}_s(\text{zigzag}) = 1 - \mathcal{W}_s(\text{bearded})$$

Nice experiments : distorted microwave lattice

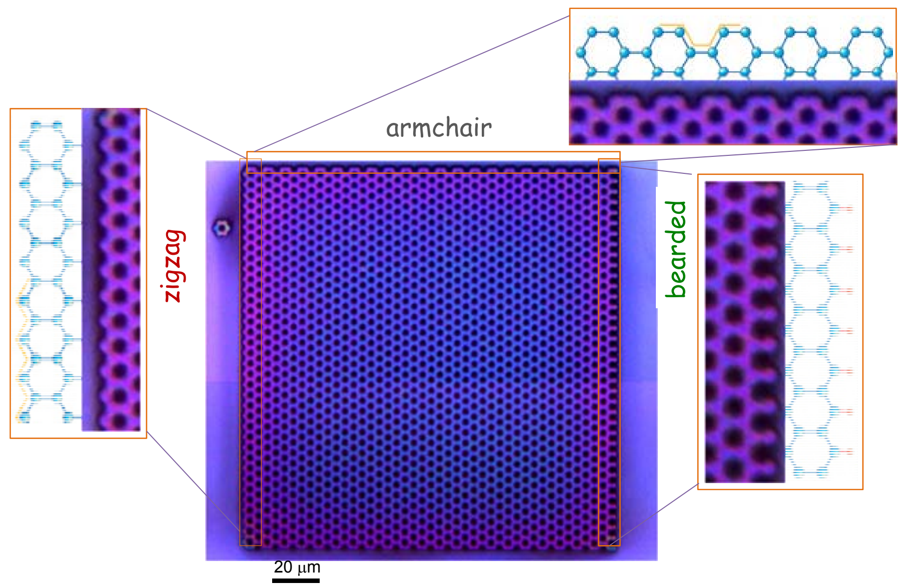




$\beta < 1$ bearded edge states at the expense of zigzag states
$\beta > 1$ zigzag edge states at the expense of bearded states

Manipulation of edge states in microwave artificial graphene
M. Bellec, U. Kuhl, G.M., F. Mortessagne
New J. Phys. ***16***, *113 023 (2014)*

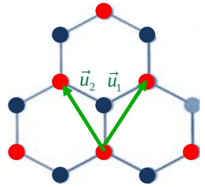
Polariton honeycomb lattice : edges



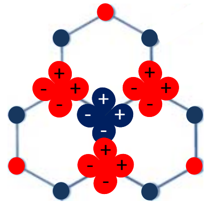
Milicevic et al, 2D Mater. ***2,*** *034012 (2015)*

s / p tight-binding Hamiltonian

$$f_s = 1 + e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}$$



$$\hat{\mathcal{H}}_s = -t_s \begin{pmatrix} 0 & f_s \\ f_s^* & 0 \end{pmatrix}$$



$$\hat{\mathcal{H}}_p = t_p \begin{pmatrix} 0 & Q \\ Q^* & 0 \end{pmatrix}$$

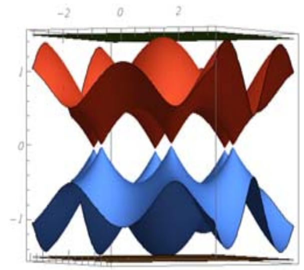
$$Q = \begin{pmatrix} \frac{3}{4}(e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}) & \frac{\sqrt{3}}{4}(e^{i\vec{k}\cdot\vec{u}_1} - e^{i\vec{k}\cdot\vec{u}_2}) \\ \frac{\sqrt{3}}{4}(e^{i\vec{k}\cdot\vec{u}_1} - e^{i\vec{k}\cdot\vec{u}_2}) & 1 + \frac{1}{4}(e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}) \end{pmatrix}$$

C. Wu & S. Das Sarma, **px,y-orbital counterpart of graphene: cold atoms in the honeycomb optical lattice**
PRB ***77***, *235107 (2008)*

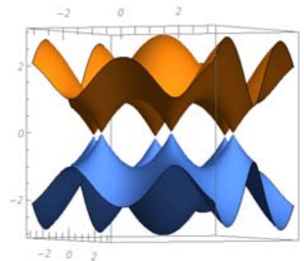
p spectrum

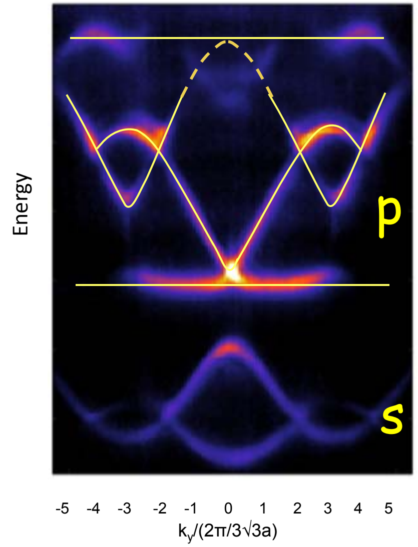
$$\hat{\mathcal{H}}_p = t_p \begin{pmatrix} 0 & Q \\ Q^* & 0 \end{pmatrix}$$

p



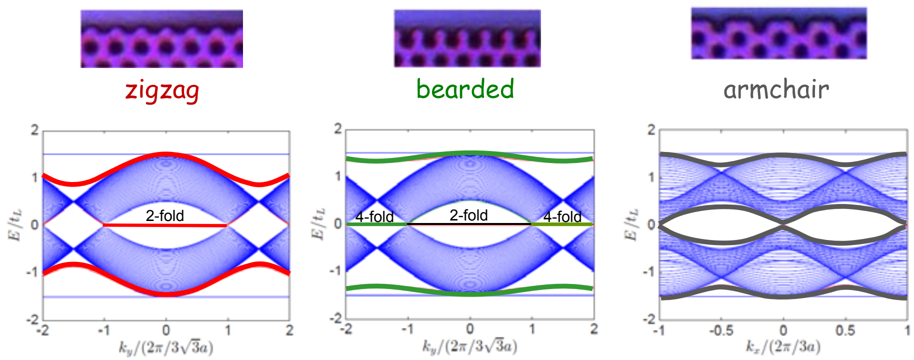
s





M. Milicevic, et al. , C2N

p-band edge states: tight-binding



Zero energy states : p-states and s-states are complementary

Topological description

One additional bearded zero energy state

New dispersive edge states

Topological properties of the s and p Hamiltonians

$$\hat{\mathcal{H}}_s = -t_s \begin{pmatrix} 0 & f_s \\ f_s^* & 0 \end{pmatrix}$$

$$\hat{\mathcal{H}}_p = t_p \begin{pmatrix} 0 & Q \\ Q^* & 0 \end{pmatrix}$$

$$f_s = 1 + e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}$$

$$Q = \begin{pmatrix} \frac{3}{4}(e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}) & \frac{\sqrt{3}}{4}(e^{i\vec{k}\cdot\vec{u}_1} - e^{i\vec{k}\cdot\vec{u}_2}) \\ \frac{\sqrt{3}}{4}(e^{i\vec{k}\cdot\vec{u}_1} - e^{i\vec{k}\cdot\vec{u}_2}) & 1 + \frac{1}{4}(e^{i\vec{k}\cdot\vec{u}_1} + e^{i\vec{k}\cdot\vec{u}_2}) \end{pmatrix}$$

$$\mathcal{W}_s(k_\parallel) = \frac{1}{2i\pi}\int \frac{\partial \ln f_s}{\partial k_\perp} dk_\perp$$

$$\mathcal{W}_p(k_\parallel) = \frac{1}{2i\pi}\int \frac{\partial \ln \det Q}{\partial k_\perp} dk_\perp$$

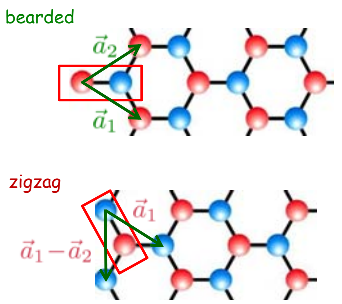
$$f_s = 1 + z_1 + z_2 \quad \Leftrightarrow \quad f_p = \det Q = \frac{3}{4}(z_1 z_2 + z_1 + z_2)$$

$$|z_1| = |z_2| = 1$$

C. Kane, T. Lubensky, Nat. Phys. (2014)

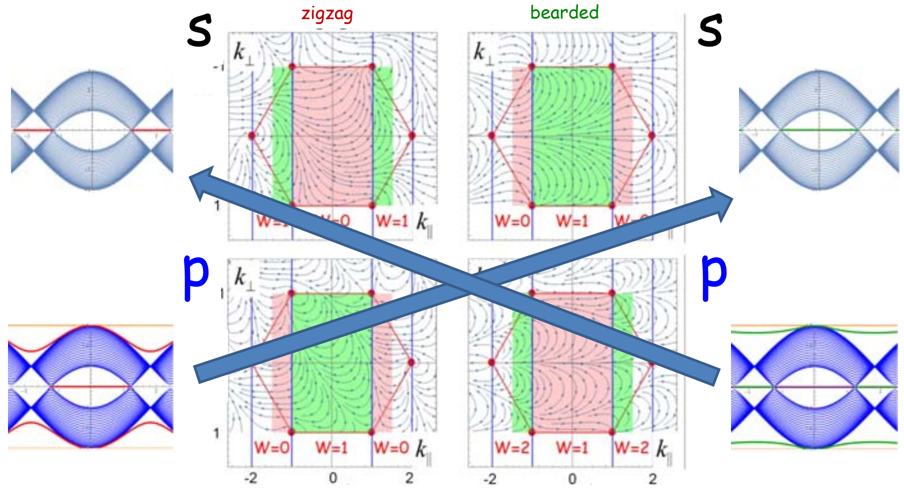
Topological properties of the s and p Hamiltonians

$$f_s = 1 + z_1 + z_2 \quad \Leftrightarrow \quad f_p = \det Q = \frac{3}{4}(z_1 z_2 + z_1 + z_2) = \frac{3}{4} z_1 z_2 \,(1 + z_1^* + z_2^*)$$



$$\mathcal{W}_p(\text{zigzag}) = 1 - \mathcal{W}_s(\text{zigzag}) = \mathcal{W}_s(\text{bearded})$$

$$\mathcal{W}_p(\text{bearded}) = 2 - \mathcal{W}_s(\text{bearded}) = 1 + \mathcal{W}_s(\text{zigzag})$$

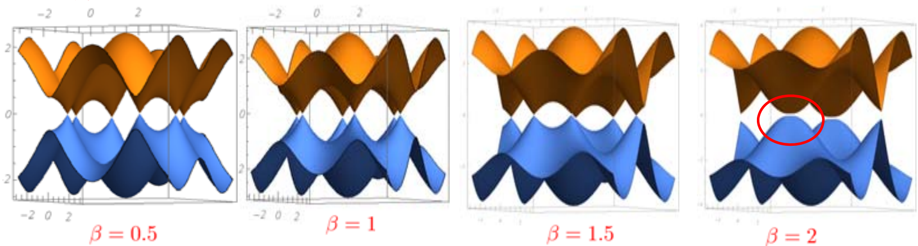


$$\mathcal{W}_p(\text{zigzag}) = \mathcal{W}_s(\text{bearded})$$

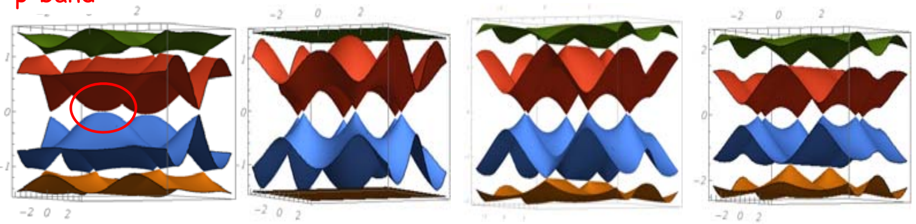
$$\mathcal{W}_p(\text{bearded}) = 1 + \mathcal{W}_s(\text{zigzag})$$

Deformation of the lattice $\beta \neq 1$

s-band



p-band



$$\varepsilon_p^{flat}(\beta)\ \varepsilon_p^{disp.}(\beta) = \frac{3}{4}\beta\ \varepsilon_s\left(\frac{1}{\beta}\right)$$

Correspondance s-band / p-band

s-band		p-band
bearded edge states	⟺	zigzag edge states
zigzag edge states	⟺	bearded edge states
increase β	⟺	decrease β
decrease β	⟺	increase β

$$f_s = 1 + z_1 + z_2 \quad \Leftrightarrow \quad f_p = \det Q = \frac{3}{4}(z_1 z_2 + z_1 + z_2)$$

$$f_s = \beta + z_1 + z_2 \quad \Leftrightarrow \quad f_p = \det Q = \frac{3}{4}\beta\left(\frac{1}{\beta} z_1 z_2 + z_1 + z_2\right)$$

$$\mathcal{W}_p(\text{zigzag}, \beta) = 1 - \mathcal{W}_s\left(\text{zigzag}, \frac{1}{\beta}\right)$$

$$\mathcal{W}_p(\text{bearded}, \beta) = 2 - \mathcal{W}_s\left(\text{bearded}, \frac{1}{\beta}\right)$$

Correspondance s-band / p-band

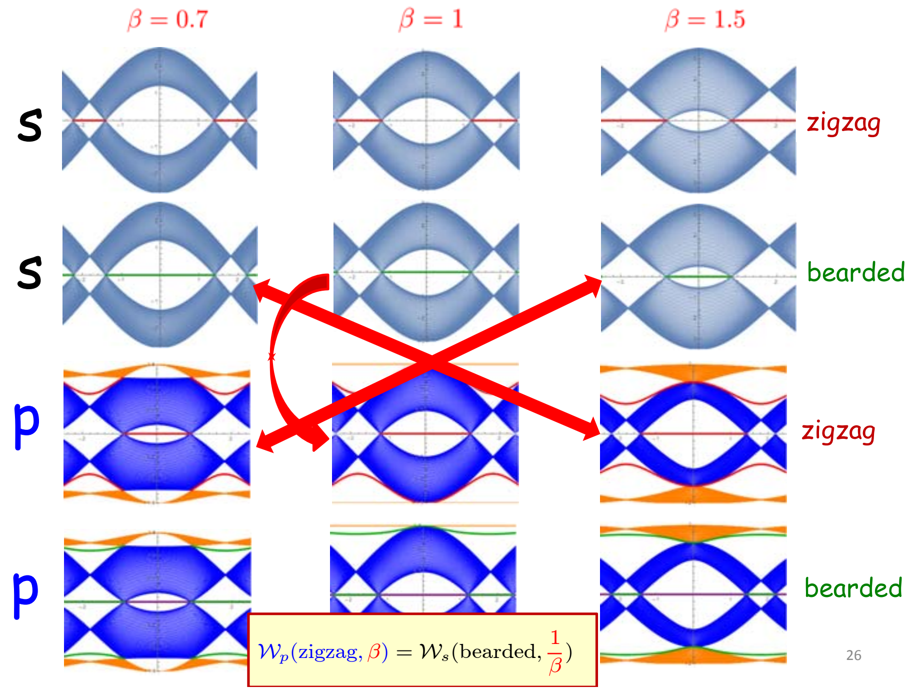
s-band		p-band
bearded edge states	⟺	zigzag edge states
zigzag edge states	⟺	bearded edge states
increase β	⟺	decrease β
decrease β	⟺	increase β

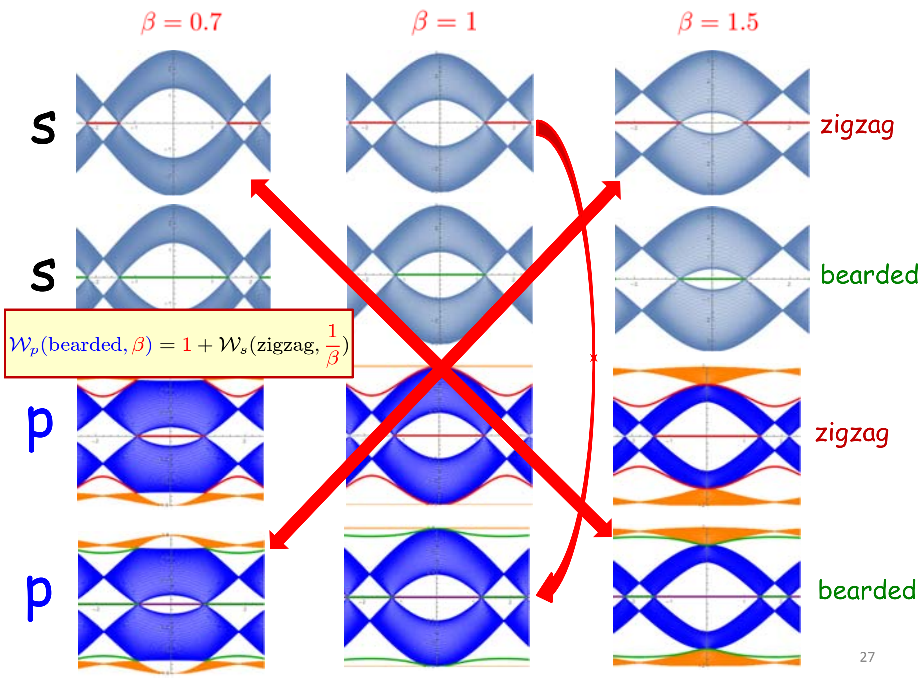
$$f_s = 1 + z_1 + z_2 \quad \Longleftrightarrow \quad f_p = \det Q = \frac{3}{4}(z_1 z_2 + z_1 + z_2)$$

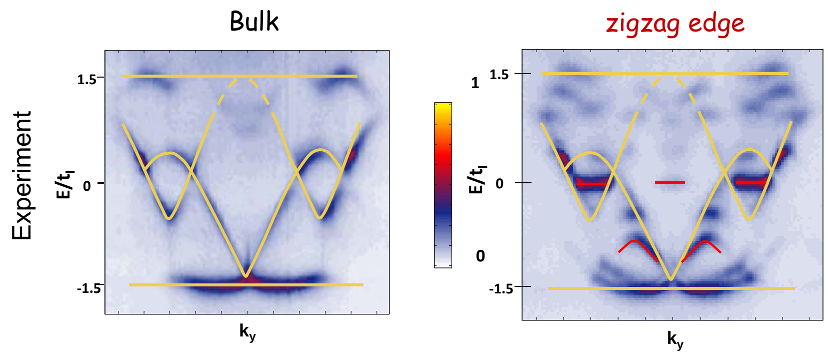
$$f_s = \beta + z_1 + z_2 \quad \Longleftrightarrow \quad f_p = \det Q = \frac{3}{4}\beta(\frac{1}{\beta} z_1 z_2 + z_1 + z_2)$$

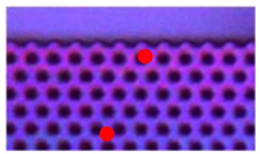
$$\mathcal{W}_p(\text{zigzag}, \beta) = 1 - \mathcal{W}_s(\text{zigzag}, \frac{1}{\beta}) = \mathcal{W}_s(\text{bearded}, \frac{1}{\beta})$$

$$\mathcal{W}_p(\text{bearded}, \beta) = 2 - \mathcal{W}_s(\text{bearded}, \frac{1}{\beta}) = 1 + \mathcal{W}_s(\text{zigzag}, \frac{1}{\beta})$$









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Phys. Rev. Lett. 118, 107403 (2017)

G.M., **Artificial graphenes: Dirac matter beyond condensed matter**, C. R. Physique **19**, 285 (2018)
arXiv:1810.07505

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