Linear and angular momentum of the quantum vacuum

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Casimir energy

1. Isotropic radiation with power spectrum ω^3 is Lorentz-invariant (Einstein, 1917);
2. Van der Waals force $1/r^6$ (London, 1930)
3. Relation to Cosmological constant (Pauli, 1934, Davies, 1984)
4. Casimir Polder Force $1/r^7$ (1947)
5. Attraction between metallic plates (Casimir, 1948) comments
6. Lifshitz theory for dielectric media (Lifshitz, 1956, Dzyalonishiniskii 1961)
7. Observation of Casimir effect (Sparnaay, 1958, Lamoureux (5%), 1997), Chan etal, (1%), 2001)
8. Unruh effect & Hawking radiation (Hawking 1974, Unruh 1976)
9. Sonoluminescence (Schwinger, 1993, Eberlein, 1996) comments!
10. Quantum friction and sheering the quantum vacuum (Pendry, 1998), comments!
11. Casimir momentum in magneto-electric media (Feigel, 2004). Comments!
12. Casimir momentum of magneto-chiral media (Donaire, BVT, 2015)

Photons in Homogeneous media

Photons interact with a magnetic field (T)

$$n^2 \frac{\omega(\boldsymbol{k},\sigma,\boldsymbol{B})^2}{c_0{}^2} = (\mathbf{k} + \sigma V \mathbf{B}\,)^2$$

Faraday effect (1846)

Photons are bent by a magnetic field

$$\boldsymbol{v}_G = \frac{c_0}{n}\left(\hat{\boldsymbol{k}} + \sigma \frac{V}{k}\boldsymbol{B}\,\right)$$

Landau & Lifshitz ECM exercice

Rikken & Van Tiggelen (1997).

Photons probe chirality of matter (P)

$$n^2 \frac{\omega(\boldsymbol{k},\sigma)^2}{c_0{}^2} = (1 + \sigma g)\boldsymbol{k}^2$$

Rotatory power (1811)

Photons probe magneto-chirality (PT)

$$n^2 \frac{\omega(\boldsymbol{k},\sigma,\boldsymbol{B})^2}{c_0{}^2} = (\mathbf{k} + V g \mathbf{B}\,)^2$$

Magneto-chiral effect. Pasteur, 1884…….
Groenewege, Mol. Phys. (1962)
Kleindienst & Wagnière, CPL 288 (1998)
Rikken & Raupach, Nature (1997)

Photons in Bi-anisotropic Media

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega) + \mathbf{g}(\omega)\cdot\mathbf{B}(\omega)$$

$$\mathbf{H}(\omega) = \mathbf{g}^T(\omega)\cdot\mathbf{E}(\omega) + \mu(\omega)^{-1}\mathbf{B}(\omega)$$

$$g_{ij}(\omega) = i\omega\, g\delta_{ij}$$

Rotatory power

10^{-2}

$$g_{ij}(\omega) = (1-\varepsilon)\,\varepsilon_{ijl}\frac{v_l}{c_0}$$

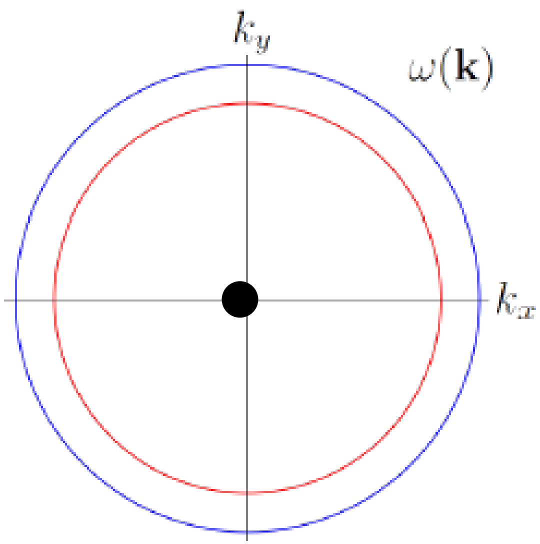
Fizeau effect

10^{-8}

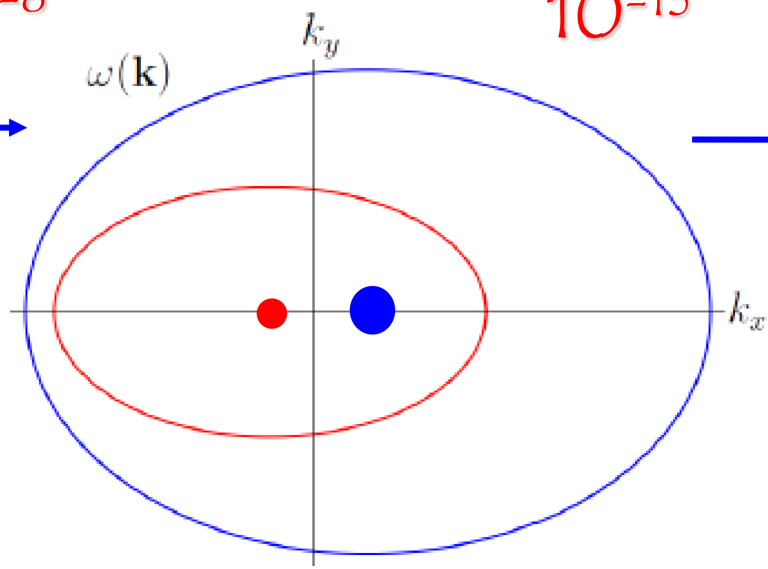
$$g_{ij}(\omega) = g\left(E_i^0 B_j^0 - B_i^0 E_j^0\right)$$

Magneto-electric birefringence

10^{-15}



V



$E_0 \times B_0$

« Momentum from Nothing »

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Quantum Vacuum Contribution to the Momentum of Dielectric Media

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Momentum transfer between matter and electromagnetic field is analyzed. The related equations of motion and conservation laws are derived using relativistic formalism. Their correspondence to various, at first sight self-contradicting, experimental data (the so-called Abraham-Minkowski controversy) is demonstrated. A new, Casimir-like, quantum phenomenon is predicted: contribution of vacuum fluctuations to the motion of dielectric liquids in crossed electric and magnetic fields. Velocities of about 50 nm/s can be expected due to the contribution of high frequency vacuum modes. The proposed phenomenon could be used in the future as an investigating tool for zero fluctuations. Other possible applications lie in fields of microfluidics or precise positioning of micro-objects, e.g., cold atoms or molecules.

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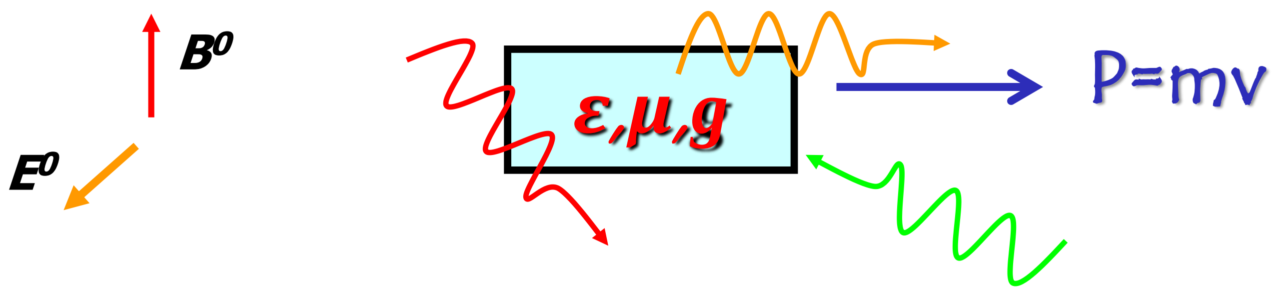


Photons in EM Semi-classical quantum vacuum

force on particle — dispersive force — Abraham force

$$\partial_t(\rho \boldsymbol{v}) + \nabla\cdot \boldsymbol{U} = +P_m \nabla E_m + M_m \nabla B_m + \frac{1}{c_0}\partial_t(\boldsymbol{P}\times\boldsymbol{B})$$

$$\frac{1}{4\pi c_0}\partial_t(\boldsymbol{E}\times\boldsymbol{B}) + \nabla\cdot \boldsymbol{T} = -P_m \nabla E_m - M_m \nabla B_m - \frac{1}{c_0}\partial_t(\boldsymbol{P}\times\boldsymbol{B})$$

$$\partial_t\left(M\boldsymbol{v} + \frac{1}{4\pi c_0}\int d^3\boldsymbol{r}\,\boldsymbol{E}\times\boldsymbol{B}\right) + \int d^2\boldsymbol{S}\cdot\boldsymbol{T}_0 = 0$$

force on particle — EM momentum — radiative force

$$\langle 0|\frac{\mathbf{E}\times\mathbf{B}}{4\pi c_0}|0\rangle \propto \begin{cases} \dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \times g(\omega)\mathbf{E}_0\times\mathbf{B}_0 = \dfrac{2}{3}\dfrac{\hbar\omega_c^4}{\pi^3 c_0^4}\, g\,\mathbf{E}_0\times\mathbf{B}_0 \\ \dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \times[\varepsilon(\omega)-1]\frac{\mathbf{v}}{c_0} = \rho_{casi}\mathbf{v} \end{cases}$$

Observation of the Abraham Force

Ex: Helium

E0=450 V/mm; B0=1 T

$\alpha(0) = 0.22\ 10^{-40}\,\mathrm{Cm^2/V}\ \ (16.6a_0^3)$

$\rho = 0.17\ \mathrm{kg/m^3}\,(\mathrm{room}\ T)$

$g = 0.017\ 10^{-22}\,\mathrm{m/VT}$

(SI units)

$$v_{\mathrm{abr}} = \frac{\varepsilon_0 \alpha(0) EB}{2m_p} \approx 0.3\ \mathrm{nm/sec}$$

Classical Abraham Force

$$F_{\mathrm{abr}} \approx 7 \cdot 10^{-32}\ N$$

$$N_{at} F_{\mathrm{abr}} \propto 10^{-13}\ N$$

$$v_{\mathrm{Feigel}} = \frac{\pi}{4} \frac{h}{\rho \lambda_c^4} gEB \approx 0.02\ \mathrm{nm/sec}$$

Semi-classical QED with cut-off 0.1 nm (Feigel)

$$v_{QED} \propto v_{\mathrm{abr}} \times (Z?\alpha)^2 \approx 0.001\ \mathrm{nm/sec}$$

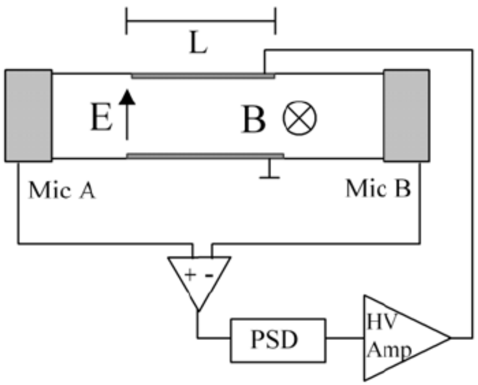
Rigorous QED (Kawka, 2012)

$$\frac{d\boldsymbol{p}}{dt} = \alpha(0)\frac{d\boldsymbol{E}}{dt} \times \boldsymbol{B}$$

Abraham force

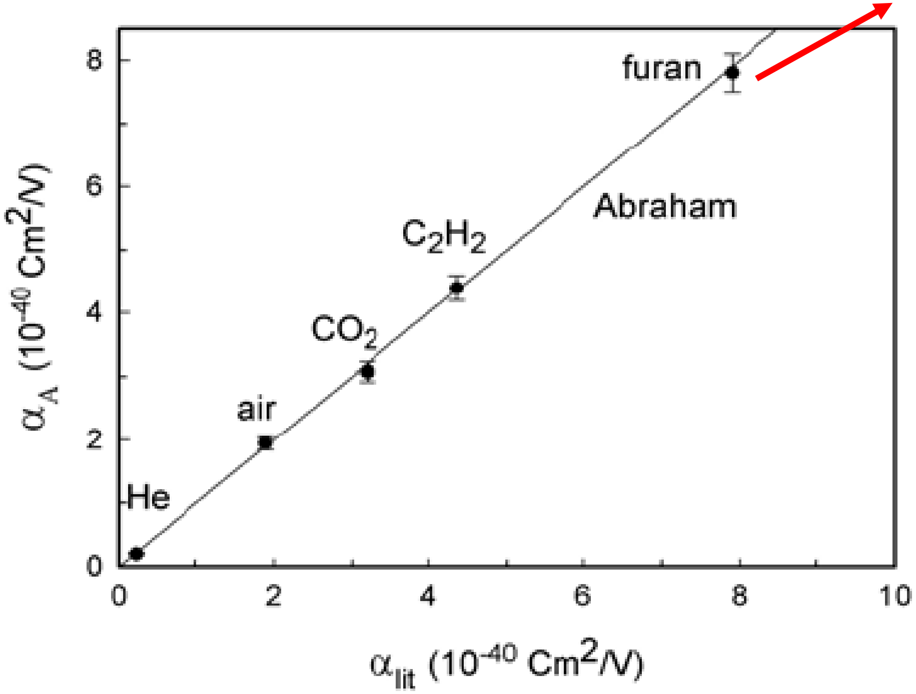
Acoustic pressure

$$P(\omega) = P_0 + \alpha(0) \times E \times B \times \omega \times \cos\omega t \times n \times L$$



V= 8 nm/sec+- 0.8
Feigel correction: 2 nm/sec
Excluded by errorbars

$\delta P/(EB)$



E=450 V/mm;
B=1 T;
f= 7.6 kHz

$\alpha(0)$

Photons in Heterogeneous, chiral media

I call any geometrical figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself

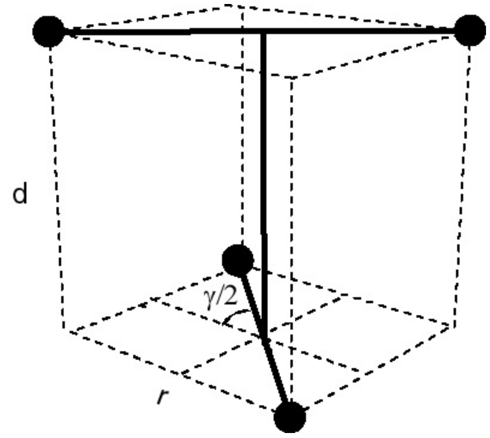
Lord Kelvin, 1904

Chirality is quantified by a rotationally invariant pseudo-scalar

$$g(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \dots) = -g(-\boldsymbol{r}_1, -\boldsymbol{r}_2, -\boldsymbol{r}_3, \dots) = g(\boldsymbol{R}\boldsymbol{r}_1, \boldsymbol{R}\boldsymbol{r}_2, \boldsymbol{R}\boldsymbol{r}_3, \dots)$$

Harris, Kamien & Lubensky, 1999

Twisted H



$$g = r^2 d \sin(2\gamma)$$

Photons in Heterogeneous, magnetochiral media

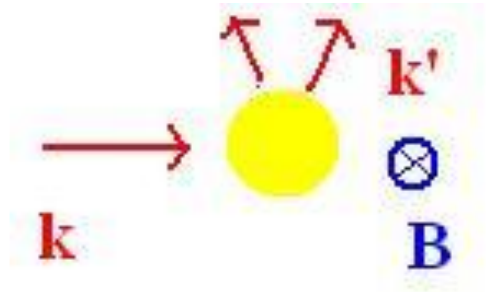
$$\frac{d\sigma}{d\Omega}(\boldsymbol{k},\boldsymbol{k}',g,\boldsymbol{B}) = \frac{d\sigma}{d\Omega}(-\boldsymbol{k}',-\boldsymbol{k},g,-\boldsymbol{B}) = \frac{d\sigma}{d\Omega}(-\boldsymbol{k},-\boldsymbol{k}',-g,\boldsymbol{B})$$

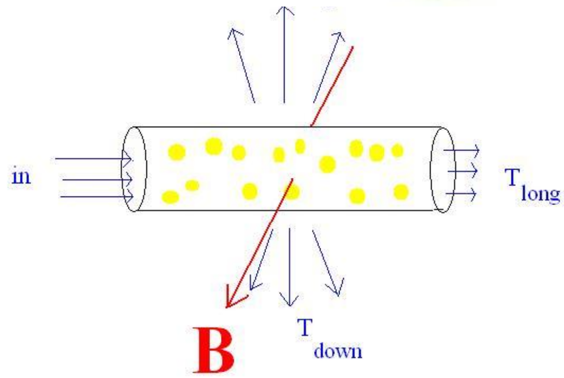
$$\frac{d\sigma}{d\Omega} = F_0(\boldsymbol{k}\cdot\boldsymbol{k}') + V\,F_H(\boldsymbol{k}\cdot\boldsymbol{k}')\det(\boldsymbol{B},\boldsymbol{k},\boldsymbol{k}') + V\,F_{MC}(\boldsymbol{k}\cdot\boldsymbol{k}')\boldsymbol{B}\cdot(\boldsymbol{k}+\boldsymbol{k}')$$

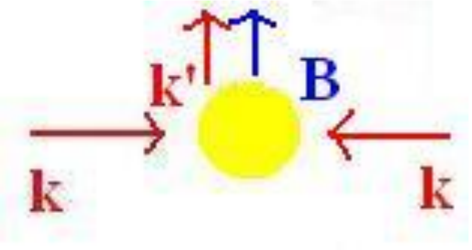
Mie scattering 1908

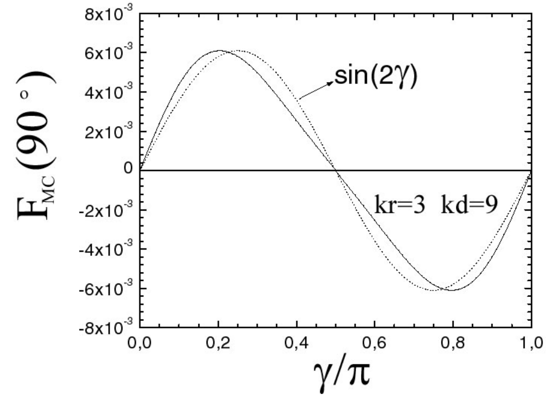
Photon Hall Effect : Rikken BVT 1996

Magneto-chiral super-current: never observed





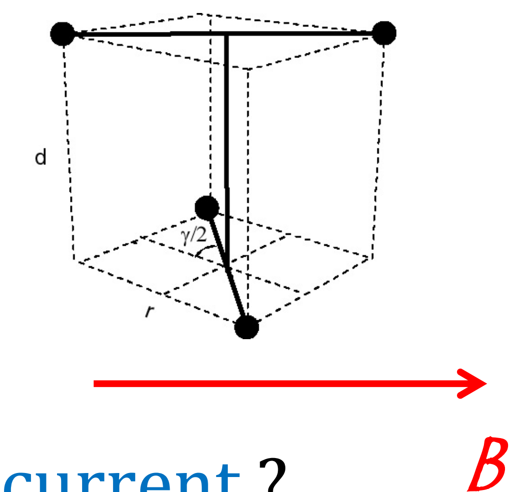




Photons in semi-classical quantum vacuum

$$\langle 0|\mathbf{E}_a(x',\omega')\mathbf{E}_b^\dagger(x,\omega)|0\rangle = \frac{\hbar\omega^2}{2i\pi c^2\epsilon_0}\delta(\omega'-\omega)$$

$$\times\left(\mathbf{G}_{ab}(x',x|\omega) - \mathbf{G}_{ba}(x,x'|\omega)^*\right),$$



an electromagnetic "London" super current ?

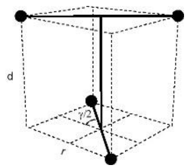
$$\boldsymbol{J} = q\rho\boldsymbol{A} \Rightarrow \langle 0|\boldsymbol{S}|0\rangle \sim gV\mathcal{E}\,\boldsymbol{B} \ ?$$

Photons in semi-classical quantum vacuum

(partially) unproven theorem:

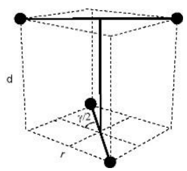
$$FDT \Rightarrow \langle 0 | \int \boldsymbol{d}^3\boldsymbol{r}\, \boldsymbol{E} \times \boldsymbol{H}\, | 0 \rangle = 0$$

twisted electric dipoles



$$\boldsymbol{P}_{EM} = \left\langle 0 \right| \frac{1}{4\pi c_0} \int d^3\boldsymbol{r}\, \boldsymbol{E} \times \boldsymbol{B} \left| 0 \right\rangle = 0 \text{ because } \boldsymbol{B} = \boldsymbol{H}$$

twisted magnetic dipoles

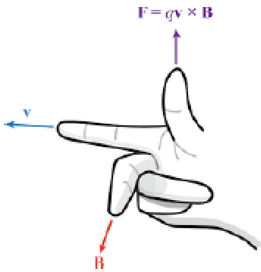


finite & nonzero

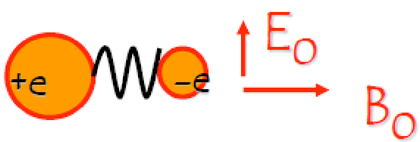
$$\boldsymbol{P}_{EM} \sim \hbar V \boldsymbol{B} \frac{\chi(0)^5}{(k_0 L)^{14}} \sin(2\gamma)$$

negligeable

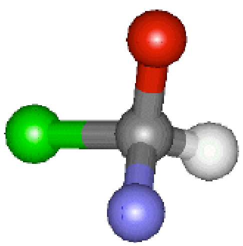
Casimir momentum in QED



$$\boldsymbol{F} = q\,(\mathrm{E} + \boldsymbol{v}\ \times \boldsymbol{B})$$ Lorentz force



$$\boldsymbol{F} = \alpha(0)\frac{\partial}{\partial t}(\mathrm{E} \times \boldsymbol{B})$$ Abraham force



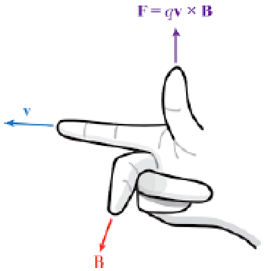
$$\boldsymbol{F} = e\frac{\beta(0)}{\alpha(0)}\frac{\partial \boldsymbol{B}}{\partial t}$$ NO Maxwell

$$\boldsymbol{F} = -\left(\frac{e^2}{\hbar c}\right)^2 \alpha(0)\frac{\partial}{\partial t}(\mathrm{E} \times \boldsymbol{B})$$

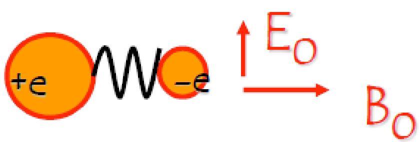
$$\boldsymbol{F} = \frac{\partial \boldsymbol{P}}{\partial t}$$

Yes Feynman !

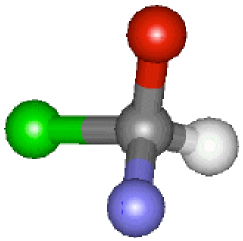
Casimir momentum in QED



$$\boldsymbol{F} = q\,(\mathrm{E} + \boldsymbol{v}\ \times \boldsymbol{B})$$ Lorentz force



$$\boldsymbol{F} = \alpha(0)\frac{\partial}{\partial t}(\mathrm{E} \times \boldsymbol{B})$$ Abraham force



$$\boldsymbol{F} = e\,\frac{\beta(0)}{\alpha(0)}\frac{\partial \boldsymbol{B}}{\partial t}$$ NO Maxwell

$$\boldsymbol{F} = -\left(\frac{e^2}{\hbar c}\right)^2 \alpha(0)\frac{\partial}{\partial t}(\mathrm{E} \times \boldsymbol{B})$$

$$\boldsymbol{F} = \frac{\partial \boldsymbol{P}}{\partial t}$$

Yes Feynman !

$$\boldsymbol{F} = e\,\frac{e^2}{\hbar c}\frac{\beta(0)}{\alpha(0)}\frac{\partial \boldsymbol{B}}{\partial t}$$

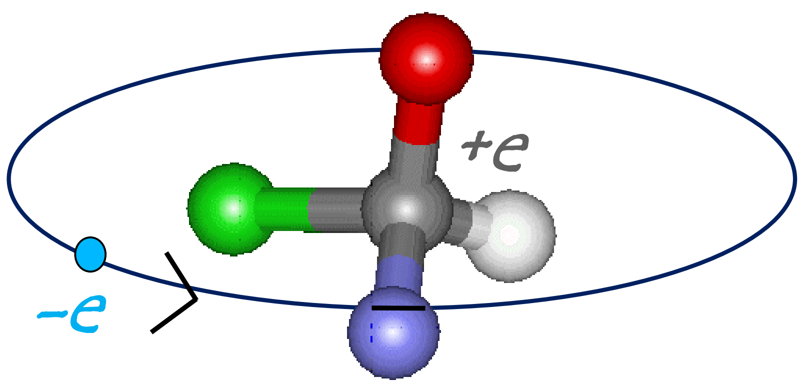
Magneto-chiral Casimir momentum in QED

A chiral group + chromophoric electron bound anisotropically
+ magnetic field + quantum vacuum beyond ED approximation

$$H_0 = \sum_{i=e,N} \frac{1}{2m_i}[\mathbf{p}_i - q_i\mathbf{A}_0(\mathbf{r}_i)]^2 + V^{HO} + V_C, \qquad (1)$$

$$H_{EM} = \sum_{\mathbf{k},\epsilon} \hbar\omega_{\mathbf{k}}(a^\dagger_{\mathbf{k}\epsilon}a_{\mathbf{k}\epsilon} + \frac{1}{2}) + \frac{1}{2\mu_0}\int \mathrm{d}^3r\mathbf{B}_0^2, \qquad (2)$$

$$W = \sum_{i=e,N} \frac{-q_i}{m_i}[\mathbf{p}_i - q_i\mathbf{A}_0(\mathbf{r}_i)] \cdot \mathbf{A}(\mathbf{r}_i) + \frac{q_i^2}{2m_i}\mathbf{A}^2(\mathbf{r}_i).$$



$$V_C = C\, xyz$$

$$V^{HO} = \frac{\mu}{2}(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$$

Momentum of Quantum Vacuum

$$
\begin{aligned}
|\tilde{\Psi}_0\rangle = & \left[1 - \frac{1}{2}\sum_{i\mathbf{Q}n}{}' \frac{|W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}|^2}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2}\right] |0\mathbf{Q}_0\{0\}\rangle \\
& + \sum_{i\mathbf{Q}n}{}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n}} |i\mathbf{Q}n\rangle \\
& + \sum_{i\mathbf{Q}n}{}' \sum_{i'\mathbf{Q}'n'}{}' \frac{W_{i\mathbf{Q}n,i'\mathbf{Q}'n'} W_{i'\mathbf{Q}'n',0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})(E_{0\mathbf{Q}_0 0} - E_{i'\mathbf{Q}'n'})} |i\mathbf{Q}n\rangle \\
& - W_{0\mathbf{Q}_0 0,0\mathbf{Q}_0 0} \sum_{i\mathbf{Q}n}{}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2} \rangle |i\mathbf{Q}n\rangle \qquad (12)
\end{aligned}
$$

$$\frac{8\pi}{3}\frac{e^2\hbar}{c_0}\sum_k \frac{1}{k}\frac{1}{\hbar\omega_k + \hbar^2 k^2/2m} = \delta m$$

Magneto-chiral Casimir momentum in QED

A chiral group + chromophoric electron bound anisotropically
+ magnetic field + quantum vacuum beyond ED approximation

$$\boldsymbol{K} = \boldsymbol{P}_{kin} + e\boldsymbol{B}_0 \times \boldsymbol{r}_{Ne} + e\boldsymbol{A}(\boldsymbol{r}_N) - e\boldsymbol{A}(\boldsymbol{r}_e) + \sum_{kg} \hbar \boldsymbol{k}\, a^*_{kg} a_{kg}$$

kinetic momentum

Longitudinal Casimir momentum

Conserved Pseudo momentum

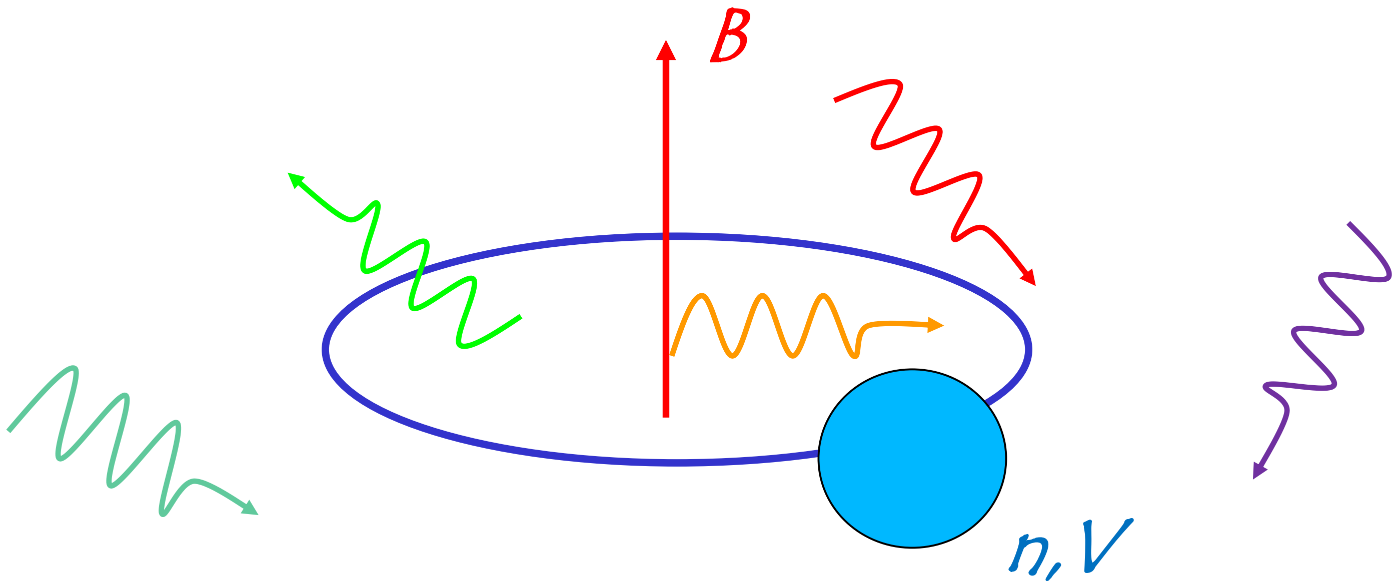
Abraham momentum

Transverse Casimir momentum

$$\langle 0|\boldsymbol{P}_{kin}|0\rangle = \frac{2\alpha}{9\pi}\left[\mathrm{Log}\left(\frac{m_N}{m_e}\right) + 1\right]\frac{\beta(0)}{\alpha(0)}\, e\boldsymbol{B}_0$$

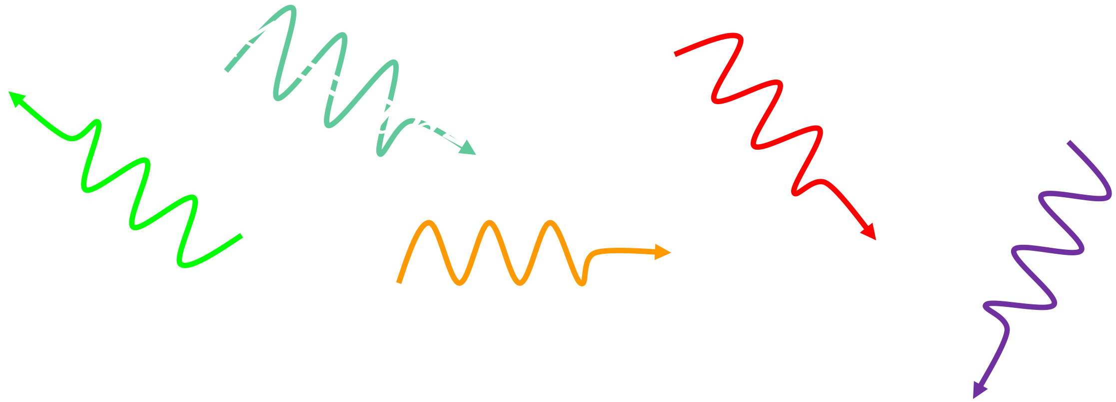
$C_8H_{18}O$: 0.06 nm/sec/Tesla

Angular Momentum of Quantum Vacuum



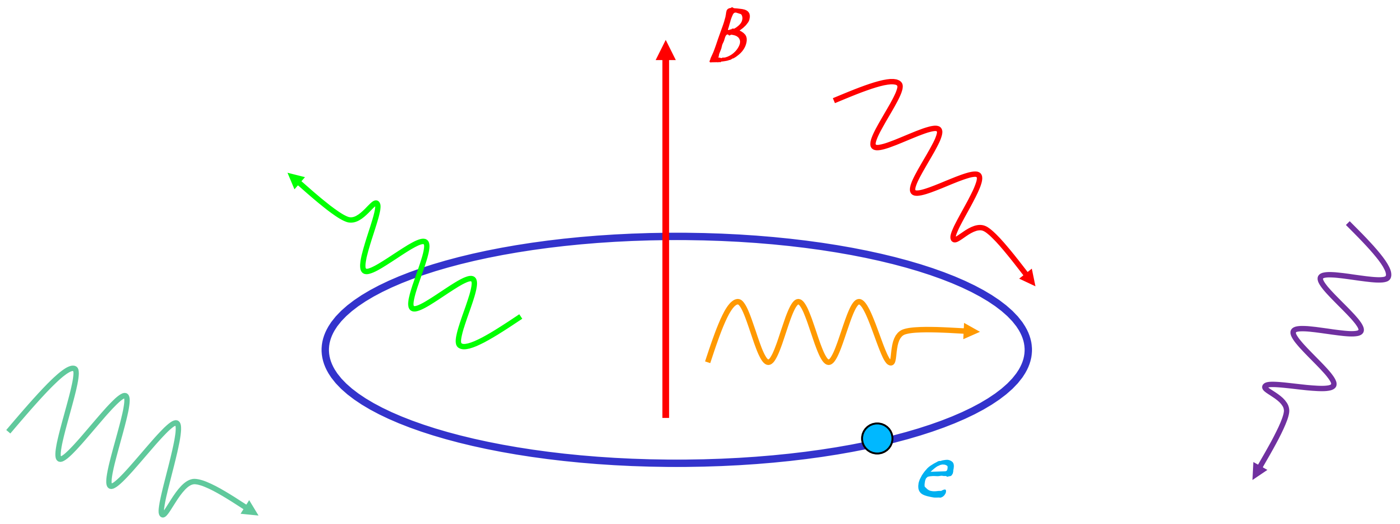
Momentum of Quantum Vacuum
Conclusions and outlook

$\hbar k$



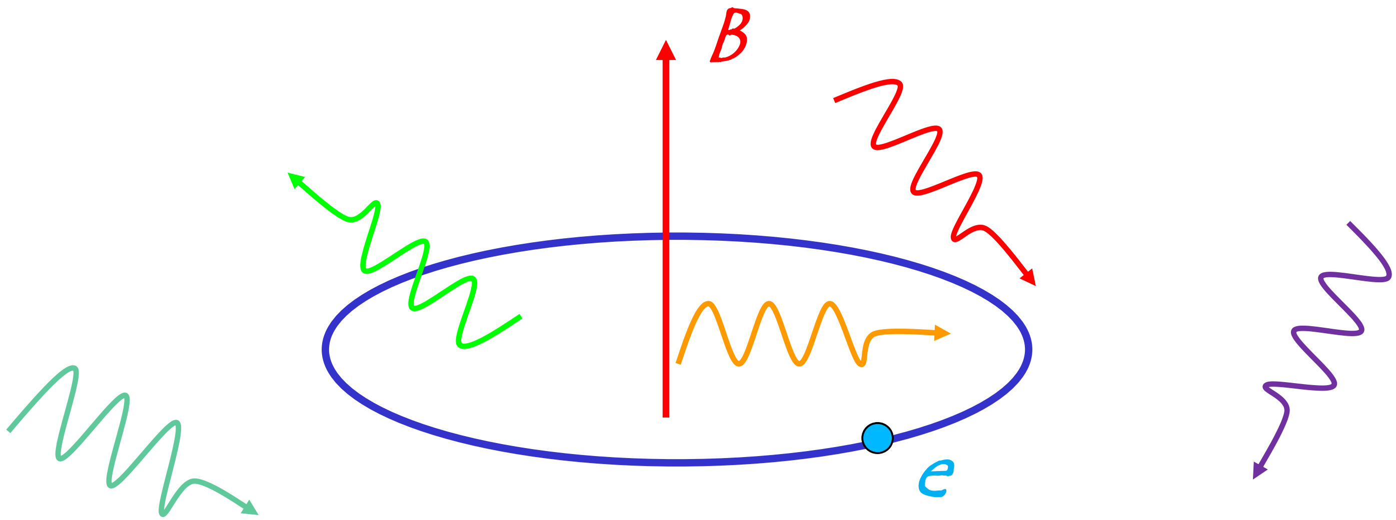
- Quantum vacuum possesses momentum and
  angular momentum in presence of chirality and magnetic fields
- Non-zero but small .
- Semi-classical approaches are inadequate
  - Unable to absorbe UV divergencies
  - Unable to include « longitudinal » momentum of gauge field
  - Unable to cope with recoil induces by vacuum photons
- Quid cavity QED? Quid excited states?

Angular Momentum of Quantum Vacuum



classically $L_z + \frac{e}{2\pi c_0}\Phi = conserved$

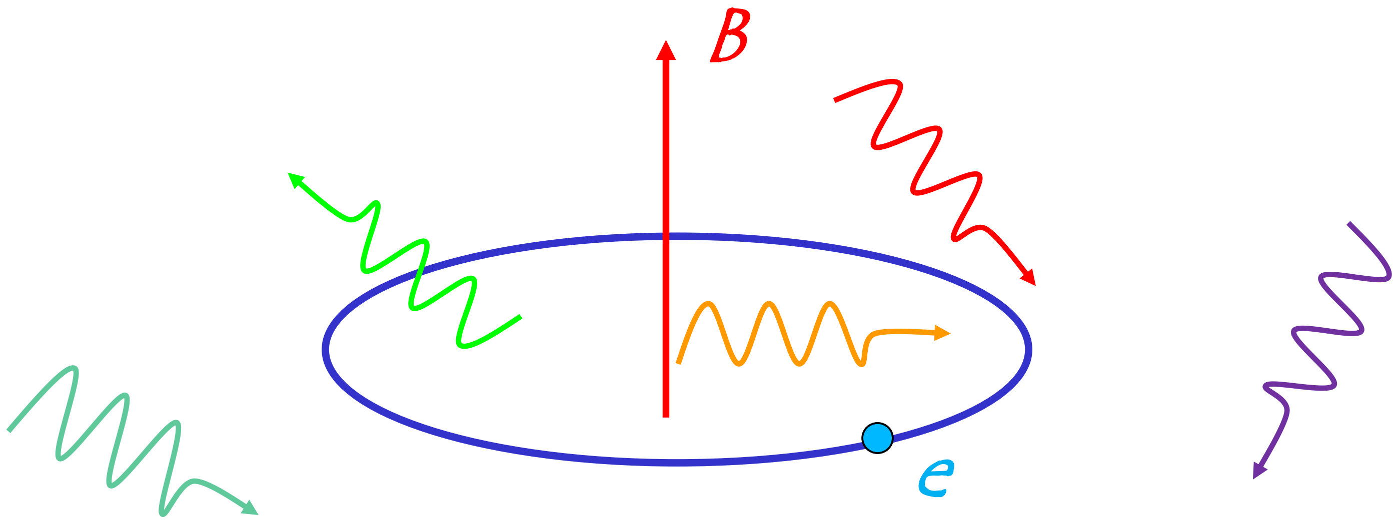
Angular Momentum of Quantum Vacuum



$$J_z = L_z(kin) + \frac{e}{c_0}\boldsymbol{r} \times \boldsymbol{A}_0(\boldsymbol{r}) + \frac{e}{c_0}\boldsymbol{r} \times \boldsymbol{A}(\boldsymbol{r})$$
$$+ \sum_{\boldsymbol{k\Pi}} \frac{1}{k^2}\left(a^\dagger_{k\boldsymbol{\Pi}} a_{k\boldsymbol{\Pi}} + a_{k\boldsymbol{\Pi}} a^\dagger_{k\boldsymbol{\Pi}}\right)\left[\Phi^*_{\boldsymbol{\Pi}}(\hat{\boldsymbol{k}}) \cdot \Upsilon_z \cdot \Phi_{\boldsymbol{\Pi}}(\hat{\boldsymbol{k}})\right]$$

$$\Upsilon_{z,\, nm} = -i\hbar\delta_{nm}(\boldsymbol{k} \times \boldsymbol{\nabla}_{\boldsymbol{k}})_z - i\hbar\epsilon_{znm}$$

Angular Momentum of Quantum Vacuum



$$\langle \mathbf{0}, N = 0, m | J_z(kin) | \mathbf{0}, N = 0, m \rangle =$$

$$\hbar \frac{3}{\pi} \frac{\hbar \omega_c}{m c_0^2} \mathrm{Log}\left(\frac{2 m c_0^2}{\hbar \omega_c} \right) + \hbar \frac{2}{5\pi} \frac{\hbar \omega_c}{m c_0^2}$$