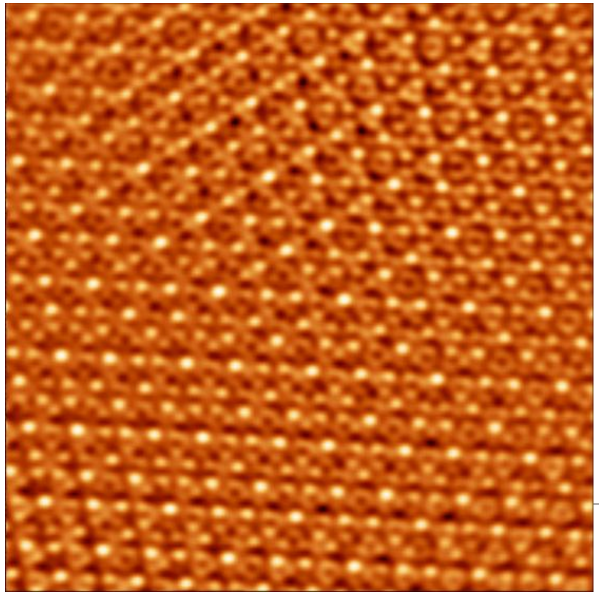
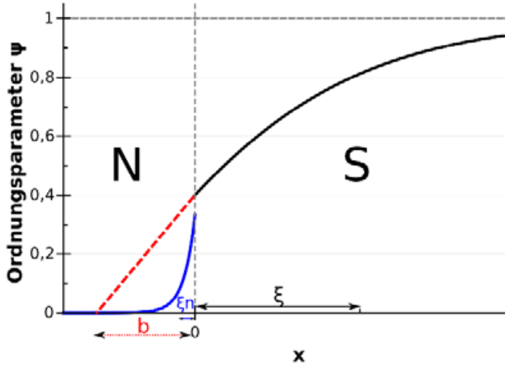
Proximity effect at a superconductor-quasicrystal interface



Pb adatoms on 5-fold AlPdMn (250x250 nm)

ANURADHA JAGANNATHAN
LABORATOIRE DE PHYSIQUE DES SOLIDES
UNIVERSITÉ PARIS-SUD - UNIVERSITY PARIS-SACLAY
&
GAUTAM RAI AND STEPHAN HAAS
UNIVERSITY OF SOUTHERN CALIFORNIA, LOS ANGELES

Why study the proximity effect ?



Consider a junction between a normal metal and a superconductor

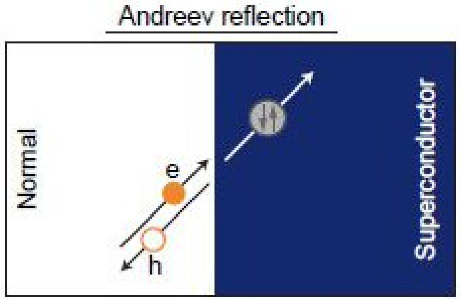
- the Cooper pairs penetrate into the normal metal over a distance ξ_n (~ $\sqrt{D/T}$:D diffusion coefficient)
- at low enough T, ξ_n can become very large → **bulk superconducting** properties of the normal metal

Recently the proximity effect used as a probe of

-- interaction effects

-- Majorana states

-- spintronics systems



-- and now in quasicrystals to probe

* Topological features

* spatial correlations

Effective dimensionality

Edge modes, chirality, Chern numbers

How do critical states change the proximity effect ?

What are the effects of edge modes ? Manifestations of topological properties ?

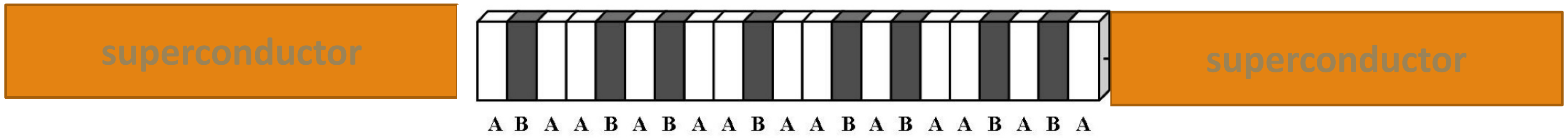
Plan of talk

I. Electronic states in quasicrystals

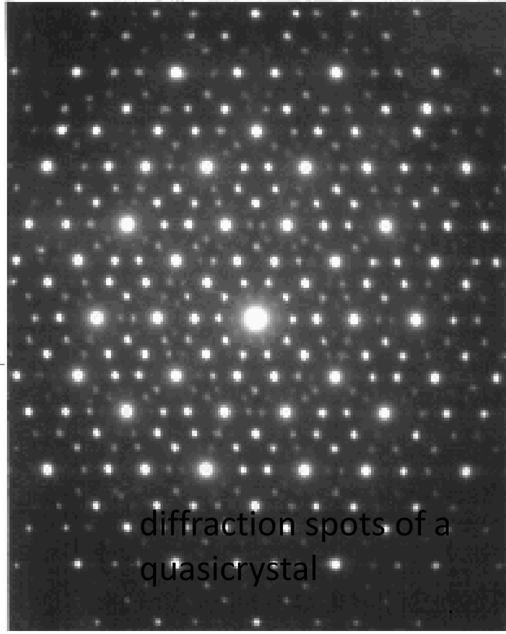
II. Model for hybrid superconductor-quasicrystal ring

III. Results

IV. Conclusions and Perspectives



Revised definition of crystals (IUCr)

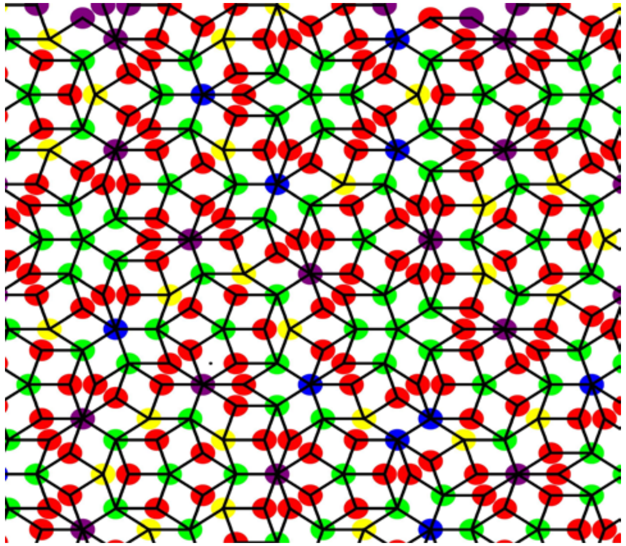


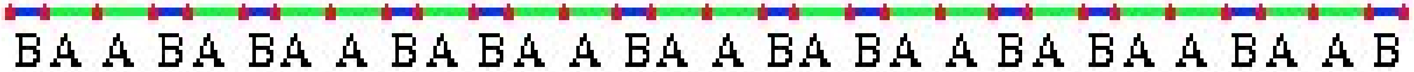
A **crystal** is a structure with sharp diffraction (**Bragg**) peaks on the nodes of a lattice.

A **periodic** d-dimensional crystal has a reciprocal lattice of d dimensions.
The structure is invariant under translations of the direct lattice.

A **quasiperiodic** crystal in d dimensions has a reciprocal lattice of dimension **D>d**.
The structure is **not invariant** under translations.

NOT translational invariant
However, identical regions – of arbitrarily large size -- occur with finite density (Conway's theorem)





The Fibonacci chain – a 1D quasicrystal

Finite chains can be built up by concatenation

B
A
AB
ABA
ABAAB
ABAABABA
ABAABABAABAAB
...

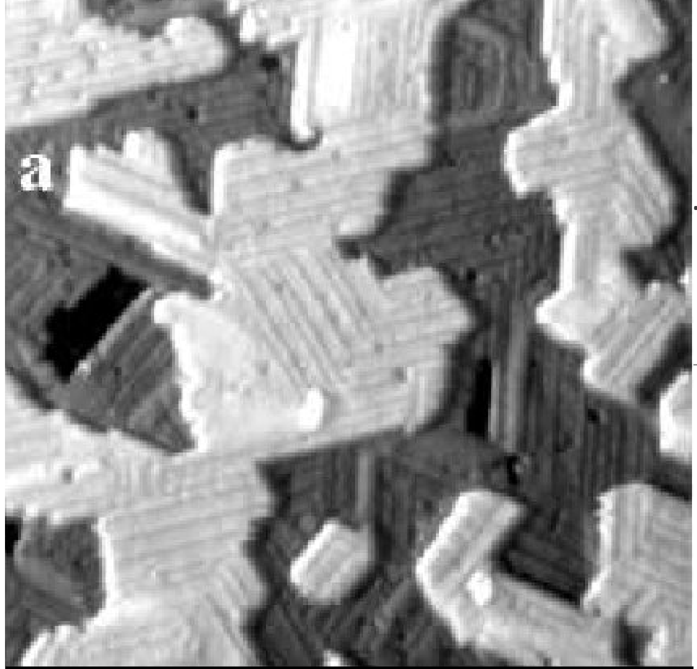
The Fibonacci numbers

$F_{n+1} = F_n + F_{n-1}$

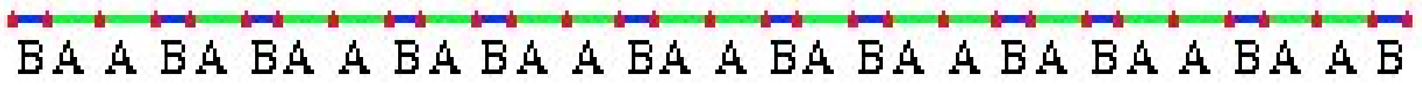
0, 1, 1, 2, 3, 5, 8, 13,…

$F_n / F_{n-1} \rightarrow \tau = (\surd 5+1)/2$

The chain can also be obtained from a 2D square lattice by cutting off a strip inclined at an angle $1/\tau$ (B:vertical bond, A: horizontal bond)



The Fibonacci chain – a 1D quasicrystal



Finite chains built up by concatenation

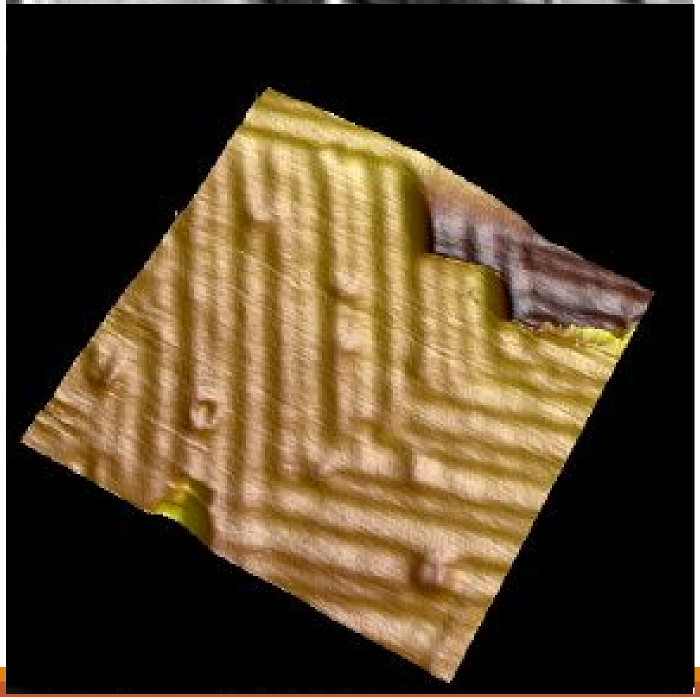
B
A
AB
ABA
ABAAB
ABAABABA
ABAABABAABAAB
...

Fibonacci numbers

$F_{n+1} = F_n + F_{n-1}$

0, 1, 1, 2, 3, 5, 8, 13,…

$F_n / F_{n-1} \rightarrow \tau = (\sqrt{5}+1)/2$



Cu on AlPdMn, Ledieu et al,2003

Fibonacci chain tight binding models

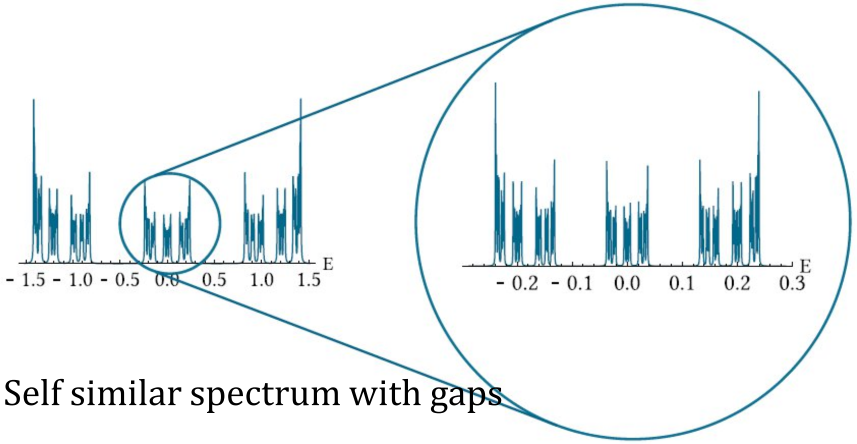
$$H = \sum_i |i\rangle \varepsilon_i \langle i| + \sum_{i,j} |i\rangle t_{ij} \langle j|$$

Studied since the early 80s (Ostlund&Pandit, Kohmoto,Kadanoff & Tang, Kalugin, Kitaev & Levitov, Niu & Nori, Bellissard, ..)

Typical features

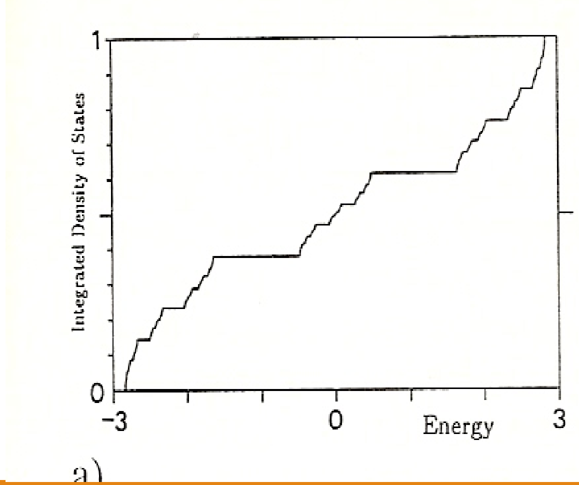
→ critical states *(see Macé et al, PRB **93** 205153 (2015))*
→ singular continuous (Cantor set) spectrum,
→ anomalous diffusion of wave packets

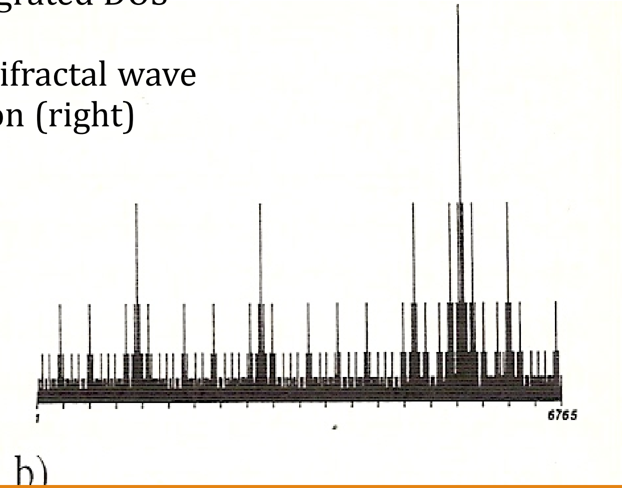
an important exact analytical result : the gap labelling theorem



Self similar spectrum with gaps

Cantor set structure of integrated DOS (left)
A multifractal wave function (right)

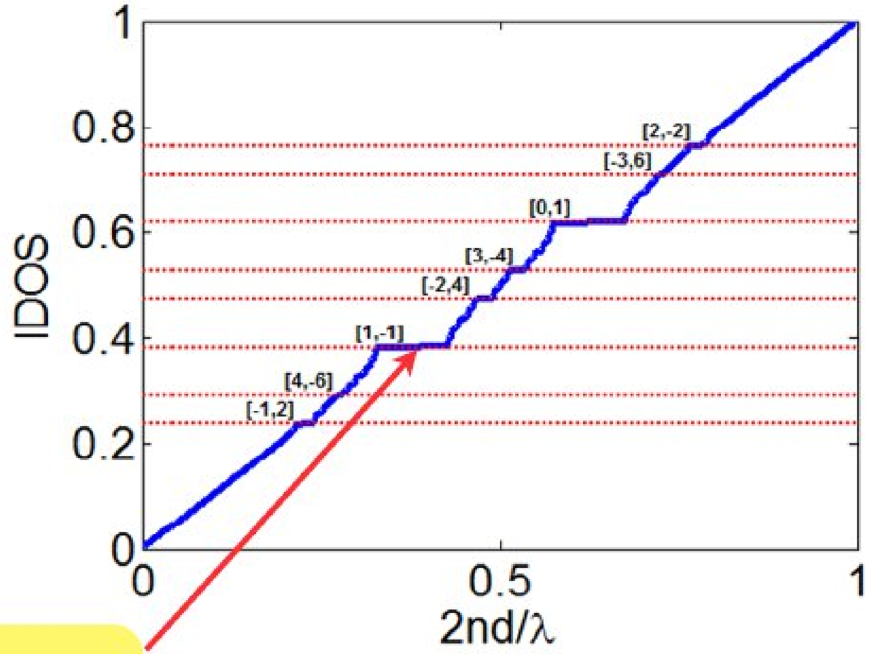




Integrated Density of States-Gap Labeling

$$\tau = \frac{\left(1+\sqrt{5}\right)}{2}$$

golden mean



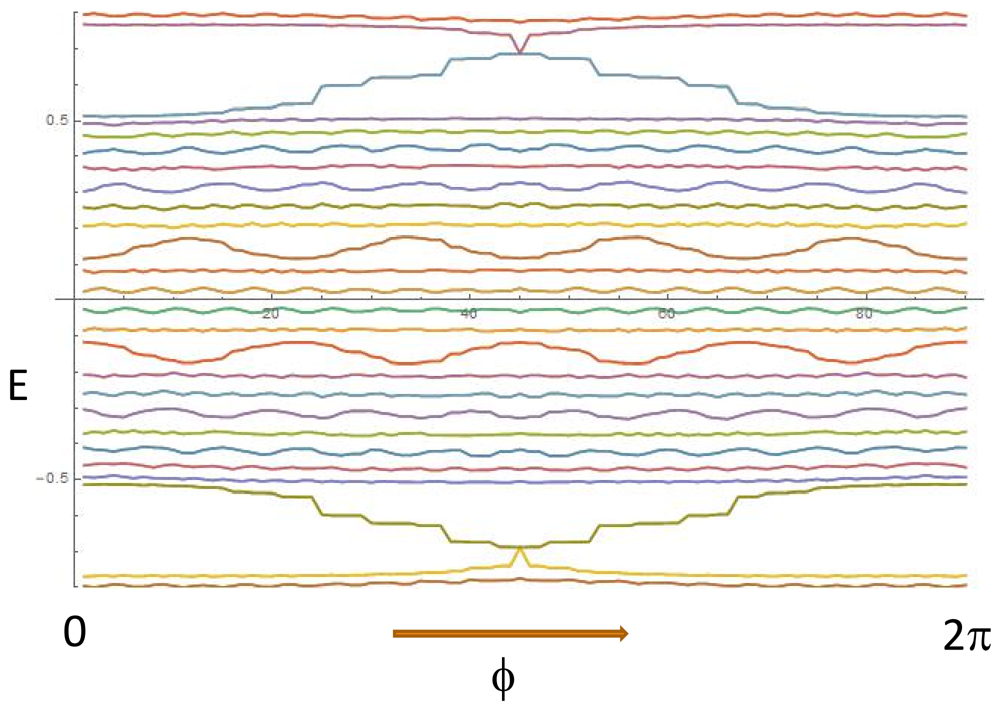
$$N\left(\omega_{gap}\right) = p + q\tau^{-1}$$

within a $[p,q]$ gap

(From E.Akkermans, cours de College de France 2015)

The ϕ parameter

Introduced in the spirit of Aubry-Andre model: V(na)= W cos(2$\pi\tau$n + ϕ)



Energy bands as a function of ϕ

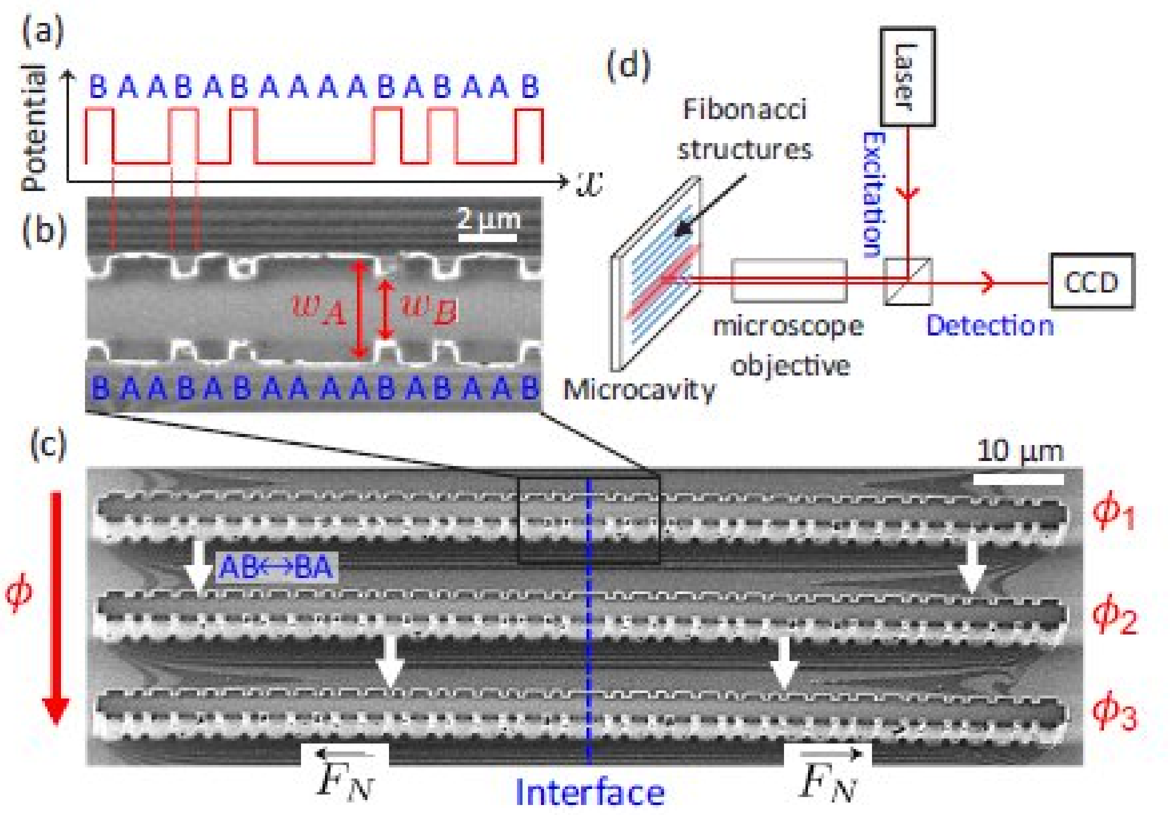
Fibonacci chains can be generated in yet another way by changing the angle ϕ in

$$\chi_n(\Phi) = \mathrm{sign}\left[\cos\left(\frac{2\pi n}{\tau} + \Phi + \Phi_0\right) - \cos\left(\frac{\pi}{\tau}\right)\right]$$

Example: 8 site chain

A	B	A	A	B	A	B	A
A	B	A	A	B	A	A	B
A	A	B	A	B	A	A	B
A	A	B	A	A	B	A	B
B	A	B	A	A	B	A	B
B	A	B	A	A	B	A	A
B	A	A	B	A	B	A	A
B	A	A	B	A	A	B	A

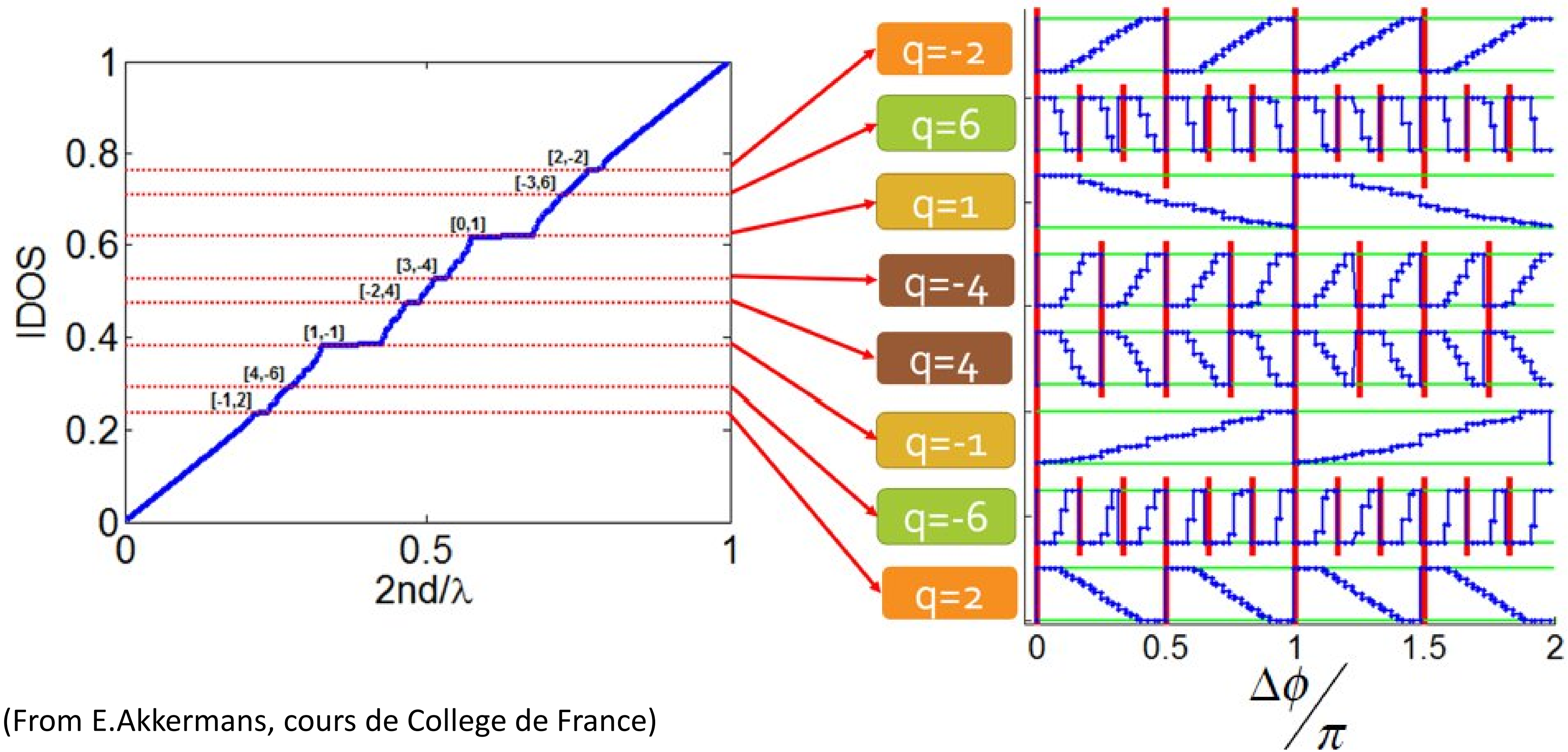
edge modes in a Fibonacci polaritonic crystal



Measuring topological invariants from generalized edge states in polaritonic quasicrystals
(Tanese et al PRL 2014, F.Baboux et al, PRB 2017)

The first experimental look at winding numbers of edge modes of a QC

Relation to the gap labelling and Chern numbers

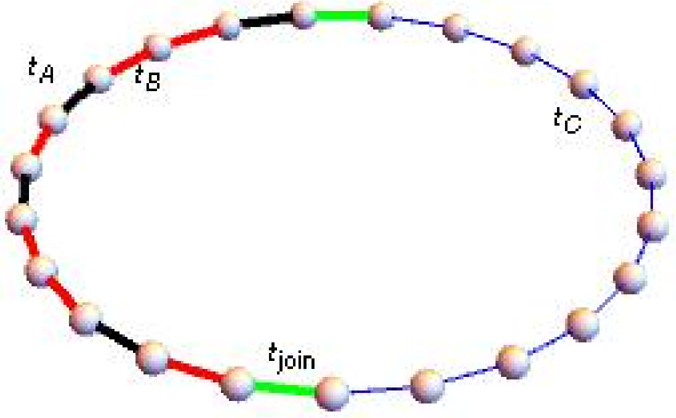


(From E.Akkermans, cours de College de France)

The model - superconducting chain coupled to a Fibonacci QC

Tight-binding Hamiltonian considered :

$$\mathcal{H} = \mathcal{H}_C + \mathcal{H}_{QC} + \mathcal{H}_{join}$$



Hamiltonian in superconducting region:

$$\mathcal{H}_C = \sum_{\substack{i=1 \\ \sigma=\pm 1}}^{N'} t_C (a^\dagger_{i+1,\sigma} a_{i,\sigma} + h.c.) + \Delta \sum_i (a_{i+1,\uparrow} a_{i,\downarrow} + h.c.)$$

Hamiltonian in quasiperiodic non-superconducting region:

$$\mathcal{H}_{QC} = \sum_{\substack{i=1 \\ \sigma=\pm 1}}^{N} t_i (c^\dagger_{i+1,\sigma} c_{i,\sigma} + h.c.)$$

Values of t_i : t_A or t_B

Coupling terms (for periodic boundary conditions):

$$\mathcal{H}_{join} = t_{join} \sum_\sigma (a^\dagger_{1,\sigma} c_{N,\sigma} + a^\dagger_{N',\sigma} c_{1,\sigma} + h.c.)$$

Self consistent solution
OP in the N segment

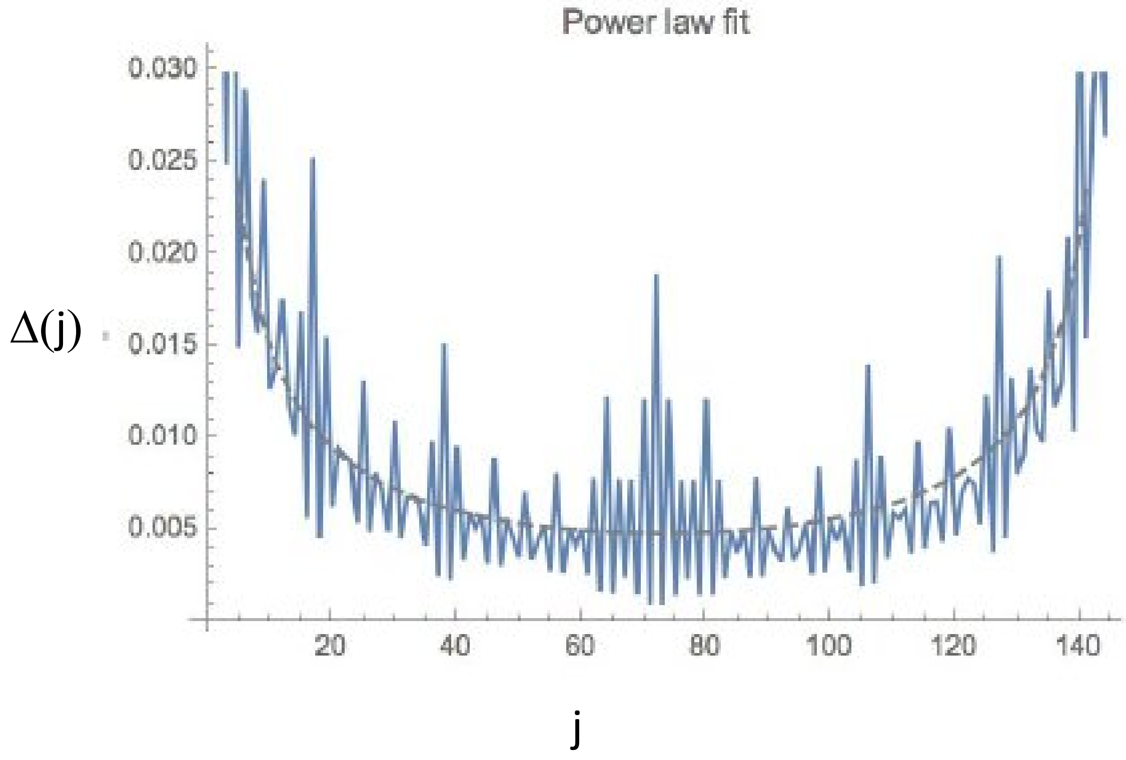
$$\Delta(\mathbf{r}_i) = -|U|\langle c_{i\downarrow} c_{i\uparrow} \rangle,$$

1. Δ(i) has large fluctuations (critical states)

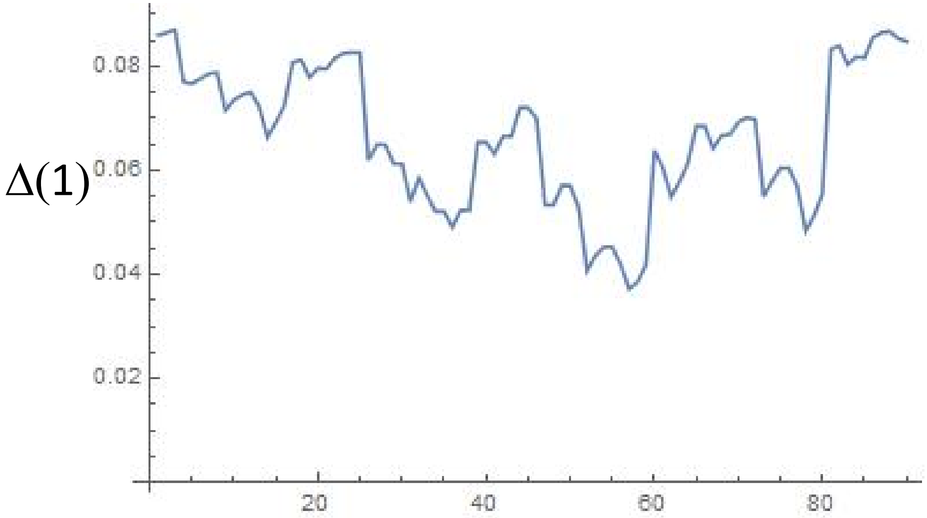
2. On average, the OP decays as a power law away from the interface, $\Delta(i) = i^{-\delta}$

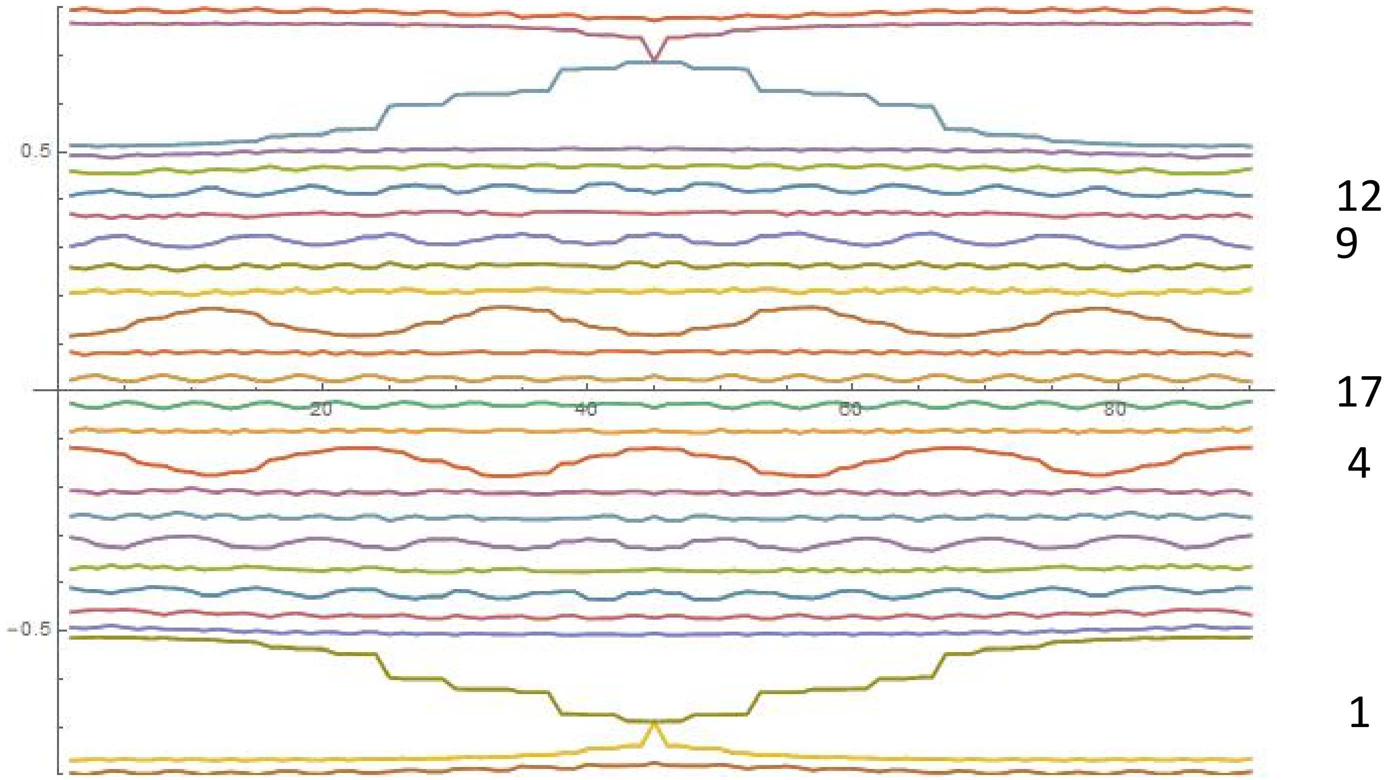
3. The nonuniversal power depends on tA/tB and on ϕ parameter

Out[445]= {a → 0.595794, b → 0.0678266, c → -0.00565879}



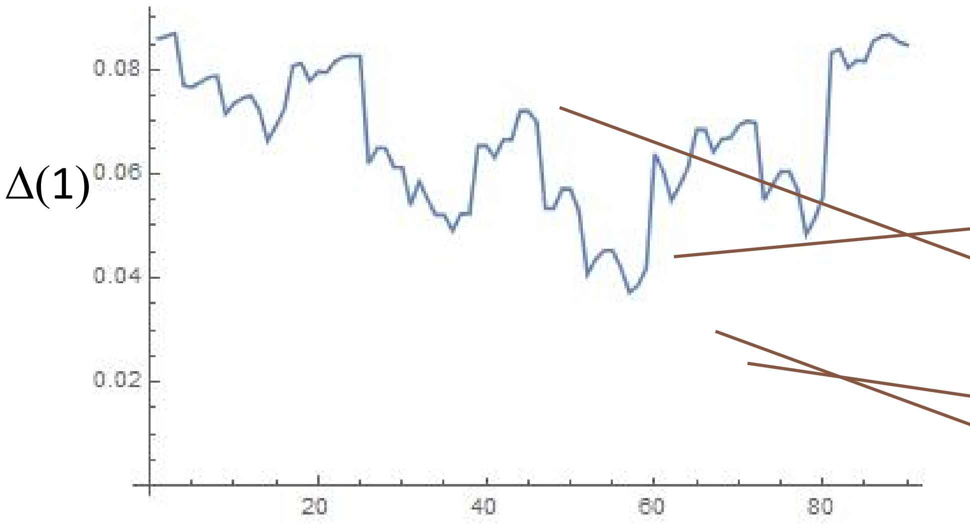
Pair amplitude on left edge

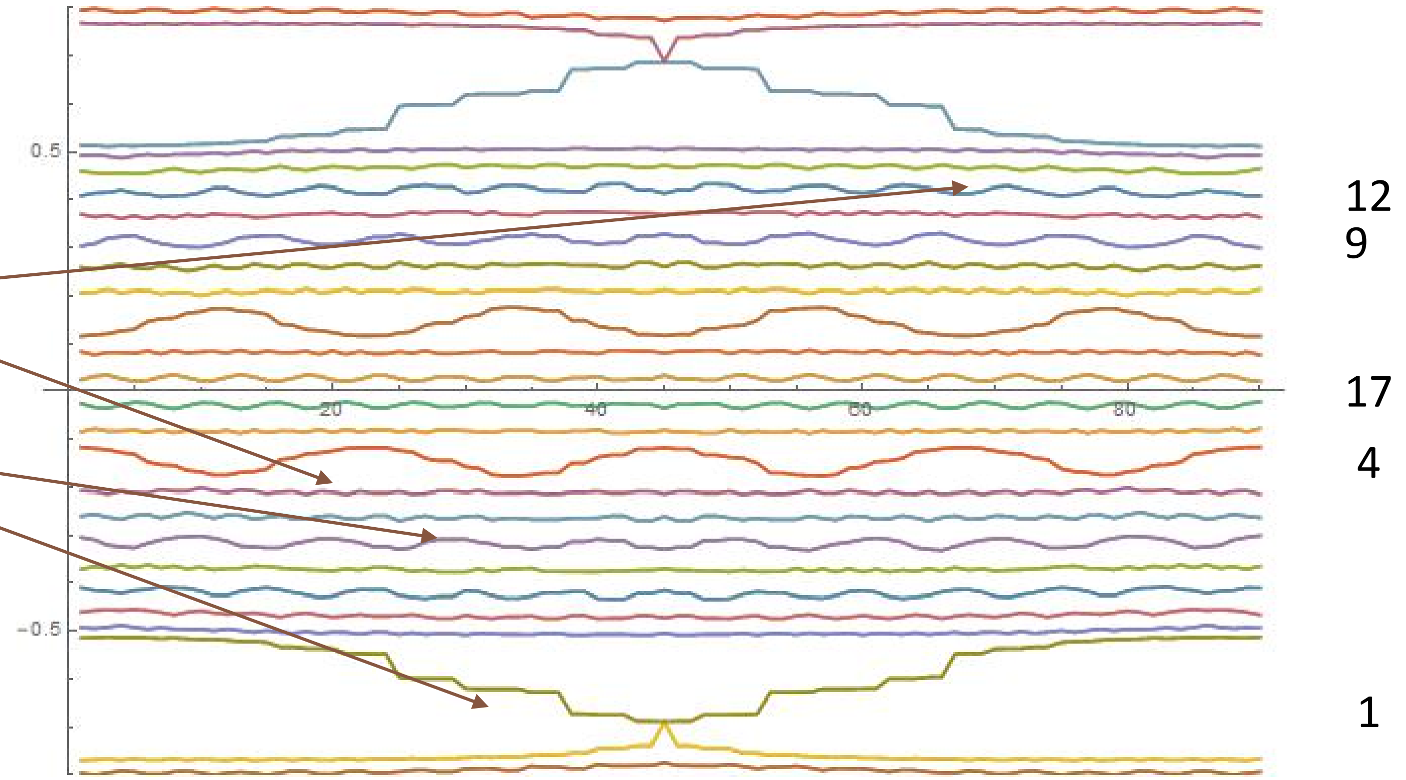




The contributions of individual edge states shows up as q-periodic oscillations!

Pair amplitude on left edge





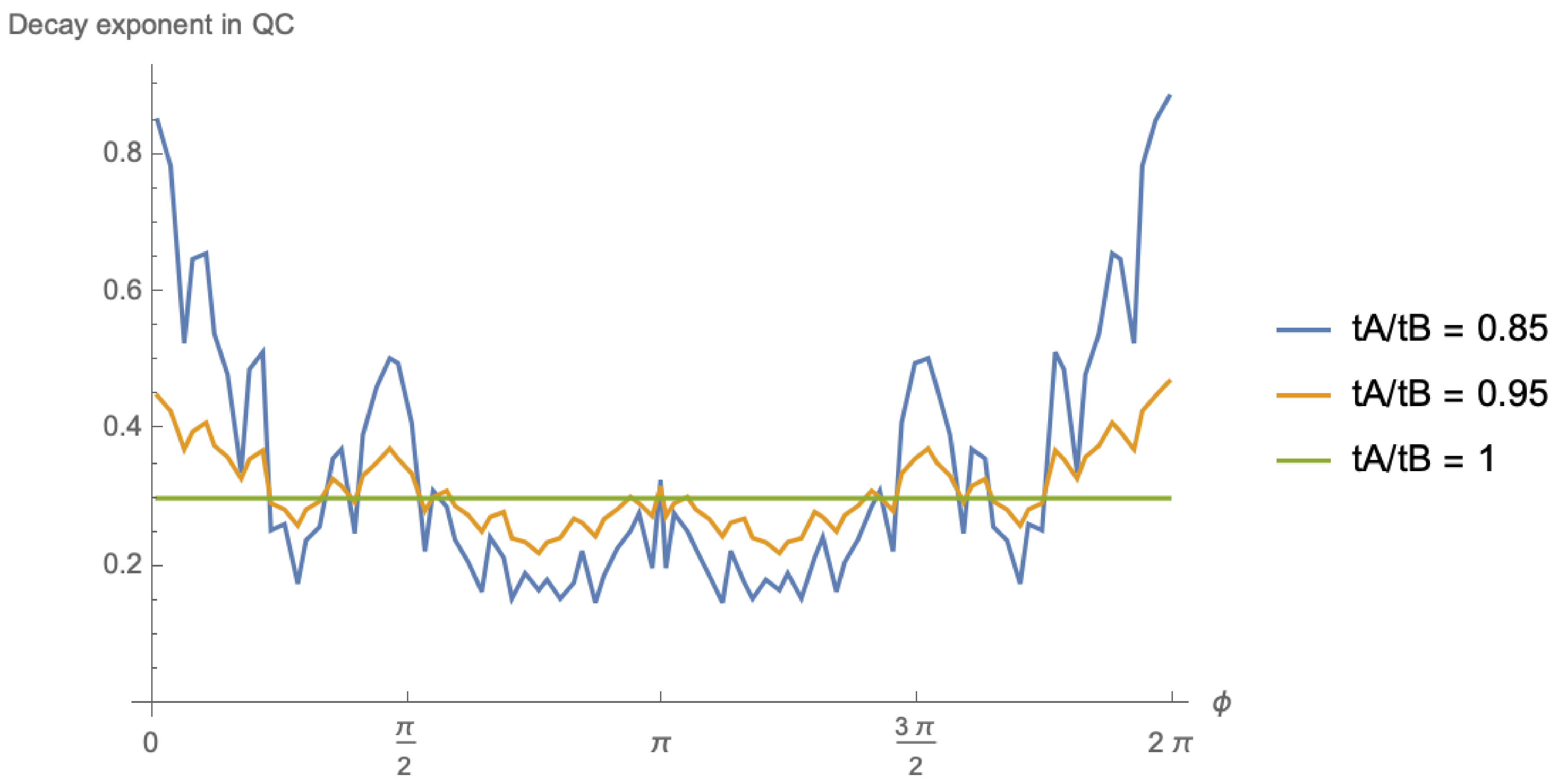
The contributions of individual edge states shows up as q-periodic oscillations!

We also see this q-periodicity in the **power law decay** of the superconducting pair correlations

$\Delta(n) = n^{-\delta}$

The decay power δ

i) *Depends on tA/tB*
ii) *Depends on f*
iii) *gets smaller as the chains approach periodic limit*



Conclusions and perspectives

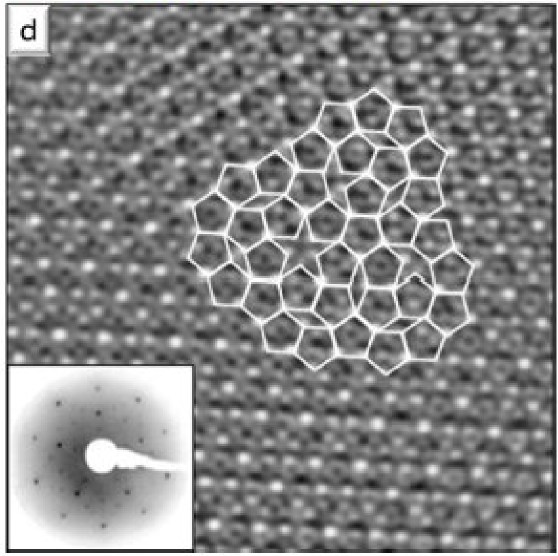
Superconductivity can be strongly enhanced in a QC compared to periodic crystals

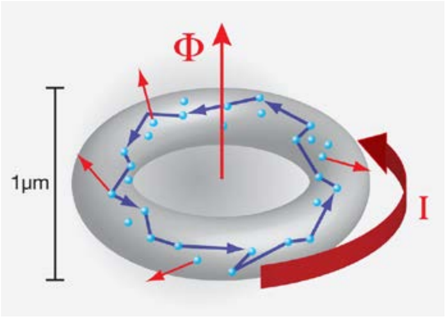
Proximity effect shows strong influence of edge modes. New way to visualize topological characteristics

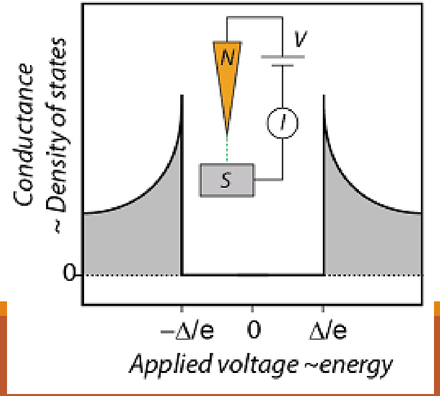
To probe chiral properties: include a magnetic flux

Experimental set-up: for adatoms on a substrate
select regions of quasiperiodic film, lay down Pb (or other superconductor) contacts
measure the induced SC gap for different sequences

Other possibilities: artificial crystals -- polaritonic ? cold atoms ?







Conclusions and perspectives

Superconductivity can be strongly enhanced in a QC compared to periodic crystals

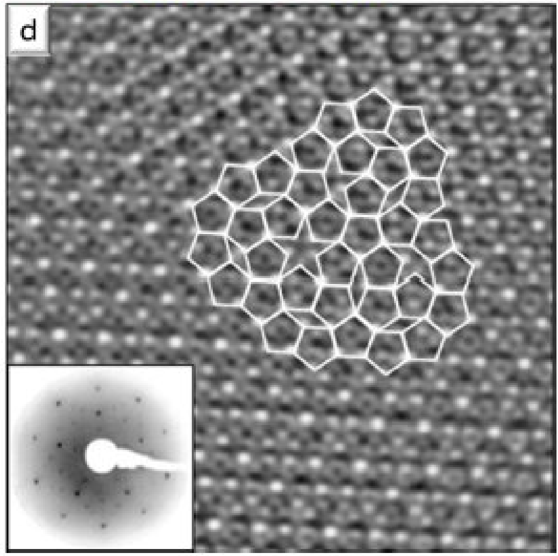
Proximity effect shows strong influence of edge modes. New way to visualize topological characteristics

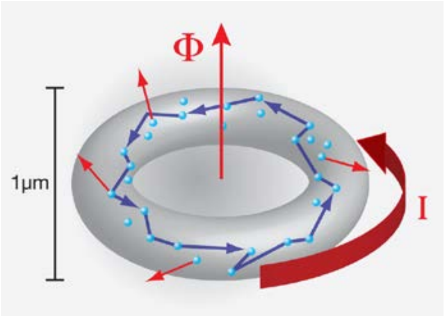
To probe chiral properties: include a magnetic flux

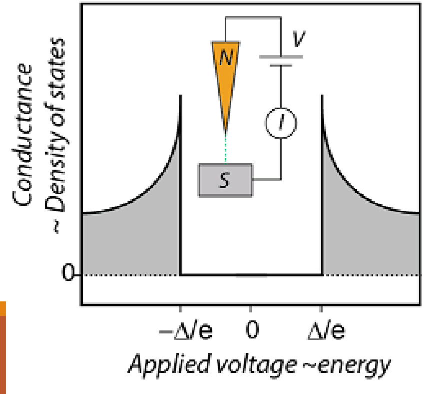
Experimental set-up: for adatoms on a substrate
select regions of quasiperiodic film, lay down Pb (or other superconductor) contacts
measure the induced SC gap for different sequences

Other possibilities: artificial crystals -- polaritonic ? cold atoms ?

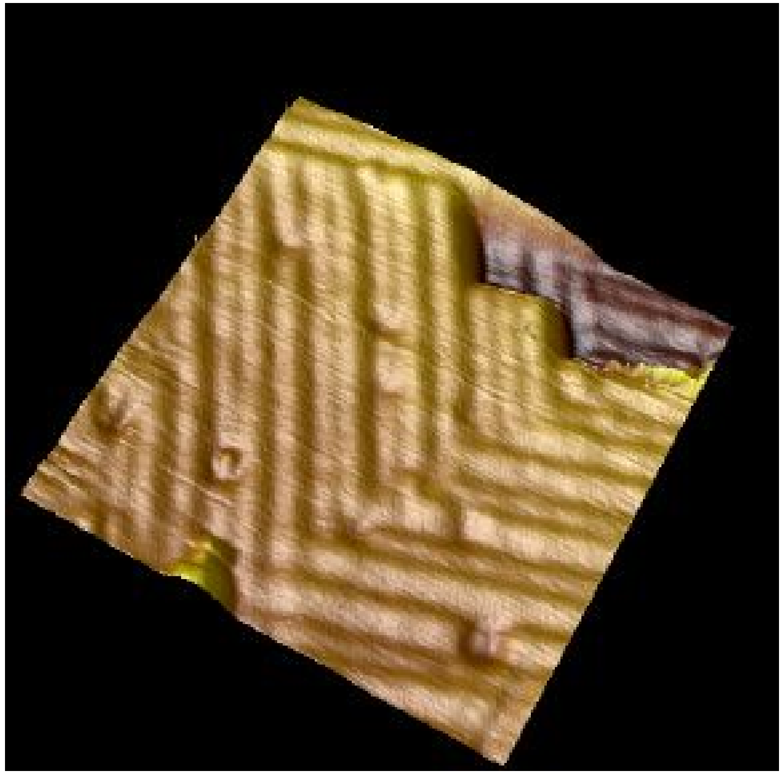
THANK YOU FOR YOUR ATTENTION!



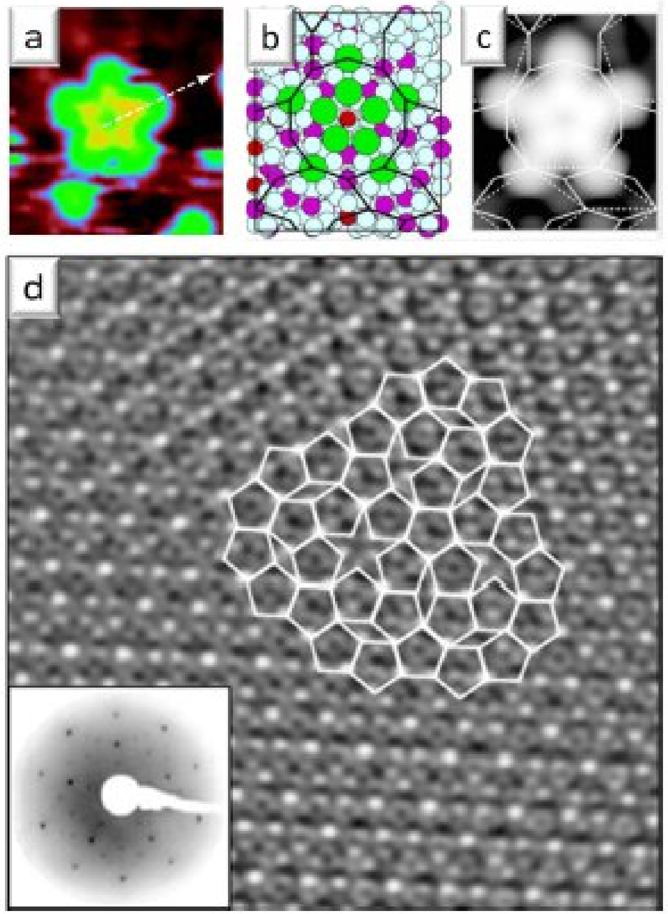




Electron states in quasicrystals



J. Ledieu, V. Fournée / C. R. Physique 15 (2014) 48–57



Spectrum and gap labels of L=89 Fibonacci chain